Course File
2022-23

STRUCTURAL DYNAMICS

(GR22D5013)

M.Tech. (Structural Enginerring)

I Year -II Semester

Instructor: Dr. V Srinivasa Reddy

Department of Civil Engineering

GOKARAJU RANGARAJU
Institute of Engineering and Technology

Vision and Mission

Gokaraju Rangaraju Institute of Engineering and Technology (GRIET) is established in 1997 by Dr. G Gangaraju as a self-financed institute under the aegis of Gokaraju Rangaraju Educational Society. GRIET is approved by AICTE, New Delhi, permanently affiliated to and autonomous under JNTUH, Hyderabad. GRIET is committed to quality education and is known for its innovative teaching practices.

Vision

To be among the best of the institutions for engineers and technologists with attitudes, skills and knowledge and to become an epicentre of creative solutions.

Mission

To achieve and impart quality education with an emphasis on practical skills and social relevance.

Department of Civil Engineering

Vision

To become a pioneering centre in Civil Engineering and technology with attitudes, skills and knowledge.

Mission

- To produce well qualified and talented engineers by imparting quality education.
- To enhance the skills of entrepreneurship, innovativeness, management and lifelong learning in young engineers.
- To inculcate professional ethics and make socially responsible engineers.

M.Tech PEOs and POs

M.Tech Programme Educational Objectives (PEOs)

PEO 1: Graduates of the program will equip with professional expertise on the theories, process, methods and techniques for building high-quality structures in a cost-effective manner.

PEO 2: Graduates of the program will be able to design structural components using contemporary softwares and professional tools with quality practices of international standards.

PEO 3: Graduates of the program will be effective as both an individual contributor and a member of a development team with professional, ethical and social responsibilities.

PEO 4: Graduates of the program will grow professionally through continuing education, training, research, and adapting to the rapidly changing technological trends globally in structural engineering.

M.Tech Programme Outcomes (POs)

Graduates of the Civil Engineering program will be able to:

PO 1: An ability to independently carry out research / investigation and development to solve practical problems.

PO2: An ability to write and present a substantial technical report / document.

PO 3: Students should be able to demonstrate a degree of mastery over the area as per the specialization of the program. The mastery should be at a level higher than the requirements in the appropriate bachelor's.

PO 4: Possesses critical thinking skills and solves core, complex and multidisciplinary structural engineering problems.

PO 5: Assess the impact of professional engineering solutions in an environmental context along with societal, health, safety, legal, ethical and cultural issues and the need for sustainable development.

PO 6: Recognize the need for life-long learning to improve knowledge and competence.

<u>COURSE FILE Enclosures</u>

The following are to be filed in each Course File:

1. Get a new file from college store for each course and file each sheet of these formats as and when it is completed.
2. Time Table
3. Syllabus copy for your course.
4. Course Plan
5. Unit Plan and
6. Lesson Plan
7. List of Program Objectives & Outcomes;
8. Course Objectives & Outcomes
9. List of various Mappings/Matrix for your Course
   a. Mapping between Course Objectives and Course Outcomes
   b. Mapping between Course Objectives and Program Outcomes(POs)
   c. Mapping between Course Outcomes and Mandatory/Program Outcomes(POs)(a–k)
   d. Mapping between Courses with titles & codes and Mandatory/Program Outcomes(POs)(a – k)
   e. Mapping between the PEOs and Course Outcomes
   f. Mapping between POs and Assignments and Assessments Methods
   g. Mapping between the Assessment Methods and PEOs
10. List of Assessments, Assignments/Seminar Topics, Projects, Experiments, etc. you have given to students and the Criteria used for evaluation
11. Assignment sheets,
12. Tutorial Sheets, and
13. Course Schedules
14. At least 1 to 3 Assessment Rubrics for your course
15. Evaluation Strategy
16. Guidelines to study the course
17. Students Roll list
18. Attach the Marks list of the students in respect of CAE -I (Continuous Assessment Exam), CAE-II, etc. and Final Exam for this Course in your course File.
19. Photocopy of the best, average and the worst answer sheets for CAE-I, & CAE-II be included in the Course File.
20. Model question papers if any, which you have distributed to the students in the beginning of the Semester for the Course may be included in the Course File.
21. Any Teaching/Learning Aids, additional resources like OHP transparencies, LCD Projection material, Soft & Hard Copies of handouts used may also be filed in it.
22. Course Completion Status
23. Grading Sheet of the Course for all students

Assessment Procedure

S. No	Component of Assessment	Marks Allotted	Type of Assessment	Scheme of Examinations
1	Theory	40	Internal Examination & Continuous Evaluation	1) Two mid semester examination shall be conducted for 30 marks each for a duration of 120 minutes. Average of the two mid exams shall be considered **i) Subjective – 20 marks** **ii) Objective – 10 marks** 2) Continuous Evaluation is by conducting Assignments and Quiz exams at the end of each unit **i) Assignment – 5 marks** **ii) Quiz/Subject Viva-voce/PPT/Poster Presentation/ Case Study on a topic in the concerned subject – 5 marks**
		60	Semester end examination	The semester-end examination is for a duration of 3 hours

I YEAR - II SEMESTER

Sl. No	Group	Course Code	Subject	Credits L	T	P	Total Credits	Hours L	T	P	Total Hours	Int. Marks	Ext. Marks	Total Marks
1	PC	GR22D5012	FEM in Structural engineering	3	0	0	3	3	0	0	3	40	60	100
2	PC	GR22D5013	Structural Dynamics	3	0	0	3	3	0	0	3	40	60	100
3	PE III	GR22D5014	1. Advanced Steel Design	3	0	0	3	3	0	0	3	40	60	100
		GR22D5015	2. Design of Formwork											
		GR22D5016	3. Principles of Bridge Engineering											
4	PE IV	GR22D5017	1. Design of Advanced Concrete Structures	3	0	0	3	3	0	0	3	40	60	100
		GR22D5018	2. Advanced Design of Foundations											
		GR22D5019	3. Earthquake Resistant Design of Buildings											
5	PC	GR22D5020	Advanced Structural Engineering Lab	0	0	2	2	0	0	4	4	40	60	100
6	PC	GR22D5021	Numerical Analysis Lab	0	0	2	2	0	0	4	4	40	60	100
7	PW	GR22D5144	Mini Project	0	0	2	2	0	0	4	4	40	60	100
			Total	**12**	**0**	**6**	**18**	**12**	**0**	**12**	**24**	**280**	**420**	**700**
8	AC		Audit Course II	0	0	0	0	2	0	0	2	40	60	100

S.No	No:	Student Name (As Per SSC)	Student Phone	Email	Phone
1	22241D2001	ADDAGATLA MAHESHKUMAR	9652205718	maheaddagatla@gmail.com	9652205718
2	22241D2002	AHMED ABDUL AZEEM	9553214459	abdulazeem17458@gmail.com	9948123715
3	22241D2003	BAIRAPAKA BHARATH	9010976868	Bairapakabharath7@gmail.com	9182443387
4	22241D2004	BARLAPUDI ACHSAH KEERTHAI	6302131589	achsahkeerthana.b@gmail.com	9553242425
5	22241D2005	CHAKALI SOWMYA	6300048204	Ch.sowmya.1311@gmail.com	9032366043
6	22241D2006	CHAPPIDI NARESH	9398916604	Chappidi.naresh88@gmail.com	9398993443
7	22241D2007	DANTHALA HARIDEEPKUMAR	6303321256	harideepdanthala@gmail.com	9618714550
8	22241D2008	DEVIREDDY ANISH	6309845262	anishdevireddy07@gmail.com	8179118516
9	22241D2009	DHARAVATH NAGENDAR	7673952028	nagendar.d99@gmail.com	8919995124
10	22241D2010	GANGAPURAM SUSHANTH REDI	9502059919	shushanthshush@gmail.com	9440054520
11	22241D2011	JEREPOTHULA RAVALIKA	9676681445	ravalikajerepothula@gmail.com	9346496095
12	22241D2012	KADABOHINA SAIPAVAN	9030300863	kadabohinasaipavan4536@gmail.com	9966358815
13	22241D2013	KASUMURU BHARATH KUMAR	9494066112	Bharathkumarkasumuru@gmail.com	9105222000
14	22241D2014	MACHARLA SRINIVAS	9959766792	macharlasrinivas111@gmail.com	9959766792
15	22241D2015	MALLI SREENIVASULU	6309432349	mallisreenu145@gmail.com	7075081569
16	22241D2016	SHAIK ABDUL MUQEED	7569656490	abdulmuqeed321@gmail.com	9515323031
17	22241D2017	SHAIK ZABI ULLAH	9640330682	shaikzabiullah2000@gmail.com	9849493634
18	22241D2018	SONWANE SAHIL SHIVAJIRAO	8328109850	sahilsss29@gmail.com	9440783546
19	22241D2019	LINGAM LAKSHMI NARAYANA	9392138942	lingam_ln@yahoo.com	9392138942

Gokaraju Rangaraju Institute of Engineering & Technology

Bachupally, Hyderabad-500090

M.Tech Structural Engg. I Yr-I Sem- GR20 2021 -22

S.No	Reg No	Student Name
1	21241D2001	ATKAPURAM PRASHANTH
2	21241D2002	BANDI SRI RAM GOPAL
3	21241D2003	CHALLA MADHAVI
4	21241D2004	PAMMI DIVYA
5	21241D2005	DUMMA UMESH KUMAR
6	21241D2006	K LATHASREE
7	21241D2007	MARIYALA VAISHNAVI
8	21241D2008	MAVOORI PRANAV
9	21241D2009	MITTAPALLI NAGA ASHWINI
10	21241D2010	R VENKATA SURAJ REDDY
11	21241D2011	REPATI MOHAN BABU
12	21241D2012	SANDHYA CHERUKU
13	21241D2013	SHAIK FEROZ
14	21241D2014	SK SAI CHANDRA
15	21241D2015	THOTA HARSHAVARDHAN
16	21241D2016	VARIKUPPALA LALITHA
17	21241D2017	Y RAMA GNANENDRA SAI
18	21241D2018	YENUMALA DEVESH GOUD
19	21241D2019	S PRASHANTH KUMAR
20	21241D2020	B THARUN TEJA
21	21241D2021	G NITISH KUMAR

22241D2001	mahesh22241d2001@grietcollege.com
22241D2002	abdul22241d2002@grietcollege.com
22241D2003	bharat22241d2003@grietcollege.com
22241D2004	keerthana22241d2004@grietcollege.com
22241D2005	sowmya22241d2005@grietcollege.com
22241D2006	naresh22241d2006@grietcollege.com
22241D2007	harideep22241d2007@grietcollege.com
22241D2008	anish22241d2008@grietcollege.com
22241D2009	nagendar22241d2009@grietcollege.com
22241D2010	sushanthreddy22241d2010@grietcollege.com
22241D2011	ravalika22241d2011@grietcollege.com
22241D2012	saipavan22241d2012@grietcollege.com
22241D2013	bharat22241d2013@grietcollege.com
22241D2014	srinivas22241d2014@grietcollege.com
22241D2015	sreenivasulu22241d2015@grietcollege.com
22241D2016	abdul22241d2016@grietcollege.com
22241D2017	zabi22241d2017@grietcollege.com
22241D2018	shivaji22241d2018@grietcollege.com
22241D2019	narayana22241d2019@grietcollege.com

	M.TECH. CIVIL (STE) 2022 Admitted		
	ROLL NO.	**STUDENT NAME**	**JOINING DATE**
1.	22241D2001	ADDAGATLA MAHESHKUMAR scholarship	26-10-2022
2.	22241D2002	AHMED ABDUL AZEEM	26-10-2022
3.	22241D2003	BAIRAPAKA BHARATH scholarship	19-11-2022
4.	22241D2004	BARLAPUDI ACHSAHKEERTHANA	26-10-2022
5.	22241D2005	CHAKALI SOWMYA scholarship	26-10-2022
6.	22241D2006	CHAPPIDI NARESH scholarship	03-11-2022
7.	22241D2007	DANTHALA HARIDEEPKUMAR scholarship	03-11-2022
8.	22241D2008	DEVIREDDY ANISH scholarship	26-10-2022
9.	22241D2009	DHARAVATH NAGENDAR scholarship	19-11-2022
10.	22241D2010	GANGAPURAM SUSHANTH REDDY*	26-10-2022
11.	22241D2011	JEREPOTHULA RAVALIKA scholarship	03-11-2022
12.	22241D2012	KADABOHINA SAIPAVAN scholarship	03-11-2022
13.	22241D2013	KASUMURU BHARATH KUMAR*	26-10-2022
14.	22241D2014	MACHARLA SRINIVAS	03-11-2022
15.	22241D2015	MALLI SREENIVASULU*	26-10-2022
16.	22241D2016	SHAIK ABDUL MUQEED scholarship	03-11-2022
17.	22241D2017	SHAIK ZABI ULLAH scholarship	26-10-2022
18.	22241D2018	SONWANE SAHILSHIVAJIRAO	03-11-2022
19	22241D2019	LINGAM LAKSHMI NARAYANA*	26-10-2022

*Management Quota
REDMARKED STUDENTS HAS ATTENDANCE BETWEEN 65 to 75%

Classes commenced from: 26-10-2022
Counselling Round 1: 12-10-2022 to 15-10-2022
Counselling Round 2: 31-10-2022 to 03-11-2022
Special Round: 15-11-2022 to 19-11-2022

DEPARTMENT OF CIVIL ENGINEERING (STRUCTURAL ENGINEERING)

I M. Tech (GR-22) - II Semester **AY: 2022-23** **wef : 03-04-2023**

Day/Hour	09:00-10:00	10:00-11:00	11:00-12:00	12:00-01:00	01:00-02:00	02:00-03:00	03:00-04:00	Room No.	
MONDAY	FEM	FEM	DM	Lunch	NA LAB			Theory/ Tutorial	4203
TUESDAY	DM	ERDB	ERDB		ASE LAB			Lab	NA Lab-4207
WEDNESDAY	DFW	DFW	SD		NA LAB				ASE Lab-4110
THURSDAY	DFW	ERDB	ERDB		ASE LAB			**M.Tech Co-ordinator**	
FRIDAY	SD	SD	DFW		Mini Projects			**Dr. V Srinivasa Reddy (1117)**	
SATURDAY	SD	FEM	FEM		Library/Sports				

Sub. Code	**Subjects**	**Faculty Name**
GR22D5012	FEM in Structural engineering	Dr. G V V Satyanarayana (842)
GR22D5013	Structural Dynamics	Dr. V Srinivasa Reddy (1117)
GR22D5015	Design of Formwork	Mrs.K.Hemalatha (1177)
GR22D5019	Earthquake Resistant Design of Buildings	Mr.V.Naresh Kumar Varma (1359)
GR22D5020	Advanced Structural Engineering Lab	Dr. G V V Satyanarayana (842)/Mr.PVVSSR Krishna (Mr.PVVSSRK-1562)
GR22D5021	Numerical Analysis Lab	Mr.C.Vivek Kumar(1500)/Dr. V Srinivasa Reddy (1117)
GR22D5143	Mini Project	Mr. Y. Kamala Raju (929)
GR22D5154	Disaster Management	Mr.T.Srikanth (1360)

Coordinator
Dr. V Srinivasa Reddy

Mr.Rathod Ravinder
Time Table Coordinator

Dr.C.Lavanya
HOD-CE

COURSE OBJECTIVES

Academic **Y**ear : 2022-23

Semester : II

Name of the Program: M.TECH. STRUCTURAL ENGINEERING

Course/Subject: STRUCTURAL DYNAMICS Course Code: GR22D5013

Name of the Faculty: DR. V SRINIVASA REDDY Dept.: CIVIL ENGINEERING

Designation: PROFESSOR.

On completion of this Subject/Course the student shall be able to:

S.No	**Objectives**
	1. To understand the importance of vibration analysis and modelling of dynamic systems 2. To analyze for dynamic response of Single Degree of Freedom System subjected to different types of loading. 3. To examine the dynamic response of Multiple Degree of Freedom System using lumped mass and distributed mass approach 4. To obtain the dynamic response of structures using numerical methods. 5. To illustrate the dynamic effects of Wind Loads, Moving Loads and Vibrations caused by Traffic, Blasting and Pile Driving

Signature of HOD Signature of faculty

Date: Date:

Gokaraju Rangaraju Institute of Engineering and Technology
(Autonomous)
Bachupally, Kukatpally, Hyderabad – 500 090. (040) 6686 4440

COURSE OUTCOMES

Academic **Y**ear : 2022-23

Semester : II

Name of the Program: M.TECH. STRUCTURAL ENGINEERING

Course/Subject: STRUCTURAL DYNAMICS Course Code: GR22D5013

Name of the Faculty: DR. V SRINIVASA REDDY Dept.: CIVIL ENGINEERING

Designation: PROFESSOR.

The expected outcomes of the Course/Subject are:

S.No	Outcomes
On successful completion of this course, it is expected that students should be able to-	
1. Comprehend and model the systems subjected to vibrations and dynamic loads	
2. Analyze and obtain dynamics response of single degree freedom system using fundamental Theory and equations of motion.	
3. Analyze and obtain dynamics response of Multi degree of freedom system idealized as lumped mass systems. Analyze and obtain dynamics response of Multi degree of freedom system idealized as distributed mass systems.	
4. Obtain dynamics response of systems using numerical methods.	
5. To explain the dynamic effects of Wind Loads, Moving Loads and Vibrations caused by Traffic, Blasting and Pile Driving.	

Signature of HOD

Date:

Signature of faculty

Date

Gokaraju Rangaraju Institute of Engineering and Technology
(Autonomous)
Bachupally, Kukatpally, Hyderabad – 500 090. (040) 6686 4440

COURSE OUTCOMES

Academic **Y**ear : 2017-18

Semester : I

Name of the Program: M.TECH. STRUCTURAL ENGINEERING

Course/Subject: THOERY OF ELASTICITY AND PLASTICITY Course Code: GR17D5152

Name of the Faculty: DR. V SRINIVASA REDDY Dept.: CIVIL ENGINEERING

Designation: PROFESSOR

The expected outcomes of the Course/Subject are:

S.No	Outcomes
After completion of this course students will be able to 1. Explain the basic concepts of stress-strain relations in theory of elasticity 2. Analyse and interpret stresses and strains in 2-D and 3-D problems of elasticity in Cartesian coordinate system. 3. Analyse and interpret stresses and strains in 2-D and 3-D problems of elasticity in polar coordinate system. 4. Apply general theorems to find solutions to problems of elasticity. 5. Find the solutions to torsional problems using principles of elasticity 6. Find the solutions to bending problems using soap film method 7. Explain various theories of failures in plasticity.	

Signature of HOD Signature of faculty

Date: Date

Gokaraju Rangaraju Institute of Engineering and Technology
(Autonomous)
Bachupally, Kukatpally, Hyderabad – 500 090, India

GRIET/DAA/1H/G/22-23 25 Oct 2022

Academic Calendar
Academic Year 2022-23

I M.Tech – First Semester

S. No.	EVENT	PERIOD	DURATION
1	Orientation Programme	26-10-2022	
2	I Spell of Instructions	26-10-2022 to 22-12-2022	8 Weeks
3	I Mid-term Examinations	23-12-2022 to 29-12-2022	1 Week
4	II Spell of Instructions	30-12-2022 to 28-02-2023	9 Weeks
5	II Mid-term Examinations	01-03-2023 to 07-03-2023	1 Week
6	Preparation / Break	08-03-2023 to 14-03-2023	1 Week
7	End Semester Examinations	15-03-2023 to 01-04-2023	3 Weeks
8	Commencement of Second Semester, AY 2022-23	03-04-2023	

I M. Tech – Second Semester

S. No.	EVENT	PERIOD	DURATION
1	Commencement of Second Semester class work	03-04-2023	
2	I Spell of Instructions	03-04-2023 to 29-04-2023	4 Weeks
3	Summer Vacation	01-05-2023 to 13-05-2023	2 Weeks
4	I Spell of Instructions Contd..	15-05-2023 to 17-06-2022	5 Weeks
5	I Mid-term Examinations	19-06-2023 to 24-06-2023	1 Week
6	II Spell of Instructions	26-06-2023 to 26-08-2023	9 Weeks
7	II Mid-term Examinations	28-08-2023 to 02-09-2023	1 Week
8	Preparation / Break	04-09-2023 to 09-09-2023	1 Week
9	End Semester Examinations	11-09-2023 to 25-09-2023	2 Weeks
10	Commencement of Second Year, First Semester, AY 2023-24	26-09-2023	

J. Praveen

Dean Academic Affairs

Copy to Principal, All HoDs, CoE

COURSE OBJECTIVES AND OUTCOMES

Course Code: GR22D5013
I Year II Semester

Course Objectives:
1. To understand the importance of vibration analysis and modelling of dynamic systems
2. To analyze for dynamic response of Single Degree of Freedom System subjected to different types of loading.
3. To examine the dynamic response of Multiple Degree of Freedom System using lumped mass and distributed mass approach
4. To obtain the dynamic response of structures using numerical methods.
5. To illustrate the dynamic effects of Wind Loads, Moving Loads and Vibrations caused by Traffic, Blasting and Pile Driving

Course Outcomes:
1. Comprehend and model the systems subjected to vibrations and dynamic loads
2. Analyze and obtain dynamics response of single degree freedom system using fundamental Theory and equations of motion.
3. Analyze and obtain dynamics response of Multi degree of freedom system idealized as lumped mass systems. Analyze and obtain dynamics response of Multi degree of freedom system idealized as distributed mass systems.
4. Obtain dynamics response of systems using numerical methods.
5. To explain the dynamic effects of Wind Loads, Moving Loads and Vibrations caused by Traffic, Blasting and Pile Driving.

SYLLABUS

Course Code: GR22D5013
I Year II Semester

UNIT I

Introduction: Objectives, Importance of Vibration Analysis, Nature of Exciting Forces, Mathematical Modeling of Dynamic Systems. Elements of vibratory system - Degrees of Freedom - Continuous System - Lumped mass idealization - Oscillatory motion - Simple Harmonic motion - Vectorial representation of S.H.M. - Free and forced vibrations - undamped and damped vibrations - critical damping - Logarithmic decrement– Phase angle.

UNIT II

Single Degree of Freedom System: Formulation of equations of motion by different methods , Free and Forced Vibration with and without Damping, Response to Harmonic Loading, Response to General Dynamic Loading using Duhamel's Integral, Fourier Analysis for Periodic Loading

UNIT III

Multiple Degree of Freedom System (Lumped parameter): Selection of the degrees of Freedom - Evaluation of structural property matrices - Formulation of the MDOF equations of motion - Undamped free vibrations - Solutions of Eigen value problem for determination of natural frequencies and mode shapes - Inverse Iteration Method for Determination of Natural Frequencies and Mode Shapes, Dynamic Response by Modal Superposition Method, Direct Integration of Equation of Motion.

UNIT IV

Numerical Solution to Response using Stodola method, Holzer method, Newmark Method and Wilson Methods.

Continuous systems: Flexural vibrations of beams - Elementary case – Derivation of governing differential equation of motion - Analysis of undamped free vibrations of beams in flexure - Natural frequencies and mode-shapes of simple beams with different end conditions.

UNIT V

Special Topics in Structural Dynamics (Concepts only): Dynamic Effects of Wind Loading, Moving Loads, Vibrations caused by Traffic, Blasting and Pile Driving, Foundations for Industrial Machinery, Excitation by rigid base translation.

Text Books:

1. Dynamics of Structures, Clough R. W. and Penzien J., McGraw-Hill Education / Asia; 2nd edition (2003) ISBN-13 : 978-0071132411
2. Dynamics of Structures: Theory and Applications to Earthquake Engineering, Anil K. Chopra, Prentice Hall international series, Pearson, 2017, ISBN 9780134555126
3. Structural Dynamics - Theory and Computation, Paz Mario, CBS Publication, 2nd Edition, 2006

Reference Books:

1. Basics of Structural Dynamics and Aseismic Design, Prentice Hall India Learning Private Limited; 5th Edition, 2009.
2. Vibration of Structures - Application in Civil Engineering Design, Smith J. W., Chapman and Hall, London,1988.
3. Dynamics of Structures, Humar J. L., CRC Press; 2nd edition, 2012.
4. Structural Dynamics for Structural Engineers, Gary C. Hart, John Wiley & Sons, 2000.
5. Structural Dynamics, CRC Press; 1st edition,2016, ISBN-10 : 9780415427326

GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

STRUCTURAL DYNAMICS

Course Code: GR20D5013 **L/T/P/C: 3/0/0/3**
I Year II Semester

UNIT I
Introduction: Objectives, Importance of Vibration Analysis, Nature of Exciting Forces, Mathematical Modeling of Dynamic Systems. Elements of vibratory system - Degrees of Freedom - Continuous System - Lumped mass idealization - Oscillatory motion - Simple Harmonic motion - Vectorial representation of S.H.M. - Free and forced vibrations - undamped and damped vibrations - critical damping - Logarithmic decrement– Phase angle.

UNIT II
Single Degree of Freedom System: Formulation of equations of motion by different methods , Free and Forced Vibration with and without Damping, Response to Harmonic Loading, Response to General Dynamic Loading using Duhamel's Integral, Fourier Analysis for Periodic Loading

UNIT III
Multiple Degree of Freedom System (Lumped parameter): Selection of the degrees of Freedom - Evaluation of structural property matrices - Formulation of the MDOF equations of motion - Undamped free vibrations - Solutions of Eigen value problem for determination of natural frequencies and mode shapes - Inverse Iteration Method for Determination of Natural Frequencies and Mode Shapes, Dynamic Response by Modal Superposition Method, Direct Integration of Equation of Motion.

UNIT IV
Numerical Solution to Response using Stodola method, Holzer method, Newmark Method and Wilson Methods.
Continuous systems**:** Flexural vibrations of beams - Elementary case – Derivation of governing differential equation of motion - Analysis of undamped free vibrations of beams in flexure - Natural frequencies and mode-shapes of simple beams with different end conditions.

UNIT V
Special Topics in Structural Dynamics(Concepts only): Dynamic Effects of Wind Loading, Moving Loads, Vibrations caused by Traffic, Blasting and Pile Driving, Foundations for Industrial Machinery, Excitation by rigid base translation.

Reference Books:

1. Dynamics of Structures, Clough R. W. and Penzien J., McGraw Hill.
2. Structural Dynamics and Introduction to Earthquake Engineering, Chopra A. K.
3. Vibration of Structures - Application in Civil Engineering Design, Smith J. W., Chapman and Hall.
4. Dynamics of Structures, Humar J. L., Prentice Hall.
5. Structural Dynamics - Theory and Computation, Paz Mario, CBS Publication.
6. Dynamics of Structures, Hart and Wong.

CO - PI - PO Mapping Table

STRUCTURAL DYNAMICS (GR22)	Program Outcomes											
Course Outcomes0	**1 (5)**		**2 (4)**		**3 (7)**		**4 (7)**		**5 (7)**		**6 (6)**	
1. Comprehend and model the systems subjected to vibrations and dynamic loads	1.1.1	M	2.1.1	L	3.3.1	L	4.1.1	H	5.1.1	L	6.3.1	L
	1.1.2		-		3.3.2		4.1.2		5.2.2		6.3.2	
	-		-		-		4.2.2				-	
	-		-		-		4.3.1				-	
					-		4.3.3				-	
	-				-		-				-	
					-		-					
2. Analyze and obtain dynamics response of single degree freedom system using fundamental Theory and equations of motion.	1.1.1	M	2.1.1	L	3.3.1	L	4.2.2	L	5.1.1	H	6.3.1	L
	1.1.2		-		3.3.2		-		5.1.2		6.3.2	
	-		-		-		-		5.1.3		-	
	-		-		-		-		5.2.1		-	
					-		-		5.2.2		-	
	-				-		-		-		-	
					-		-		-			
3. Analyze and obtain dynamics response of Multi degree of freedom system idealized as lumped mass systems. Analyze and obtain dynamics response of Multi degree of freedom system idealized as distributed mass systems.	1.1.1	H	2.1.1	M	3.1.1	H	4.2.2	L	5.1.1	H	6.3.1	L
	1.1.2		2.1.2		3.2.1		4.3.3		5.1.2		6.3.2	
	1.2.1		-		3.2.2		-		5.1.3		-	
	1.2.3				3.3.1		-		5.2.1		-	
	-				3.3.2		-		5.2.2		-	
			-		-		-		-		-	
					-		-		-			
4. Obtain dynamics response of systems using numerical methods.	1.2.3	L	2.2.1	L	3.2.2	L	-	-	5.1.3	M	6.3.1	L
	-		-		3.3.2		-		5.2.1		6.3.2	
	-		-		-		-		5.2.2		-	
	-				-		-		5.3.2		-	
					-		-		-		-	
	-		-		-		-		-		-	
					-		-		-			
5. To explain the dynamic effects of Wind Loads, Moving Loads and Vibrations caused by Traffic, Blasting and Pile Driving.	1.1.1	M	2.1.1	L	3.3.1	L	4.1.1	H	5.1.1	L	6.3.1	L
	1.1.2		-		-		4.1.2		5.2.2		6.3.2	
	1.2.3		-		-		4.2.1		-		-	
	-				-		4.3.1		-		-	
					-		4.3.2		-		-	
	-		-		-		4.3.3		-		-	
					-							

Note:

1. If more than 67% of PIs match with CO, then CO-PO mapping is HIGH (H)
2. If the number of PIs matching with CO is between 34% & 67%, then CO-PO mapping is MEDIUM (M)
3. If the number of PIs matching with CO is less than 34%, then CO-PO mapping is LOW (L)

M.Tech Structural Engineering Program Program Outcomes – List of Competencies – Associated Performance Indicators	
PO 1: Conduct Investigations of Complex Problems: An ability to independently carry out research /investigation and development to solve practical problems.	
Competencies	**Performance Indicators (PI)**
1.1 Demonstrate an ability to conduct investigations of technical issues	1.1.1 Define a problem, its scope and importance for purposes of investigation 1.1.2 Use appropriate procedures, tools and techniques to conduct experiments and arrive at solution.
1.2 Demonstrate an ability to design experiments to solve open-ended problems	1.2.1 Design and develop an experimental approach, specify appropriate equipment and procedures 1.2.2 Choose an appropriate experimental design plan based on the study objectives. 1.2.3 Analyze data for trends and correlations, stating possible errors and limitations
PO 2: Technical Communication: An ability to write and present a substantial technical report/document.	
Competencies	**Performance Indicators (PI)**
2.1 Demonstrate an ability to comprehend technical literature and document project work	2.1.1 Read, understand and interpret technical and non-technical information 2.1.2 Produce clear, well-constructed, and well-supported written engineering documents with a logical progression of ideas.
2.2 Demonstrate an ability to integrate different modes of communication	2.2.1 Create engineering-standard figures, reports and drawings to complement writing and presentations 2.2.2 Use a variety of media effectively to convey a message in a document or a presentation
PO 3: Modern Engineering Tools and Project Management: Students should be able to demonstrate a degree of mastery over the area as per thespecialization of the program. The mastery should be at a level higher than the requirements in the appropriate bachelor's program.	
Competencies	**Performance Indicators (PI)**
3.1 Demonstrate an ability to evaluate the economic and financial performance of an engineering activity and plan/manage an engineering activity within time and budget constraints	3.1.1 Identify the tasks required to complete an engineering activity, and the resources required to complete the tasks. 3.1.2 Analyze and select the most appropriate engineering project based on economic and financial considerations. 3.1.3 Use project management tools to schedule an engineering project, so as to complete on time and within budget.
3.2 Demonstrate an ability to identify/ create modern	3.2.1 Identify/create/adapt/modify/extend tools such as STAAD Pro, ETABS, MIDAS, SAP 2000, ANSYS and techniques to solve structural engineering problems.

engineering tools, techniques and resources.	3.2.2 Demonstrate proficiency in using Structural engineering-specific tools and verify the credibility of results from tool use with reference to theaccuracy and limitations.
3.3 Demonstrate an ability to formulate and interpret a model.	3.3.1 Combine scientific principles and engineering concepts to formulate model(s) of a system or process that is appropriate in terms of applicability and required accuracy. 3.3.2 Apply engineering mathematics and computations to solve mathematical models.

PO 4: Solutions to Multidisciplinary Problems:
Possess critical thinking skills and solve core, complex and multidisciplinary structural engineering problems.

Competencies	Performance Indicators (PI)
4.1 Demonstrate an ability to identify and formulate a methodology and find solution to core and complex engineering problems	4.1.1 Articulate problem statements and identify objectives 4.1.2 Reframe complex problems into interconnected sub-problems 4.1.3 Identify existing processes/ methods for solving the problem, including forming justified approximations and assumptions
4.2 Demonstrate an ability to analyze data and reach a valid conclusion	4.2.1 Represent data (in tabular and/or graphical forms) so as to facilitate analysis and explanation of the data, and drawing of conclusions 4.2.2 Synthesize information and knowledge about the problem from the raw data to reach appropriate conclusions
4.3 Demonstrate an ability to advance a multidisciplinary engineering design to defined end state	4.3.1 Refine a conceptual design into a detailed design within the existing constraints (of the resources) 4.3.2 Generate information through appropriate tests to improve or revise the design

PO 5: Ethics, Environment and Sustainability:
Assess the impact of professional engineering solutions in an environmental context along with societal, health, safety, legal, ethical and cultural issues and the need for sustainabledevelopment.

Competencies	Performance Indicators (PI)
5.1 Demonstrate an understanding of the impact of engineering and industrial practices on the society and environment.	5.1.1 Identify risks/impacts in the life-cycle of an engineering product or activity related to design and construction of structures. 5.1.2 Understand the relationship between the technical, societal, health, safety and cultural issues.
5.2 Demonstrate an ability to apply principles of sustainable design and development.	5.2.1 Describe management techniques for sustainable development 5.2.2 Apply principles of sustainable development to an engineering activity or product relevant to the discipline.
5.3 Demonstrate an ability to apply the code of ethics and understanding of professional engineering regulations,	5.3.1 Identify situations of unethical professional conduct and propose ethical alternatives as per ICE(I), ECI, NSPE. 5.3.2 Examine and apply moral & ethical principles to known case studies 5.3.3 Interpret legislation, regulations, codes, and standards such as

legislation and standards.	ASCE, ASTM, BIS, ISO etc. which are relevant to Structural Engineering and its contribution to the protection of the public.

PO 6: Lifelong Learning:
Recognize the need for life-long learning to improve knowledge and competence.

Competencies	Performance Indicators (PI)
6.1 Demonstrate an ability to identify gaps in knowledge and a strategy to close these gaps	6.1.1 Describe the rationale for the requirement for continuing professional development 6.1.2 Identify deficiencies or gaps in knowledge and demonstrate an ability to source information to close this gap
6.2 Demonstrate an ability to identify changing trends in engineering knowledge and practice	6.2.1 Identify historic points of technological advance in engineering that required practitioners to seek education in order to stay current. 6.2.2 Recognize the need and be able to clearly explain why it is vitally important to keep current regarding new developments in the field of structural Engineering.
6.3 Demonstrate an ability to identify and access sources for new information.	6.3.1 Comprehend technical literature and other credible sources of information. 6.3.2 Analyze sourced technical and popular information for feasibility, viability, sustainability, etc.

	Competencies	Performance Indicators (PI)	CO1	CO2	CO3	CO4	CO5
PO 1	1.1 Demonstrate an ability to conduct investigations of technical issues	1.1.1 Define a problem, its scope and importance for purposes of investigation	Yes	Yes	Yes		Yes
		1.1.2 Use appropriate procedures, tools and techniques to conduct experiments and collect data	Yes	Yes	Yes		Yes
	1.2 Demonstrate an ability to design experiments to solveopen-ended problems	1.2.1 Design and develop an experimental approach, specify appropriate equipment and procedures			Yes		
		1.2.2 Choose an appropriate experimental design plan based on the study objectives.					
		1.2.3 Analyze data for trends and correlations, stating possible errors and limitations			Yes	Yes	Yes
PO 2	2.1 Demonstrate an ability to comprehend technical literature and document project work	2.1.1 Read, understand and interpret technical and non-technical information	Yes	Yes	Yes		Yes
		2.1.2 Produce clear, well-constructed, and well-supported written engineering documents with a logical progression of ideas.			Yes		
	2.2 Demonstrate an ability to integrate different modes of communication	2.2.1 Create engineering-standard figures, reports and drawings to complement writing and presentations				Yes	
		2.2.2 Use a variety of media effectively to convey a message in a document or a presentation					
PO 3	3.1 Demonstrate an ability to evaluate the economic and financial performance of an engineering activity and plan/manage an engineering activity within time and budget constraints	3.1.1 Identify the tasks required to complete an engineering activity,and the resources required to complete the tasks.			Yes		
		3.1.2 Analyze and select the most appropriate engineering project based on economic and financial considerations.					
		3.1.3 Use project management tools to schedule an engineering project, so as to complete on time and within budget.					
	3.2 Demonstrate an ability to identify/ create modern engineering tools, techniques and resources.	3.2.1 Identify/create/adapt/modify/extend tools and techniques tosolve structural engineering problems.			Yes		
		3.2.2 Demonstrate proficiency in using discipline-specific tools and verify the credibility of results from tool use with reference to the accuracy and limitations			Yes	Yes	

	3.3 Demonstrate an ability to formulate and interpret a model.	3.3.1 Combine scientific principles and engineering concepts to formulate model(s) of a system or process that is appropriate in terms of applicability and required accuracy.	Yes	Yes	Yes		Yes
		3.3.2 Apply engineering mathematics and computations to solve mathematical models	Yes	Yes	Yes	Yes	
PO4	4.1 Demonstrate an understanding of the impact of engineering and industrial practices on society and environment.	4.1.1 Identify risks/impacts in the life-cycle of an engineering product or activity.	Yes				Yes
		4.1.2 Understand the relationship between the technical, societal,health, safety and cultural issues.	Yes				Yes
	4.2 Demonstrate an ability to apply principles of sustainable design and development.	4.2.1 Describe management techniques for sustainable development					Yes
		4.2.2 Apply principles of sustainable development to an engineering activity or product relevant to the discipline.	Yes	Yes	Yes		
	4.3 Demonstrate an ability to apply the Code of Ethics and understanding of professional engineering regulations, legislation and standards.	4.3.1 Identify situations of unethical professional conduct andpropose ethical alternatives	Yes				Yes
		4.3.2 Examine and apply moral & ethical principles to known case studies					Yes
		4.3.3 Interpret legislation, regulations, codes, and standards such asASCE, ASTM, BIS, ISO etc. which are relevant to Structural Engineering and its contribution to the protection of the public.	Yes		Yes		Yes
PO 5	5.1 Demonstrate an ability to identify and formulate a methodology and find solution to complex engineering problems	5.1.1 Articulate problem statements and identify objectives	Yes	Yes	Yes		Yes
		5.1.2 Reframe complex problems into interconnected sub-problems		Yes	Yes		
		5.1.3 Identify existing processes/ methods for solving the problem, including forming justified approximations and assumptions		Yes	Yes	Yes	
	5.2 Demonstrate an ability to analyze data and reach a valid conclusion	5.2.1 Represent data (in tabular and/or graphical forms) so as tofacilitate analysis and explanation of the data, and drawing of conclusions		Yes	Yes	Yes	
		5.2.2 Synthesize information and knowledge about the problem from the raw data to reach appropriate conclusions	Yes	Yes	Yes	Yes	Yes

	5.3 Demonstrate an ability to advance a multidisciplinary engineering design to defined end state	5.3.1 Refine a conceptual design into a detailed design within the existing constraints (of the resources)					
		5.3.2 Generate information through appropriate tests to improve or revise the design				Yes	
PO 6	6.1 Demonstrate an ability to identify gaps in knowledge and a strategy to close these gaps	6.1.1 Describe the rationale for the requirement for continuing professional development					
		6.1.2 Identify deficiencies or gaps in knowledge and demonstrate an ability to source information to close this gap					
	6.2 Demonstrate an ability to identify changing trends in engineering knowledge and practice	6.2.1 Identify historic points of technological advance in engineering that required practitioners to seek education in order tostay current.					
		6.2.2 Recognize the need and be able to clearly explain why it is vitally important to keep current regarding new developments in your field.					
	6.3 Demonstrate an ability to identify and access sources for new information.	6.3.1 Comprehend technical literature and other credible sources of information	Yes	Yes	Yes	Yes	Yes
		6.3.2 Analyze sourced technical and popular information for feasibility, viability, sustainability, etc.	Yes	Yes	Yes	Yes	Yes

Program outcomes

PO 1: Research on practical and complex problems: An ability to independently carry out research /investigation and development to solve practical problems.
PO 2: Technical communication: An ability to write and present a substantial technical report/document.
PO 3: Modern engineering tools and project management: Students should be able to demonstrate a degree of mastery over the area as per the specialization of the program. The mastery should be at a level higher than the requirements in the appropriate bachelors.
PO 4: Ethics, Environment and Sustainability: Assess the impact of professional engineering solutions in an environmental context along with societal, health, safety, legal, ethical and cultural issues and the need for sustainable development.
PO 5: Analysis and solutions to complex and multidisciplinary problems: Possess critical thinking skills and solve core, complex and multidisciplinary structural engineering problems.
PO 6: Lifelong learning: Recognize the need for life-long learning to improve knowledge and competence.

Course Outcomes: At the end of the course, students will be able to

1. Comprehend and model the systems subjected to vibrations and dynamic loads
2. Analyze and obtain dynamics response of single degree freedom system using fundamental Theory and equations of motion.
3. Analyze and obtain dynamics response of Multi degree of freedom system idealized as lumped and distributed mass systems.
4. Obtain dynamics response of systems using numerical methods.
5. To explain the dynamic effects of Wind Loads, Moving Loads and Vibrations caused by Traffic, Blasting and Pile Driving.

STRUCTURAL DYNAMICS (GR22)		
CO	**PI**	**Exam (Marks)**
1. Comprehend and model the systems subjected to vibrations and dynamic loads	1.1.1	EE(4)
	2.1.1	M1(4)
	3.3.1	EE(5) ,M1(10),A1(2)
	3.3.2	EE(10) ,A1(1)
	4.1.1	EE(5)
	4.1.2	A1(2)
	4.1.3	M1(3) ,A1(1)
	4.2.1	EE(5)
	4.2.2	EE(10)
	5.1.1	M1(5)
2. Analyze and obtain dynamics response of single degree freedom system using fundamental Theory and equations of motion.	1.2.2	EE(4)
	2.1.1	EE(5), M1(2)
	2.2.1	EE(5)
	3.2.1	A2(1)
	3.3.1	A2(3)
	3.3.2	EE(10) ,A2(1)
	4.1.1	M1(5)
	4.1.2	M1(5)
	4.1.3	EE(5), M1(5) ,A2(1)
	4.2.2	EE(5)
3. Analyze and obtain dynamics response of Multi degree of freedom system idealized as lumped mass	2.1.1	EE(4), M1(1)
	3.2.1	A3(1)
	3.3.1	EE(5), M1(5) ,A3(1)
	3.3.2	EE(10) ,A3(1)

End exam (EE)
Mid 1 Exam (M1)
Mid 2 Exam (M2)
Assignment 1 (A1)
Assignment 2 (A2)
Assignment 3 (A3)
Assignment 4 (A4)
Assignment 5 (A5)

systems. Analyze and obtain dynamics response of Multi degree of freedom system idealized as distributed mass systems.	4.1.2	M1(10)
	4.1.3	EE(15)
	4.2.2	A3(2)
4. Obtain dynamics response of systems using numerical methods.	1.1.1	EE(2)
	3.3.1	EE(2) ,A4(2)
	3.3.2	EE(20) ,A4(1)
	4.1.1	A4(1)
	4.1.2	EE(20) ,A4(1)
5. To explain the dynamic effects of Wind Loads, Moving Loads and Vibrations caused by Traffic, Blasting and Pile Driving.	1.2.3	EE(5)
	2.1.1	EE(4)
	2.1.2	EE(5)
	3.3.1	EE(5) ,A5(2)
	3.3.2	A5(1)
	4.1.3	EE(5)

	PO 1				PO 2				PO 3				PO 4				PO 5				PO 6			
CO	**Total PIs**	**Mapped PIs**	**% PI**		**Total PIs**	**Mapped PIs**	**% PI**		**Total PIs**	**Mapped PIs**	**% PI**		**Total PIs**	**Mapped PIs**	**% PI**		**Total PIs**	**Mapped PIs**	**% PI**		**Total PIs**	**Mapped PIs**	**% PI**	
CO1	5	2	40	M	4	1	25	L	7	2	29	L	7	5	71	H	7	2	29	L	6	2	33	L
CO2	5	2	40	M	4	1	25	L	7	2	29	L	7	1	14	L	7	5	71	H	6	2	33	L
CO3	5	4	80	H	4	2	40	M	7	5	71	H	7	2	29	L	7	5	71	H	6	2	33	L
CO4	5	1	20	L	4	1	25	L	7	2	29	L	7	0	0	L	7	4	59	M	6	2	33	L
CO5	5	3	60	M	4	1	25	L	7	1	15	L	7	6	86	H	7	2	29	L	6	2	33	L

Note:

1. If more than 67% of PIs match with CO, then CO-PO mapping is HIGH (H)
2. If the number of PIs matching with CO is between 34% & 67%, then CO-PO mapping is MEDIUM (M)
3. If the number of PIs matching with CO is less than 34%, then CO-PO mapping is LOW (L)

CO_PO Mapping

STRUCTURAL DYNAMICS (GR20D5013)	Program Outcomes					
CO	**1**	**2**	**3**	**4**	**5**	**6**
1	M	L	L	H	L	L
2	M	L	L	L	H	L
3	H	M	H	L	H	L
4	L	L	L	L	M	L
5	M	L	L	H	L	L

OLD CO_PO Mapping

STRUCTURAL DYNAMICS (GR20D5013)	Program Outcomes					
CO	**1**	**2**	**3**	**4**	**5**	**6**
1	M	M	H	M		
2	M	M	H	M	H	M
3	M		H		M	
4	M	M	H		M	
5	M	M	H	M		

Gokaraju Rangaraju Institute of Engineering and Technology
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Bachupally, Kukatpally, Hyderabad – 500 090. (040) 6686 4440

GUIDELINES TO STUDY THE COURSE / SUBJECT

Academic **Y**ear : 2022-23

Semester : II

Name of the Program: M.TECH. STRUCTURAL ENGINEERING

Course/Subject: STRUCTURAL DYNAMICS Course Code: GR20D5013

Name of the Faculty: DR. V SRINIVASA REDDY Dept.: CIVIL ENGINEERING

Designation: PROFESSOR.

Guidelines to study the Course/ Subject: STRUCTURAL DYNAMICS

Course Design and Delivery System (CDD):

- The Course syllabus is written into number of learning objectives and outcomes.
- These learning objectives and outcomes will be achieved through lectures, assessments, assignments, experiments in the laboratory, projects, seminars, presentations, etc.
- Every student will be given an assessment plan, criteria for assessment, scheme of evaluation and grading method.
- The Learning Process will be carried out through assessments of Knowledge, Skills and Attitude by various methods and the students will be given guidance to refer to the text books, reference books, journals, etc.

The faculty be able to –

- Implement principles of Learning
- Comprehend the psychology of students
- Develop instructional objectives for a given topic
- Prepare course, unit and lesson plans
- Demonstrate different methods of teaching and learning
- Use appropriate teaching and learning aids
- Plan and deliver lectures effectively
- Provide feedback to students using various methods of Assessments and tools of Evaluation
- Act as a guide, advisor, counselor, facilitator, motivator and not just as a teacher alone

Signature of HOD Signature of faculty

Date: Date:

Date of commencement of classes: 03-04-2023

SESSION PLAN

Subject: **Structural Dynamics** Teacher: **Dr. V Srinivasa Reddy**

Internal Marks: 40 End Exam Marks: 60 Total: 100

Session	Unit No	Date	Topics
1.	Unit I	05-04-2023	Theory of vibrations: Introduction / Elements of vibratory system - Degrees of Freedom
2.	Unit I	07-04-2023	Continuous System - Lumped mass idealization/ Oscillatory motion - Simple Harmonic motion
3.	Unit I	07-04-2023	Vectorial representation of S.H.M./ Free vibrations of single degree of freedom system
4.	Unit I	08-04-2023	undamped and damped vibrations
5.	Unit I	12-04-2023	critical damping - Logarithmic decrement /Forced vibration of SDOF systems
6.	Unit I	14-04-2023	Harmonic excitation
7.	Unit I	14-04-2023	Dynamic magnification factor – Phase angle – Bandwidth
8.	Unit I	15-04-2023	Introduction to Structural Dynamics
9.	Unit I	19-04-2023	Fundamental objectives of dynamic analysis -Types of prescribed loading
10.	Unit I	21-04-2023	Methods of discretization
11.	Unit I	21-04-2023	Formulation of equations of motion by different methods –
12.	Unit I	26-04-2023	Direct equilibration using Newton’s law of motion / D’Alembert’s principle
13.	Unit I	28-04-2023	Principle of virtual work and Hamilton principle.
14.	Unit II	28-04-2023	Single Degree of Freedom Systems:
15.	Unit II	29-04-2023	Formulation and solution of the equation of motion
16.	Unit II	17-05-2023	Formulation and solution of the equation of motion
17.	Unit II	19-05-2023	Formulation and solution of the equation of motion
18.	Unit II	19-05-2023	Formulation and solution of the equation of motion
19.	Unit II	20-05-2023	Free vibration response
20.	Unit II	24-05-2023	Duhamel integral
21.	Unit II	26-05-2023	Duhamel integral
22.	Unit II	26-05-2023	Response to Harmonic, Periodic, Impulsive and general dynamic loadings
23.	Unit II	27-05-2023	Response to Harmonic, Periodic, Impulsive and general dynamic loadings
24.	Unit II	31-05-2023	Response to Harmonic, Periodic, Impulsive and general dynamic loadings
25.	Unit II	02-06-2023	Response to Harmonic, Periodic, Impulsive and general dynamic loadings
26.	Unit II	02-06-2023	Response to Harmonic, Periodic, Impulsive and general dynamic loadings
27.	Unit II	03-06-2023	Tutorial
28.	Unit II	07-06-2023	Tutorial
29.	Unit III	09-06-2023	Multi Degree of Freedom Systems
30.	Unit III	09-06-2023	Selection of the degrees of Freedom
31.	Unit III	10-06-2023	Evaluation of structural property matrices
32.	Unit III	14-06-2023	Formulation of the MDOF equations of motion
33.	Unit III	16-06-2023	Solutions of Eigen value problem for natural frequencies and mode shapes - Analysis of Dynamic response
34.	Unit III	16-06-2023	Solutions of Eigen value problem for natural frequencies and mode shapes - Analysis of Dynamic response
35.	Unit III	17-06-2023	Solutions of Eigen value problem for natural frequencies and mode shapes - Analysis of Dynamic response
36.	Unit III	28-06-2023	Solutions of Eigen value problem for natural frequencies and mode shapes - Analysis of D response
37.	Unit III	30-06-2023	Normal co-ordinates - Uncoupled equations of motion
38.	Unit III	30-06-2023	Orthogonal properties of normal modes - Mode superposition procedure.
39.	Unit III	01-07-2023	Normal co-ordinates - Uncoupled equations of motion
40.	Unit III	05-07-2023	Tutorial
41.	Unit IV	07-07-2023	Practical Vibration Analysis: Introduction
42.	Unit IV	07-07-2023	Stodola method
43.	Unit IV	08-07-2023	Fundamental mode analysis - Analysis of second and higher modes

Date of commencement of classes: 03-04-2023

44.	Unit IV	12-07-2023	Holzer method - Basic procedure.
45.	Unit IV	14-07-2023	Continuous Systems: Introduction
46.	Unit IV	14-07-2023	Flexural vibrations of beams - Elementary case
47.	Unit IV	15-07-2023	Derivation of governing differential equation of motion
48.	Unit IV	19-07-2023	Natural frequencies and mode-shapes of simple beams with different end conditions
49.	Unit IV	21-07-2023	Natural frequencies and mode-shapes of simple beams with different end conditions
50.	Unit IV	21-07-2023	Natural frequencies and mode-shapes of simple beams with different end conditions
51.	Unit IV	22-07-2023	Natural frequencies and mode-shapes of simple beams with different end conditions
52.	Unit IV	26-07-2023	Principles of application to continuous beams.
53.	Unit IV	28-07-2023	Principles of application to continuous beams.
54.	Unit IV	28-07-2023	Principles of application to continuous beams.
55.	Unit IV	29-07-2023	Numerical methods using MATLAB
56.	Unit IV	02-08-2023	Numerical methods using MATLAB
57.	Unit IV	04-08-2023	Numerical methods using MATLAB
58.	Unit V	04-08-2023	Introduction to Earthquake Analysis: Introduction
59.	Unit V	05-08-2023	Introduction to Earthquake Analysis: Introduction
60.	Unit V	09-08-2023	Earthquake analysis methods
61.	Unit V	11-08-2023	Earthquake IS code discussion
62.	Unit V	11-08-2023	Excitation by rigid base translation
63.	Unit V	12-08-2023	Dynamic effect of wind loading
64.	Unit V	14-08-2023	Dynamic effect of wind loading
65.	Unit V	16-08-2023	Dynamic effect of wind loading
66.	Unit V	18-08-2023	Dynamic effect of vibration caused by traffic
67.	Unit V	18-08-2023	Dynamic effect of blasting
68.	Unit V	19-08-2023	Dynamic effect of pile driving
69.	Unit V	23-08-2023	Dynamic effect of pile driving
70.	Unit V	25-08-2023	Foundation for industrial machinery
71.	Unit V	25-08-2023	Foundation for industrial machinery
72.	Unit V	26-08-2023	Foundation for industrial machinery

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SCHEDULE OF INSTRUCTIONS

UNIT PLAN

Academic Year : 2022-23
Semester : II UNIT NO.: 1
Name of the Program: M.TECH. STRUCTURAL ENGINEERING
Course/Subject: STRUCTURAL DYNAMICS Course Code: GR22D5013
Name of the Faculty: DR. V SRINIVASA REDDY Dept.: CIVIL ENGINEERING
Designation: PROFESSOR.

Lesson No.	Date	No. of Periods	Topics / Sub - Topics	Objective & Outcome Nos.	References (Text Book, Journal…) Page Nos.: ____to ____	Bloom's Knowledge Levels
1.	05-04-2023	1	Theory of vibrations: Introduction / Elements of vibratory system - Degrees of Freedom	COB-2 CO-2	Lecture Notes	Level 2
2.	07-04-2023	1	Continuous System - Lumped mass idealization/ Oscillatory motion - Simple Harmonic motion	COB-2 CO-2,3	Lecture Notes	Level 2
3.	07-04-2023	1	Vectorial representation of S.H.M./ Free vibrations of single degree of freedom system	COB-2 CO-2,3,4	Lecture Notes	Level 2
4.	08-04-2023	1	undamped and damped vibrations	COB-2 CO-2,3,4	Lecture Notes	Level 2
5.	12-04-2023	1	critical damping - Logarithmic decrement /Forced vibration of SDOF systems	COB-2 CO-2,3,4	Lecture Notes	Level 3
6.	14-04-2023	1	Harmonic excitation	COB-2 CO-2,3,4	Lecture Notes	Level 3
7	14-04-2023	1	Dynamic magnification factor – Phase angle – Bandwidth	COB-2 CO-2,3,4	Lecture Notes	Level 3
8	15-04-2023	1	Introduction to Structural Dynamics	COB-2 CO-2,3,4	Lecture Notes	Level 3
9	19-04-2023	1	Fundamental objectives of dynamic analysis -Types of prescribed loading	COB-2 CO-2,3,4	Lecture Notes	Level 3
10	21-04-2023	1	Methods of discretization	COB-2 CO-2,3,4	Lecture Notes	Level 3
11	21-04-2023	1	Formulation of equations of motion by different methods –	COB-2 CO-2,3,4	Lecture Notes	Level 3
12	21-04-2023	1	Direct equilibration using Newton's law of motion / D'Alembert's principle	COB-2 CO-2,3,4	Lecture Notes	Level 3
13	21-04-2023	1	Principle of virtual work and Hamilton principle.	COB-2 CO-2,3,4	Lecture Notes	Level 3

Signature of HOD
Date:

Signature of faculty
Date:

Gokaraju Rangaraju Institute of Engineering and Technology (Autonomous)

Bachupally, Kukatpally, Hyderabad – 500 090. (040) 6686 4440

SCHEDULE OF INSTRUCTIONS

UNIT PLAN

Academic Year : 2022-23
Semester : II UNIT NO.: II
Name of the Program: M.TECH. STRUCTURAL ENGINEERING
Course/Subject: STRUCTURAL DYNAMICS Course Code: GR22D5013
Name of the Faculty: DR. V SRINIVASA REDDY Dept.: CIVIL ENGINEERING
Designation: PROFESSOR.

Lesson No.	Date	No. of Periods	Topics / Sub - Topics	Objective & Outcome Nos.	References (Text Book, Journal…) Page Nos.: ____to ____	Bloom's Knowledge Levels
1.	28-04-2023	1	Single Degree of Freedom Systems:	COB-2 CO-2	Lecture Notes	Level 2
2.	29-04-2023	1	Formulation and solution of the equation	COB-2 CO-2,3	Lecture Notes	Level 2
3.	17-05-2023	1	Formulation and solution of the equation	COB-2 CO-2,3,4	Lecture Notes	Level 2
4.	19-05-2023	1	Formulation and solution of the equation of motion	COB-2 CO-2,3,4	Lecture Notes	Level 2
5.	19-05-2023	1	Formulation and solution of the equation of motion	COB-2 CO-2,3,4	Lecture Notes	Level 3
6.	20-05-2023	1	Free vibration response	COB-2 CO-2,3,4	Lecture Notes	Level 3
7	24-05-2023	1	Duhamel integral	COB-2 CO-2,3,4	Lecture Notes	Level 3
8	26-05-2023	1	Duhamel integral	COB-2 CO-2,3,4	Lecture Notes	Level 3
9	26-05-2023	1	Response to Harmonic, Periodic, Impulsi general dynamic loadings	COB-2 CO-2,3,4	Lecture Notes	Level 3
10	27-05-2023	1	Response to Harmonic, Periodic, Impuls general dynamic loadings	COB-2 CO-2,3,4	Lecture Notes	Level 3
11	31-05-2023	1	Response to Harmonic, Periodic, Impulsi general dynamic loadings	COB-2 CO-2,3,4	Lecture Notes	Level 3
12	02-06-2023	1	Response to Harmonic, Periodic, Impulsi general dynamic loadings	COB-2 CO-2,3,4	Lecture Notes	Level 3
13	02-06-2023	1	Response to Harmonic, Periodic, Impulsi general dynamic loadings	COB-2 CO-2,3,4	Lecture Notes	Level 3

Signature of HOD
Date:

Signature of faculty
Date:

Gokaraju Rangaraju Institute of Engineering and Technology (Autonomous)

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SCHEDULE OF INSTRUCTIONS

UNIT PLAN

Academic Year : 2022-23
Semester : II UNIT NO.: 1II
Name of the Program: M.TECH. STRUCTURAL ENGINEERING
Course/Subject: STRUCTURAL DYNAMICS Course Code: GR22D5013
Name of the Faculty: DR. V SRINIVASA REDDY Dept.: CIVIL ENGINEERING
Designation: PROFESSOR.

Lesson No.	Date	No. of Periods	Topics / Sub - Topics	Objective & Outcome Nos.	References (Text Book, Journal…) Page Nos.: ____to ____	Bloom's Knowledge Levels
1.	09-06-2023	1	Multi Degree of Freedom Systems	COB-2 CO-2	Lecture Notes	Level 2
2.	09-06-2023	1	Selection of the degrees of Freedom	COB-2 CO-2,3	Lecture Notes	Level 2
3.	10-06-2023	1	Evaluation of structural property matrices	COB-2 CO-2,3,4	Lecture Notes	Level 2
4.	14-06-2023	1	Formulation of the MDOF equations of motion	COB-2 CO-2,3,4	Lecture Notes	Level 2
5.	16-06-2023	1	Solutions of Eigen value problem for natural frequencies and mode shapes - Analysis of Dynamic response	COB-2 CO-2,3,4	Lecture Notes	Level 3
6.	16-06-2023	1	Solutions of Eigen value problem for natural frequencies and mode shapes - Analysis of Dynamic response	COB-2 CO-2,3,4	Lecture Notes	Level 3
7	17-06-2023	1	Solutions of Eigen value problem for natural frequencies and mode shapes - Analysis of Dynamic response	COB-2 CO-2,3,4	Lecture Notes	Level 3
8	28-06-2023	1	Solutions of Eigen value problem for natu frequencies and mode shapes - Analysis c response	COB-2 CO-2,3,4	Lecture Notes	Level 3
9	30-06-2023	1	Normal co-ordinates - Uncoupled equations of motion	COB-2 CO-2,3,4	Lecture Notes	Level 3
10	30-06-2023	1	Orthogonal properties of normal modes - superposition procedure.	COB-2 CO-2,3,4	Lecture Notes	Level 3

Signature of HOD
Date:

Signature of faculty
Date:

Gokaraju Rangaraju Institute of Engineering and Technology (Autonomous)

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SCHEDULE OF INSTRUCTIONS

UNIT PLAN

Academic Year : 2022-23
Semester : II UNIT NO.: 1V
Name of the Program: M.TECH. STRUCTURAL ENGINEERING
Course/Subject: STRUCTURAL DYNAMICS Course Code: GR22D5013
Name of the Faculty: DR. V SRINIVASA REDDY Dept.: CIVIL ENGINEERING
Designation: PROFESSOR.

Lesson No.	Date	No. of Periods	Topics / Sub - Topics	Objective & Outcome Nos.	References (Text Book, Journal…) Page Nos.: ____to ____	Bloom's Knowledge Levels
1.	07-07-2023	1	Practical Vibration Analysis: Introduction	COB-2 CO-2	Lecture Notes	Level 2
2.	07-07-2023	1	Stodola method	COB-2 CO-2,3	Lecture Notes	Level 2
3.	08-07-2023	1	Fundamental mode analysis - Analysis of higher modes	COB-2 CO-2,3,4	Lecture Notes	Level 2
4.	12-07-2023	1	Holzer method - Basic procedure.	COB-2 CO-2,3,4	Lecture Notes	Level 2
5.	14-07-2023	1	Continuous Systems: Introduction	COB-2 CO-2,3,4	Lecture Notes	Level 3
6.	14-07-2023	1	Flexural vibrations of beams - Elementary case	COB-2 CO-2,3,4	Lecture Notes	Level 3
7	15-07-2023	1	Derivation of governing differential equation of motion	COB-2 CO-2,3,4	Lecture Notes	Level 3
8	19-07-2023	1	Natural frequencies and mode-shapes of simple beams with different end conditions	COB-2 CO-2,3,4	Lecture Notes	Level 3
9	21-07-2023	1	Natural frequencies and mode-shapes of simple beams with different end conditions	COB-2 CO-2,3,4	Lecture Notes	Level 3
10	21-07-2023	1	Natural frequencies and mode-shapes of simple beams with different end conditions	COB-2 CO-2,3,4	Lecture Notes	Level 3
11	22-07-2023	1	Natural frequencies and mode-shapes of simple beams with different end conditions	COB-2 CO-2,3,4	Lecture Notes	Level 3

Signature of HOD
Date:

Signature of faculty
Date:

Gokaraju Rangaraju Institute of Engineering and Technology (Autonomous)

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SCHEDULE OF INSTRUCTIONS

UNIT PLAN

Academic Year : 2022-23
Semester : II UNIT NO.: V
Name of the Program: M.TECH. STRUCTURAL ENGINEERING
Course/Subject: STRUCTURAL DYNAMICS Course Code: GR22D5013
Name of the Faculty: DR. V SRINIVASA REDDY Dept.: CIVIL ENGINEERING
Designation: PROFESSOR.

Lesson No.	Date	No. of Periods	Topics / Sub - Topics	Objective & Outcome Nos.	References (Text Book, Journal…) Page Nos.: ____to ____	Bloom's Knowledge Levels
1.	09-08-2023	1	Earthquake analysis methods	COB-2 CO-2	Lecture Notes	Level 2
2.	11-08-2023	1	Earthquake IS code discussion	COB-2 CO-2,3	Lecture Notes	Level 2
3.	11-08-2023	1	Excitation by rigid base translation	COB-2 CO-2,3,4	Lecture Notes	Level 2
4.	12-08-2023	1	Dynamic effect of wind loading	COB-2 CO-2,3,4	Lecture Notes	Level 2
5.	14-08-2023	1	Dynamic effect of wind loading	COB-2 CO-2,3,4	Lecture Notes	Level 3
6.	16-08-2023	1	Dynamic effect of wind loading	COB-2 CO-2,3,4	Lecture Notes	Level 3
7.	18-08-2023	1	Dynamic effect of vibration caused by traffic	COB-2 CO-2,3,4	Lecture Notes	Level 3
8.	18-08-2023	1	Dynamic effect of blasting	COB-2 CO-2,3,4	Lecture Notes	Level 3
9.	19-08-2023	1	Dynamic effect of pile driving	COB-2 CO-2,3,4	Lecture Notes	Level 3
10.	23-08-2023	1	Dynamic effect of pile driving	COB-2 CO-2,3,4	Lecture Notes	Level 3
11.	25-08-2023	1	Foundation for industrial machinery	COB-2 CO-2,3,4	Lecture Notes	Level 3
12.	25-08-2023	1	Foundation for industrial machinery	COB-2 CO-2,3,4	Lecture Notes	Level 3

Signature of HOD
Date:

Signature of faculty
Date:

Course Objectives – Course Outcomes Relationship Matrix

Course - outcomes / Course Objectives	1	2	3	4	5
1	X				
2		X			
3			X		
4				X	
5					X

Course Objectives – Program Outcomes (POs) Relationship Matrix

Program -outcomes / Course objectives	1	2	3	4	5	6
1		X				
2						
3	X		X	X		X
4	X		X			
5	X			X	X	X

Course Outcomes – Program Outcomes (POs) Relationship Matrix

Program -Outcomes / Course-Outcomes	1	2	3	4	5	6
1		X				
2						
3	X		X	X		X
4	X		X			
5	X			X	X	X

Program Outcomes (POs) Relationship Matrix

Course: STRUCTURAL DYNAMICS **Course Code: GR22D5013**

Program -Outcomes / Course	1	2	3	4	5	6
STRUCTURAL DYNAMICS	X	X	X	X	X	X

Program Educational Objectives (PEOs) – Course Outcomes [CO] Relationship Matrix

PEOs / Course Outcomes	1	2	3	4
1	X			
2	X			
3	X	X	X	X
4	X	X	X	X
5	X		X	X

Mapping between POs and Assessment methods

Assessments:

1. ASSIGNMENT
2. INTERNAL EXAMINATION
3. EXTERNAL EXAMINATION
4. PRACTICAL PROJECTS/ CASE STUDIES
5. VIVA

Course outcomes / Assessments	1	2	3	4	5
1	X	X	X	X	X
2	X	X	X	X	X
3	X	X	X	X	X
4			X		X
5			X		X

Assessments – Program Educational Objectives (PEOs) Relationship matrix

PEOs / Assessments	1	2	3	4
1	X	X	X	X
2	X	X	X	X
3	X	X	X	X
4		X		X
5				X

GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

Department of Civil Engineering

I M.Tech. II Semester (Mid- II Examination August 2022)

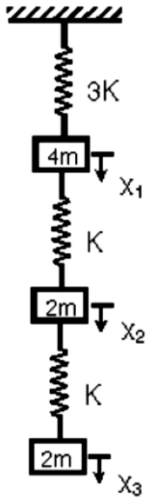
STRUCTURAL DYNAMICS (GR20D5013)

(Subjective)

Time: 75 Minutes `` *Max. Marks: 15 Marks*

Name : ___________________________ Roll No. | | | | | | D | | | | |

Answer all questions

	M	CO	BL	PIs
1. a) Derive the governing differential equation for transverse flexural vibration of a continuous system subjected to dynamic loading. Apply the above equation for a beam with both ends fixed	5	4	3	1.1.1 3.3.1
OR				
b) Using normal mode theory, explain the method for uncoupling the equations of motion of MDOF system.	5	3	4	3.3.2
2. a) For the multistory building shown in fig.. Obtain frequencies and modes of vibration using STODOLA method. Assume *m* = 5 x 10^4 kg, k= 5 x 10^4 kN/cm.  	5	4	2	3.3.1
OR				
b) For the multistory building shown in fig above.. Obtain frequencies and modes of vibration using HOLZER method. Assume *m* = 5 x 10^4 kg, k= 5 x 10^4 kN/cm.	5	4	4	5.1.3
3. a) Discuss the (i) Orthogonal property of normal modes (ii) Dynamic effects of wind loading	5	3	3	3.3.1 5.1.2
OR				
b) Describe how the equation of motion can be set up using Newton's second law of motion for the system subjected to earthquake horizontal ground acceleration u_g.	5	5	2	3.2.1

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GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

Department of Civil Engineering

I M.Tech. II Semester (Mid-II Examination June 2022)

STRUCTURAL DYNAMICS (GR20D5013)

(Objective)

Time: 15 Minutes *Date of examination 07-06-2022* *Max. Marks: 5 Marks*

Name: ______________________ Roll No. | | | | | | D | | | | |

1. IS Code used for seismic analysis?
 a) IS 1893 b) IS 456 c) IS 1983 d) IS 875
2. Which one is linear dynamic analysis?
 a) Equivalent static method b) Push over analysis c) Response spectrum method d) Time history method
3. IS Code for wind analysis?
 a) IS 875-Part III b) IS 875-Part II c) IS 875-Part I d) IS 875-Part IV
4. Wind Load Explanatory Hand Book
 a) SP 34 b) SP 64 c) SP 16 d) SP 40
5. Phenomenon which affects the design of a tall building is
 a) Vortex shedding b) Gust c) Galloping d) Flutter
6. Single high pressure impulses acting directly on the exterior envelope over milliseconds causing localized damage is called
 a) Blast loads b) Seismic loads c) Impact loads d) Live loads
7. Design of machine foundations can be categorized into three approaches
 a) Static Analysis with Rule of Thumb b) Natural Frequency Analysis
 c) Forced Vibration Analysis d) Time history analysis
8. Only fundamental natural frequency and model vector of vibration are found in
 a) Stodola method b) Holzer method c) Wilson method d) Newmark method
9. A pattern of motion in which all parts of the system move sinusoidally with the same frequency and with a fixed phase relation is called
 a) Normal mode b) eigen value c) eigen vector d) General mode
10. The most general motion of a system is a superposition of its normal modes.
 a) True b) False c) if not orthogonal d) not always

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GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

Department of Civil Engineering

I M.Tech. II Semester (Mid-I Examination March 2022)

STRUCTURAL DYNAMICS (GR20D5013)

(Subjective)

Time: 75 Minutes *Date of examination 07-06-2022* *Max. Marks: 15 Marks*

Name : ______________________ Roll No. | | | | | | D | | | | |

Answer all questions

	M	CO	BL	PIs
1. a) One of the construction companies hires you to determine the dynamic properties of a frame system for which it has lost the original blue prints. Being a civil engineer, you were assigned to do a free vibration test of the frame system. Supplied with a hydraulic jack, you were able to apply a jacking force to displace the frame. With a jacking force of 134 kN, you noted down that the frame has displaced 0.76 cm. On the first return swing after release, the frame did not come back to the release point but rather it stopped at 0.64 cm towards it. You recorded time between the release and the first return as 2 sec. Determine the following a. Natural frequency c. Logarithmic decrement d. Damping ratio e. Damping frequency f. Amplitude of the frame after 6 cycles	5	1	3	1.1.1 3.3.1
OR				
b) Examine whether the log – decrement is also given by the equation $\delta = 1/n \log (U_0/U_n)$ represents the amplitude after n cycles have elapsed.	5	1	4	3.3.2
2. a) Formulate equations of motion for SDOF systems using 1) D'Alembert's Principle 2) Newton's Law and 3) Energy method	5	2	2	3.3.1
OR				
b) Find the solution for the vibration of a SDOF system with viscous damping	5	2	4	5.1.3
3. a) Model multi degrees of freedom discrete parameter system and formulate its equation of motion.	5	3	3	3.3.1 5.1.2
OR				
b) Derive an expression for the force transmitted to the foundation and phase angle for a damped oscillator idealized as a SDOF system subjected to harmonic force.	5	2	2	3.2.1

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GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

Department of Civil Engineering

I M.Tech. II Semester (Mid- I Examination March 2022)

STRUCTURAL DYNAMICS (GR20D5013)

(Objective)

Time: 15 Minutes *Date of examination 07-06-2022* *Max. Marks: 5 Marks*

Name: ________________________ Roll No. | | | | | | D | | | | |

1. Mass = 10 kg, K = 10 N/m, C = 10 Ns/m, The system is
a) Overdamped b) Underdamped c) Critically damped d) Viscous damped [ ]
2. Damping ratio is 0.5%, Logarithmic decrement is
a) 0.03 b)0.5 c) 36.2 d) 2.5 [ ]
3. Natural undamped frequency is 20 rad/s, Damping ratio is 50%, Damped frequency is
a) 20 rad/sb) 10 rad/s c) 5 rad/s d) 17.3 rad/s [ ]
4. ________ is the graphical representation of the relative amplitudes of the two coordinates and their phase angle relationship.
a) Stiffness b) Mode shape c) Node d) Flexibility [ ]
5. For which case there is an oscillatory motion
a) Overdamped system b) Underdamped system c) Critically damped system d) undulated damped system [ ]
6. Response due to dynamic load is (Chose multiple answers)
a) Displacement b) Velocity c) Acceleration d) Momentum [ ]
7. The number of independent coordinates required to specify configuration of vibrating system in space at any instant of time
a) Degrees of freedom b) Nodes c) global coordinates d) Degree of redundancy[ ]
8. In mass-spring-damper model of the structural mass, the mass element signifies
a) the mass and the inertial characteristics
b) the potential energy of and the stiffness characteristics
c) the energy dissipation characteristics
d) excitation force, F(t) which is obviously a function of time. [ ]
9. If all the springs displace by the same amount, then the springs are in
a) Parallel b) Series c) out of phase d) phase [ ]
10. To transform the building structure into a discrete number of degree of freedom with lumped masses at the floor level, following assumptions are necessary.
(Chose multiple answers)
a) The entire mass of the building is concentrated at the floor levels.
b) The axial forces do not contribute significantly for the deformation of structures and hence the stiffness
c) The floors with slabs and beams are infinitely rigid as compared to the columns and remain horizontal without rotation.
d) The entire mass of the building is treated as continuous system. [ ]

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CODE:GR20D5013 | GR 20 | SET - 4

M.Tech I Year II Semester Regular Examinations, September 2022

STRUCTURAL DYNAMICS

(Structural Engineering)

Time: 3 hours Max Marks: 70

Instructions:

1. Question paper comprises of **Part-A** and **Part-B**
2. **Part-A** (for 20 marks) must be answered at one place in the answer book.
3. **Part-B** (for 50 marks) consists of **five questions with internal choice,** answer all questions.

PART – A

(Answer ALL questions. All questions carry equal marks)

10 * 2 = 20 Marks

1. a. Define free and forced vibrations of single degree of freedom. CO1 BL1 [2]

b. Write differences between damped and undamped vibrations of SDOF. CO1 BL2 [2]

c. Define single degree of freedom of vibrations. CO2 BL1 [2]

d. Write a formula for SDOF with damped forced vibrations. CO2 BL1 [2]

e. Write difference between static and dynamic loading. CO1 BL2 [2]

f. Define natural frequency and mode shape. CO3 BL1 [2]

g. Write steps involved in stodola method. CO4 BL2 [2]

h. List out possible end conditions of beams in continuous system. CO4 BL1 [2]

i. Write difference between wind load and moving load. CO5 BL2 [2]

j. Define blasting. CO5 BL1 [2]

PART – B

(Answer ALL questions. All questions carry equal marks)

5 * 10 = 50 Marks

2. A SDOF system consists of a mass 420 kg and a spring stiffness of 250 kN/m. By testing it was found that a force of 100 N produces a relative velocity 12 cm/s. Determine: (i) Damping ratio. (ii) Damped frequency. (iii) Logarithmic decrement. (iv) Ratio of two consecutive amplitudes. [10] CO1 BL3

OR

3. **(a)** Derive an expression for Logarithmic decrement of free longitudinal vibrations of a spring-mass-damper SDOF system. [10] CO1 BL3

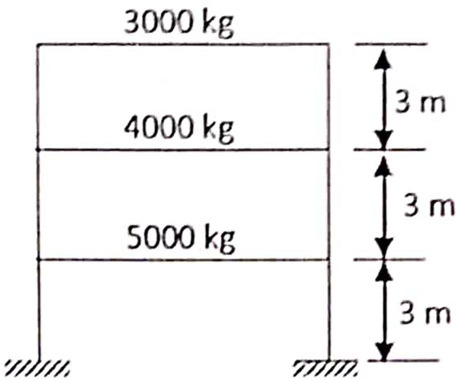
(b) Explain about Duhamel integral. CO2 BL2

4. Derive an expression for the steady state response of an SDOF un-damped system subjected to harmonic excitation force. [10] CO2 BL3

OR

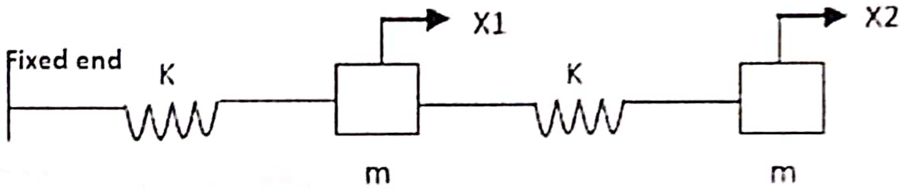
5. Determine the natural frequency and natural period of the system consisting of a mass of 120 kg attached to a horizontal cantilever beam through the linear spring K. The cantilever beam has a thickness of 0.8 cm and a width of 1.2 cm. Take E=2.1×10^6 kg/cm^2, L=70 cm and K= 10 kg/cm. [10] CO2 BL3

6. Determine the natural frequencies and mode shapes of the given MDOF system. Take EI = 4.5×10^6 N-m^2 for all the columns. [10] CO3 BL5

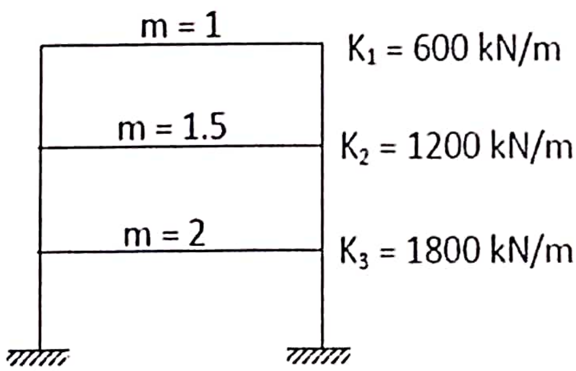


OR

7. Determine the natural frequencies and mode shapes for the system shown. Sketch the mode shapes and check the orthogonality conditions with respect to the stiffness and mass matrices. [10] CO3 BL5

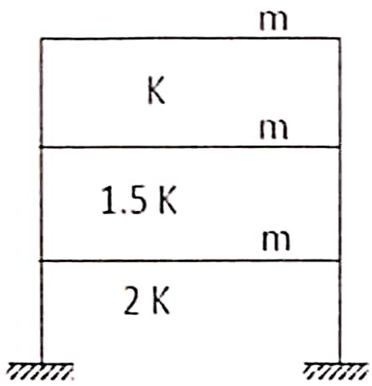


8. Determine the natural frequency and mode shape for the shear building by Stodola method. [10] CO4 BL5



OR

9. Determine the fundamental natural frequency and mode shape using Holzer method. [10]



CO4
BL5

10. Explain about dynamic effects of wind and moving load on the structure and illustrate with example. [10]

CO5
BL4

OR

11. Explain the concept of an excitation by rigid base translation with example. [10]

CO5
BL4

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GR 22 | SET - 1

M.Tech I Year II Semester Regular Examinations, July 2022

STRUCTURAL DYNAMICS

(M.Tech. Structural Engineering)

Time: 3 hours | **MODEL PAPER** | **Max Marks: 70**

< **Note: T**ype the questions in the given format only, Times New Roman font , size 12 >

Instructions:

1. Question paper comprises of **Part-A** and **Part-B**
2. **Part-A** (for 20 marks) must be answered at one place in the answer book.
3. **Part-B** (for 50 marks) consists of **five questions with internal choice,** answer all questions.

PART – A

(Answer ALL questions. All questions carry equal marks)

2 * 10 = 20 Marks

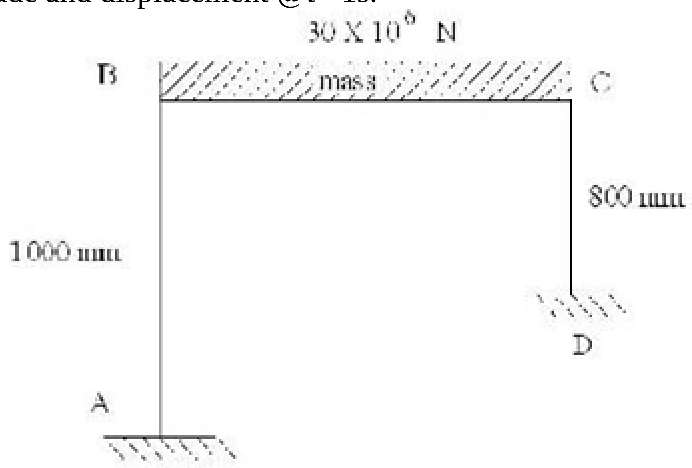
		Marks	CO	BL	PI
1. a.	Define Logarithmic Decrement. Sketch and show how Logarithmic decrement varies with damping ratio.	[2]	1	1	1.1.1
b.	State the D’Alembert’s principle	[2]	1	1	1.1.1
c.	What is dynamic degrees-of-freedom? What are the essential characteristics of a dynamic loading?	[2]	2	2	1.1.2
d.	Write the general equation of motion for an SDOF system.	[2]	2	1	1.1.2
e.	Write the expression for Impulsive loading of Duhamel integral?	[2]	3	1	2.1.1
f.	What are basic assumptions made in STODLA and HOLZER methods?	[2]	3	1	2.1.1
g.	Derive the equation of motion for flexural vibrations of beams.	[2]	4	2	1.1.1
h.	Explain Normalization of modes.	[2]	4	2	3.3.1
i.	What are the methods of combining storey shear force mentioned in the IS 1893 part-1:2002?	[2]	5	1	2.1.1
j.	What do you understand about lumped mass approach?	[2]	5	2	2.1.1

PART – B

(Answer ALL questions. All questions carry equal marks)

10 * 5 = 50 Marks

2.	A Vibrating system consisting of a weight of W= 10lb and a spring with stiffness k= 20lb/in in viscously damped so that the ratio of two consecutive amplitudes is 1.00to 0.85. Determine: a) the natural frequency of the undamped system, b) the logarithmic decrement c) the damping ratio.	[10]	**1**		

OR					
3.		**[10]**	**1**		
4.	A SDF system having viscous damping has a spring of stiffness 500 N/m. When the weight is displaced and released the period of vibration is 2 s and ratio of successive amplitudes is 4 to 1. Determine the Amplitude of the motion and the Phase-angle when a force is applied to the system?	**[10]**	**3**		
OR					
5.	Evaluate the algorithm for step-by-step solution for elasto plastic single degree of freedom system.	**[10]**	**3**	**5**	
6.	a) Calculate the natural angular frequency of the frame shown in figure. Compute also natural period of vibration. If the initial displacement is 25 mm and initial velocity is 25 mm/s what is the amplitude and displacement @t =1s.  **b**) Explain the numerical evaluation of 'DUHAMEL INTEGRAL' for an undamped system.	**[10]**	**3** **3**		
OR					
7.	a.) Discuss the orthogonality conditions for the modes of vibration of a system. b.) How does a non- linear MDOF system respond to dynamic forces? Explain	**[10]**	**4** **5**		
8.	Create a Nonlinear MDOF model and evaluate it by Wilson – θ method and New-mark Beta method.	**[10]**	**4**	**6**	
OR					
9.	Derive the governing differential equation for transverse flexural vibration of a continuous system subjected to dynamic loading. Apply the above equation for a beam with one end fixed and other end is free.	**[10]**	**4**		
10.	A three storey RC building is situated in Delhi. The D.L and L.L is lumped at respective floors the soil below the foundation is assumed to be hard rock. Assume the building is used for office purpose. Determine the total base shear and base moment and distribute the	**[10]**	**5**		

	base shear along the height of the building in Fig.1 (m1 = 1.0 $kN.s^2/cm$; m2 = 1.5 $kN.s^2/cm$; m3 = 2.0 $kN.s^2/cm$; k 1 = 600 kN/cm; k 2 = 1200 kN/cm; k 3 = 1800 kN/cm)				
	OR				
11.	Detail the procedures of Response Spectrum analysis and Time History methods of seismic analysis for obtaining response of multi-storied buildings?	**[10]**	**5**		

*****

Class Test 1

1 One of the construction companies hires you to determine the dynamic properties of a frame system for which it has lost the original blue prints. Being a civil engineer, you were assigned to do a free vibration test of the frame system. Supplied with a hydraulic jack, you were able to apply a jacking force to displace the frame. With a jacking force of 134 kN, you noted down that the frame has displaced 0.76 cm. On the first return swing after release, the frame did not come back to the release point but rather it stopped at 0.64 cm towards it. You recorded time between the release and the first return as 2 sec. Determine the following

a. Natural frequency
c. Logarithmic decrement
d. Damping ratio
e. Damping frequency
f. Amplitude of the frame after 6 cycles

2 Derive the equation of motion for the 3-degree of freedom system.

1. The number of cycles completed in a unit time is called?
   a) Frequency b) Time Period c) Amplitude d) Resonance

2. When frequency of the exciting force is equal to the natural frequency of the system it is called?
   a) Frequency b) Time Period c) Amplitude d) Resonance

3. The number of independent coordinates which are required to define the motion of the body or system at given instant.?
   a) Frequency b) Mode shapes c) Degrees of Freedom d) Degrees of redundancy

4. Simple Harmonic Motion is represented by an expression
   a) x = X sin ωt b) x = X cos ωt c) x = X sin ωt cos ωt d) aω

5. Methods to analyse an undamped system is based on
   a) Newton's II Law b) D'Alembert's Principle c) Energy Method d) Rayleigh's Method

6. Critical damping coefficient Cc is equal to
   a) 2√ km b) $2m\omega_n$ c) 2km d) C/ζ

7. The ratio of damping coefficient (c) to the critical damping coefficient is called
   a) Damping factor b) safety factor c) amplification factor d) mass factor

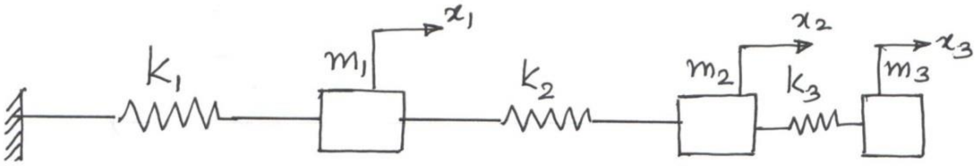
8. $\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$ **is called**

a) Logarithmic decrement b) logarithmic increment c) semi-logarithmic decrement d) damping ratio

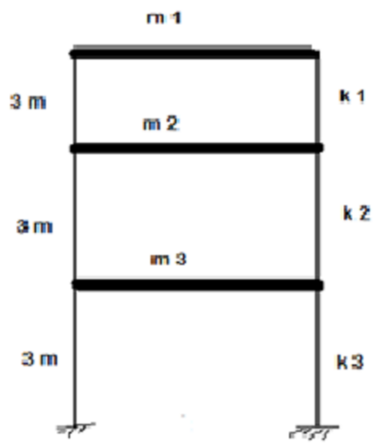
9. The vibrations of a system that take place due to the application of an excitation (or) dynamic load are called a) forced vibrations b) free vibrations c) Resonance d) damped vibrations

Long Answer Questions

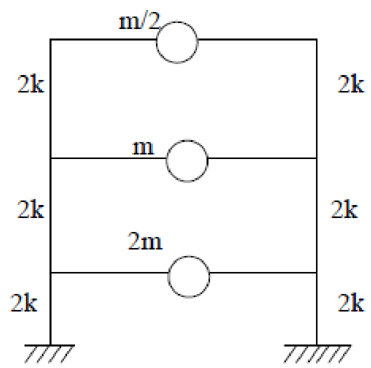
1. One of the construction companies hires you to determine the dynamic properties of a frame system for which it has lost the original blue prints. Being a civil engineer, you were assigned to do a free vibration test of the frame system. Supplied with a hydraulic jack, you were able to apply a jacking force to displace the frame. With a jacking force of 134 kN, you noted down that the frame has displaced 0.76 cm. On the first return swing after release, the frame did not come back to the release point but rather it stopped at 0.64 cm towards it. You recorded time between the release and the first return as 2 sec. Determine the following
   a. Natural frequency
   c. Logarithmic decrement
   d. Damping ratio
   e. Damping frequency
   f. Amplitude of the frame after 6 cycles
2. Model multi degrees of freedom discrete parameter system and formulate its equation of motion.
3. Examine whether the log – decrement is also given by the equation δ = 1/n log (U0/ Un) represents the amplitude after n cycles have elapsed or Derive an expression for the Logarithmic Decrement.
4. Formulate equations of motion for SDOF systems using 1) D'Alembert's Principle 2) Newton’s Law and 3) Energy method
5. Derive the equation for the Displacement of Damped Free Vibrations for SDOF system. Derive the expressions for the following cases.
   i) Over Damped ii) Under Damped iii) Critically Damped
6. Determine the Natural Frequencies of the system shown in figure below. Assume $m_1 = m_2 = m_3 = 1kg$ $K_1 = K_2 = K_3 = 1N/m$ .Use Holzer‘s Method.



7. State and prove the orthogonality property of mode shapes.
8. Derive the governing differential equation for transverse flexural vibration of a continuous system subjected to dynamic loading. Apply the above equation for a beam with one end fixed and other end is free. Sketch the three mode shapes
9. Discuss the Uncoupling of the MDOF equations of motion or Using normal mode theory, explain the method for uncoupling the equations of motion of MDOF system.
10. Using STODOLA method of vibration analysis, determine the fundamental frequency of vibration and the corresponding mode shapes for idealized 3 storey shear building for the given data : m1 = 1.0 kN.s^2/cm; m2 = 1.5 kN.s^2/cm; m3 = 2.0 kN.s^2/cm; k 1 = 600 kN/cm; k 2 = 1200 kN/cm; k 3 = 1800 kN/cm



11. Explain the mode superposition procedure of obtaining the response of the MDOF system
12. Describe how the equation of motion can be set up using Newton's second law of motion for the system subjected to earthquake horizontal ground acceleration u_g.
13. Obtain the expression for dynamic magnification factor for damped harmonic excitation
14. A structure is modelled as a viscously damped oscillator with a spring constant k=5900 kN/m and un-damped natural frequency ω_n =25 rad/s. Experimentally it was found that a force of 0.5 kN produced a relative velocity of 50 mm/s in the damping element. Determine: (i) the damping ratio ξ; (ii) the damped period T_d; (iii) the logarithmic decrement of damping δ; and (iv) the ratio between two consecutive amplitudes.
15. Discuss the solutions of Eigen value/vector problems to determine natural frequencies and mode shapes
16. Solve for Natural frequencies, Time periods and Modes of Vibration of the multistory building shown in Fig (a) as an Eigen value problem. Assume $m = 5 \times 10^4$ kg, k= 5×10^4 kN/cm.



17. Write short notes on
a) dynamic effects of Wind Loads
b) dynamics effects of Moving Loads and Vibrations caused by Traffic
c) dynamic effects of Blasting and Pile Driving

Short Answer questions

1. What are the elements of vibratory system?
2. Determine the equivalent stiffness of a typical spring-mass system having springs are in series and parallel combination
3. Explain about Critical damped, Over damped and under damped SDOF systems
4. Define ‘Dynamic Magnification Factor’
5. Define ‘Phase Angle’
6. What are Normal coordinates and Geometric coordinates?
7. Differentiate the continuous mass and lumped mass or discrete approach
8. Define Logarithmic Decrement.
9. What are Forced and free vibrations ?
10. Explain the D’Alembert’s principle
11. What is Dynamic degrees-of-freedom mean?
12. What are various types of Dynamic loadings.
13. What are SDOF and MDOF systems mean?
14. Expression for Duhamal integral
15. What is Natural frequency and Time period?
16. What is Simple harmonic motion mean?
17. What is Mode shape>
18. Represent SHM vectorially

TEST YOUR KNOWLEDGE 1

1. If 1500 g of water is required to have a cement paste 1875 g of normal consistency, the percentage of water is,
2. W_p and W_f are the weights of a cylinder containing partially compacted and fully compacted concrete. If the compaction factor   is 0.95, the workability of concrete is
3. To obtain cement dry powder, lime stones and shales or their slurry, is burnt in a rotary kiln at a temperature between
4. What percentage of aggregates are required in concrete in terms of volume?
5. What is factor of safety and partial factor of safety?
6. Why Partial safety factor for concrete and steel are 1.5 and 1.15 respectively?
7. What are the 4 sources of errors in the finite element method?
8. What does homogeneous and isotropic material mean?
9. What is the maximum strain of concrete before reaching failure?
10. What is the maximum strain value in concrete in case of limit state of flexure?
11. Does stress cause strain or does strain cause stress? Explain
12. How membrane analogy (soap film) is useful in torsional analysis?
13. Why stiffness method is preferred over flexibility method?
14. What is stress and strain tensor?
15. Who is the Chairman of disaster management?
16. Why does water reduce concrete strength?
17. Why is the working stress method still used for the construction of a water tank?
18. How is underwater concreting done?
19. Do hidden beams really transfer the load?
20. How does transfer beam work?
21. What is the difference between drop beam and inverted beam?
22. What is the main benefit of trapezoidal footing then rectangular footing?
23. Why hollow section is stronger than I section in a column?
24. What is depth of deep beam?
25. What are the advantages and disadvantages of waffle slab wrt to solid slab?

Note: Can Google for information but don't copy as it is write what you understood.

NO CUT AND PASTE or Copying as it is.

Happy Learning!!!

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GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

Department of Civil Engineering

I M.Tech. II Semester (Mid- I Examination)

STRUCTURAL DYNAMICS

(Subjective)

Time: 75 Minutes **MODEL PAPER** *Max. Marks: 15 Marks*

Name : ____________________________ Roll No. | | | | | | D | | | | |

Answer all questions

	M	CO	BL	PI
1. a) One of the construction companies hires you to determine the dynamic properties of a frame system for which it has lost the original blue prints. Being a civil engineer, you were assigned to do a free vibration test of the frame system. Supplied with a hydraulic jack, you were able to apply a jacking force to displace the frame. With a jacking force of 134 kN, you noted down that the frame has displaced 0.76 cm. On the first return swing after release, the frame did not come back to the release point but rather it stopped at 0.64 cm towards it. You recorded time between the release and the first return as 2 sec. Determine the following a. Natural frequency c. Logarithmic decrement d. Damping ratio e. Damping frequency f. Amplitude of the frame after 6 cycles	5	1	3	3.3.1 5.1.1
OR				
b) Examine whether the log – decrement is also given by the equation $\delta = 1/n \log (U_0/ U_n)$ represents the amplitude after n cycles have elapsed.	5	1	4	3.3.1
2. a) Formulate equations of motion for SDOF systems using 1) D'Alembert's Principle 2) Newton's Law and 3) Energy method	5	2	2	4.1.1
OR				
b) Find the solution for the vibration of a SDOF system with viscous damping	5	2	4	4.1.3
3. a) Model multi degrees of freedom discrete parameter system and formulate its equation of motion.	5	3	3	3.3.1 4.1.2
OR				
b) Derive an expression for the force transmitted to the foundation and phase angle for a damped oscillator idealized as a SDOF system subjected to harmonic force.	5	2	2	4.1.2

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GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

Department of Civil Engineering

I M.Tech. II Semester (Mid- I Examination)

STRUCTURAL DYNAMICS

(Objective)

Time: 15 Minutes **MODEL PAPER** *Max. Marks: 5 Marks*

Name: ___________________________ Roll No. | | | | | | D | | | | |

	M	CO	BL	PI
1. Mass = 10 kg, K = 10 N/m, C = 10 Ns/m. The system is a) Overdamped b) Underdamped c) Critically damped d) Viscous damped	1	1	3	4.1.3
2. Damping ratio is 0.5%, Logarithmic decrement is a) 0.03 b)0.5 c) 36.2 d) 2.5	1	1	4	4.1.3
3. Natural undamped frequency is 20 rad/s, damping ratio is 50%, Damped frequency is a) 20 rad/sb) 10 rad/s c) 5 rad/s d) 17.3 rad/s	1	1	4	4.1.3
4. ________ is the graphical representation of the relative amplitudes of the two coordinates and their phase angle relationship. a) Stiffness b) Mode shape c) Node d) Flexibility	1	1	2	2.1.1
5. For which case there is an oscillatory motion a) Overdamped system b) Underdamped system c) Critically damped system d) undulated damped system	1	2	2	2.1.1
6. Response due to dynamic load is (Chose multiple answers) a) Displacement b) Velocity c) Acceleration d) Momentum	1	1	2	2.1.1
7. The number of independent coordinates required to specify configuration of vibrating system in space at any instant of time a) Degrees of freedom b) Nodes c) global coordinates d) Degree of redundancy	1	1	2	2.1.1
8. In mass-spring-damper model of the structural mass, the mass element signifies a) the mass and the inertial characteristics b) the potential energy of and the stiffness characteristics c) the energy dissipation characteristics d) excitation force, F(t) which is obviously a function of time	1	1	2	2.1.1
9. If all the springs displace by the same amount, then the springs are in a) Parallel b) Series c) out of phase d) phase	1	2	2	2.1.1
10. To transform the building structure into a discrete number of degree of freedom with lumped masses at the floor level, following assumptions are necessary. (Chose multiple answers) a) The entire mass of the building is concentrated at the floor levels. b) The axial forces do not contribute significantly for the deformation of structures and hence the stiffness c) The floors with slabs and beams are infinitely rigid as compared to the columns and remain horizontal without rotation. d) The entire mass of the building is treated as continuous system.	1	3	2	2.1.1

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GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

Department of Civil Engineering

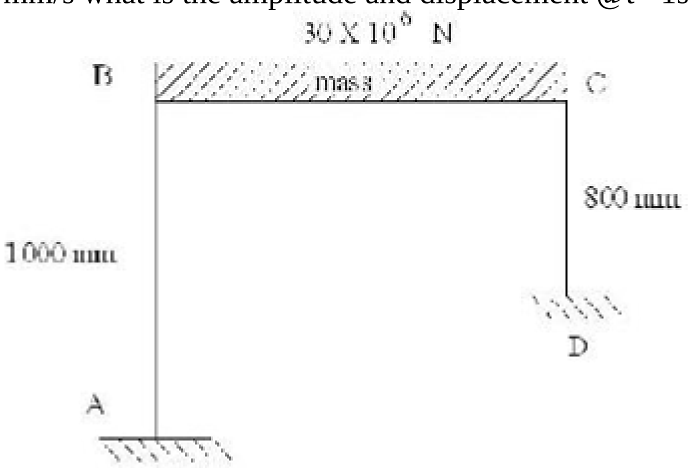
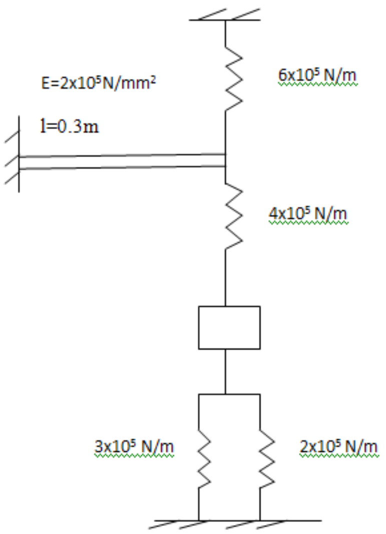
I M.Tech. II Semester

STRUCTURAL DYNAMICS

Assignment 1

Name : ____________________________ Roll No. | | | | | | D | | | | |

Answer all questions

<table>
<tr><th></th><th>M</th><th>CO</th><th>BL</th><th>PI</th></tr>
<tr><td>1. A Vibrating system consisting of a weight of W= 10lb and a spring with stiffness k= 20lb/in in viscously damped so that the ratio of two consecutive amplitudes is 1.00to 0.85. Determine: a) the natural frequency of the undamped system, b) the logarithmic decrement c) the damping ratio.</td><td>1</td><td>1</td><td>3</td><td>3.3.1</td></tr>
<tr><td>2. Calculate the natural angular frequency of the frame shown in figure. Compute also natural period of vibration. If the initial displacement is 25 mm and initial velocity is 25 mm/s what is the amplitude and displacement @t =1s. [CO2]

</td><td>1</td><td>1</td><td>3</td><td>3.3.2</td></tr>
<tr><td>3. Find the effective stiffness for the following system.

</td><td>1</td><td>1</td><td>3</td><td>4.1.2</td></tr>
</table>

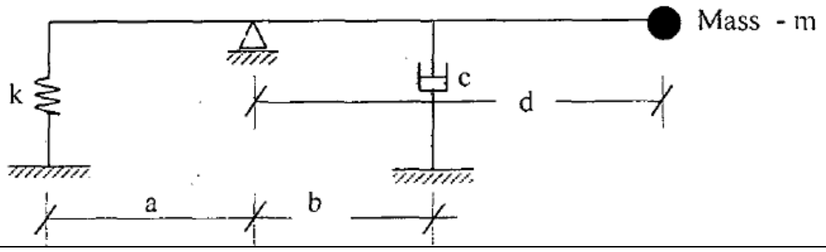
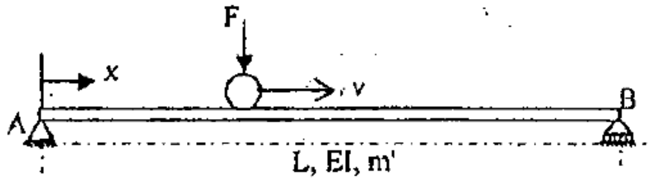
<table>
<tr>
<td>4. Derive the Equilibrium equation of motion for the structural system shown in Fig. below. Find out the natural frequency of the system.

</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>4.1.3</td>
</tr>
<tr>
<td>5.
For the simplified analysis of the response of a bridge to moving loads the bridge deck is idealized as a simply supported beam of span L, mass per unit length m' and flexural rigidity EI. A single wheel load of magnitude F (neglect the mass of the wheel) traverses the bridge at a speed of v. Assume a displacement shape function given by $\varphi(x) = \sin(\pi x/L)$, obtain the equation of motion for flexural vibration of the deck.


FIGURE -1</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>3.3.1
4.1.2</td>
</tr>
</table>

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GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

Department of Civil Engineering

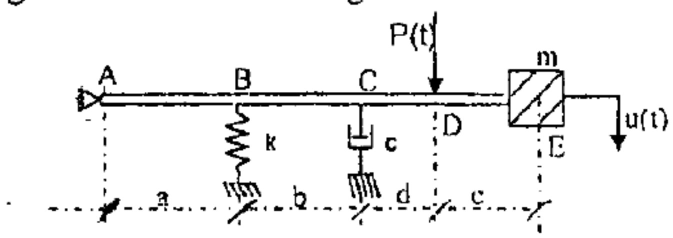
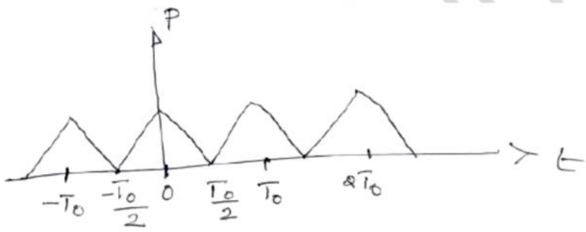
I M.Tech. II Semester

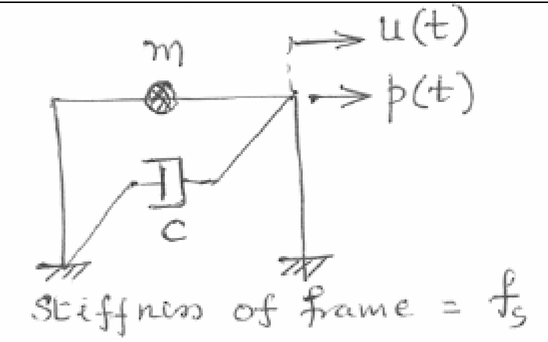
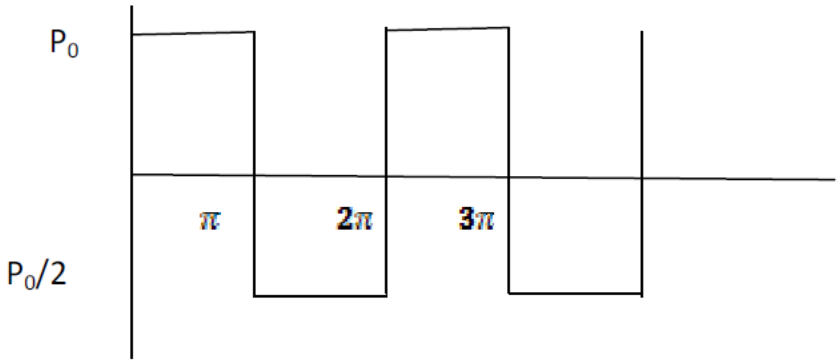
STRUCTURAL DYNAMICS

Assignment 2

Name : ____________________________   Roll No. | | | | | | D | | | | |

Answer all questions

	M	CO	BL	PI
1. Use the Laplace Transform method to determine the response of a damped SDOF system with natural frequency ω_n, damping factor ξ and mass m, initially at rest in equilibrium and subject to step excitation force $F(t) = F_0$.	1	2	3	3.3.2
2. Develop the equation of motion for the system shown below in figure 2 modeling as SDOF system. Rigid mass less bar of length ℓ:  FIGURE -2	1	2	3	3.2.1
3. An SDF system with natural period T_n and damping ratio 3 is subjected to a periodic force as shown in figure, with an amplitude P_0 and period T_0. (i) Evaluate the forcing function in its Fourier series. (ii) Determine the steady state response of an un-damped system. 	1	2	4	3.3.1 4.1.3
4. Describe how the equation of motion can be set up using Newton's second law of motion for the system shown in Fig1 below (Assume the stiffness of the system is F_s)	1	2	3	3.3.1

 Fig 1				
5. Derive the Fourier series expression for periodic loading and write the expression for steady state response.  $p(t) = \begin{cases} p_0 & 0 < t < \pi \\ \frac{p_0}{2} & \pi < t < 2\pi \end{cases}$	1	2	3	3.3.1

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GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

Department of Civil Engineering

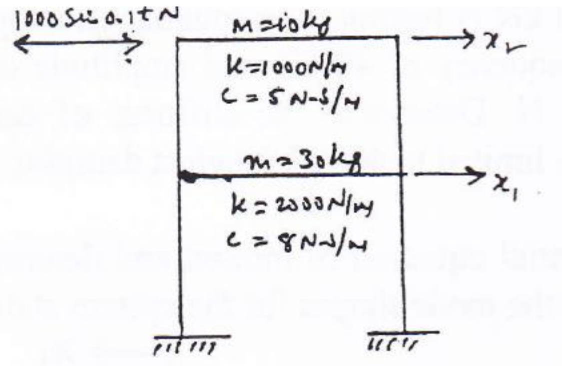
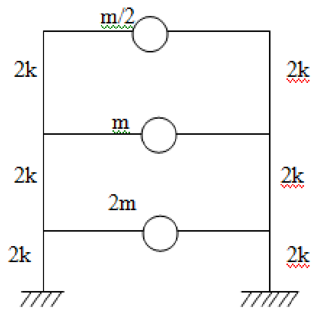
I M.Tech. II Semester

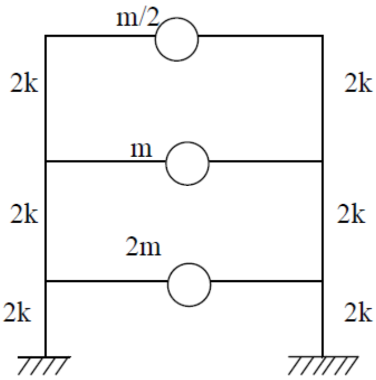
STRUCTURAL DYNAMICS

Assignment 3

Name : ____________________________ Roll No. | | | | | | D | | | | |

Answer all questions

	M	CO	BL	PI
1. For the two storey damped shear building shown in Fig.7, determine the response of all the floors using modal super position method. The data is as follows : $\begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix} = \begin{Bmatrix} 6.500 \\ 12.56 \end{Bmatrix}$ rad/sec $[M] = \begin{bmatrix} 30 & 0 \\ 0 & 10 \end{bmatrix}$ kg $[C] = \begin{bmatrix} 13 & -5 \\ -5 & 5 \end{bmatrix} \frac{N-sec}{M}$ $[K] = \begin{bmatrix} 3000 & -1000 \\ -1000 & 1000 \end{bmatrix} N/M$ $[\phi] = \begin{bmatrix} 1000 & 1000 \\ 1.732 & -1.732 \end{bmatrix}$. Fig.7 	1	3	4	3.3.2
2. For the multistory building shown in fig, Obtain the natural frequencies .Assume m =1kN -sec^2/cm, k= 600 kN/cm. 	1	3	3	4.2.2
3. (i) Derive for the response of a triangular impulse by differential equation method. (ii) Using Duhamel's integral derive for displacement of a rectangular impulse.	1	3	3	3.3.1
4. For the multistoried building shown in fig.2. Obtain the frequencies and modes of vibration using HOLZER method. Assume $m = 5 \times 10^4$ kg, k= 5×10^4 kN/cm.	1	3	4	4.2.2

				
5. Explain the mode superposition procedure of obtaining the response of the MDOF system. Derive the orthogonality condition of natural modes of vibration	1	3	3	3.2.1

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GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

Department of Civil Engineering

I M.Tech. II Semester

STRUCTURAL DYNAMICS (GR20D

Assignment 4

Name : ___________________________ Roll No. | | | | | | D | | | | |

Answer all questions

	M	CO	BL	PI
1. Find the first three natural frequencies of vibrations and the corresponding mode shapes for a fixed beam of span 'L' if it has uniform cross section	1	4	3	3.3.2
2. Discuss the orthogonality conditions for the modes of vibration of a system.	1	4	2	4.1.1
3. Derive the governing differential equation for transverse flexural vibration of a continuous system subjected to dynamic loading. Apply the above equation for a beam with one end fixed and other end is free	1	4	3	4.1.2
4. A weight W is suspended from the mid span of a simply supported beam of span L, flexural rigidity EI and mass per unit length m. If the wire by which the weight w is suspended suddenly snaps. (a) Describe the subsequent vibration of the beam. Neglect damping. (b) Find its deflection expression at the centre.	1	4	4	3.3.1
5. Derive the governing differential equation for transverse flexural vibration of a continuous system subjected to dynamic loading.	1	4	3	3.3.1

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GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

Department of Civil Engineering

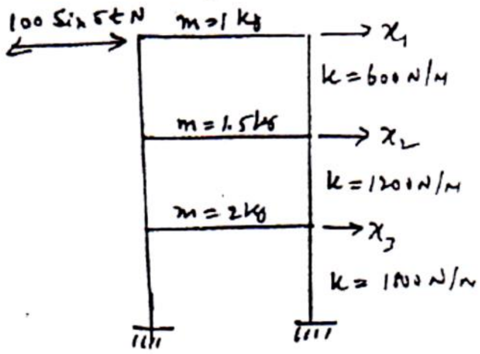
I M.Tech. II Semester

STRUCTURAL DYNAMICS

Assignment 5

Name : ___________________________ Roll No. | | | | | | D | | | | |

Answer all questions

	M	CO	BL	PI
1.A three storey RC building is situated in Delhi. The D.L and L.L is lumped at respective floors the soil below the foundation is assumed to be hard rock. Assume the building is used for office purpose. Determine the total base shear and base moment and distribute the base shear along the height of the building in Fig.1 (m1 = 1.0 kN.s2/cm; m2 = 1.5 kN.s2/cm; m3 = 2.0 kN.s2/cm; k 1 = 600 kN/cm; k 2 = 1200 kN/cm; k 3 = 1800 kN/cm)	1	5	4	3.3.1
2.A 5-story building is to be constructed in the area of seismic zone III having medium soil. The dimension of the building is 15m × 20m. The height of each story is 3.5m. The live and dead load on each floor is 2.5 kN/m2 and 10 kN/m2, respectively. The live and dead load on the roof is 1.5 kN/m2 and 5 kN /m2, respectively. Take importance factor as 1 and response reduction factor as 5. Determine the seismic shear force in each story and overturning moment at the base as per IS: 1893 (Part 1)-2002. Take the value of Z=0.16 for Zone III and spectral acceleration for medium soil from IS: 1893 (Part 1)-2002 as $\frac{S_a}{g} = \begin{cases} 1+15T \ \ for \quad 0 \le T \le 0.1 \\ 2.5 \quad 0.1 \le T \le 0.55 \\ 1.36/T \quad 0.55 \le T \le 4 \end{cases}$	2	5	4	3.3.2
3 For the three storey shear building shown in Fig. , determine the response x_1 using modal superposition method. The natural frequencies, mass matrix and stiffness matrix are as follows : $\begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix} = \begin{Bmatrix} 14.522 \\ 31.048 \\ 46.100 \end{Bmatrix}$ rad/sec $\quad [M] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ kg $\quad [K] = \begin{bmatrix} 600 & -600 & 0 \\ -600 & 1800 & -1200 \\ 0 & -1200 & 3000 \end{bmatrix}$ N/M Modal matrix $[Q] = \begin{bmatrix} 1 & 1 & 1 \\ 0.6486 & -0.6066 & -2.5405 \\ 0.3018 & -0.6790 & 2.4382 \end{bmatrix}$. 	2	5	4	3.3.1

GR22

GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

Department of Civil Engineering

I M.Tech. II Semester (Mid- II Examination)

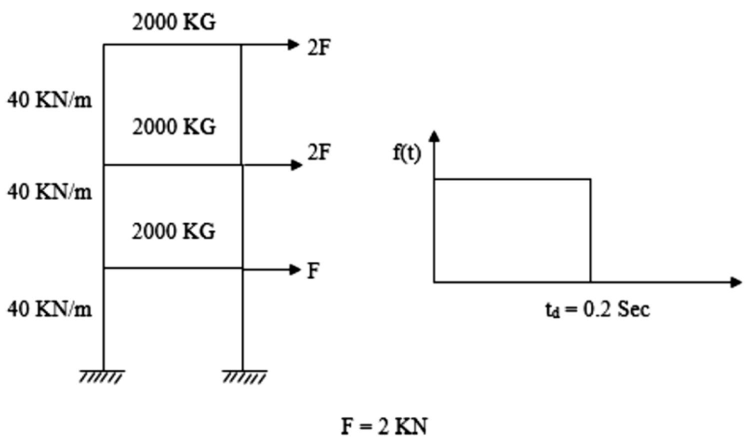
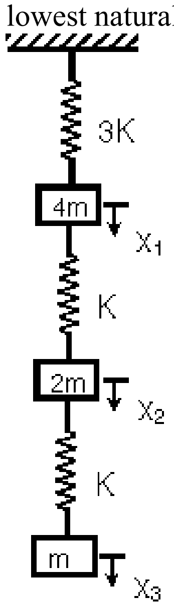
STRUCTURAL DYNAMICS

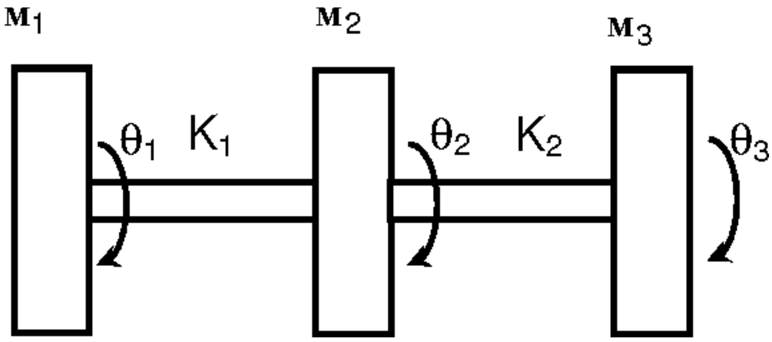
(Subjective)

Time: 75 Minutes **MODEL PAPER** *Max. Marks: 15 Marks*

Name : ____________________________ Roll No. | | | | | | D | | | | |

Answer all questions

	M	CO	BL	PI
1. a) Draw the mode shapes for given problem. 	5	3	3	3.3.2
OR				
b) Derive the natural frequency and mode shapes for uniform beam having both end simply supported.	5	4	3	3.3.1
2. a) For the given system, find the lowest natural frequency by Stodola's method 	5	4	4	4.1.3
OR				
b) Derive the natural frequency and mode shapes for uniform beam having one end fixed other end simply supported.	5	4	3	4.1.2

3. a) For the system shown in figure, obtain natural frequencies using Holzer's method? 	5	4	4	4.1.3
OR				
b) A three storeyed symmetrical RC school building situtated at Bhuj with following data: Plan dimension : 7 m Storey height : 3.5 m Total weight of beams in a storey : 130 kN Total weight of slab in a storey : 250 kN Total weight of columns in a storey : 50 kN Total weight of walls in a storey : 530 kN Live load : 130 kN Weight of terrace floor : 655 kN The structure is resting on hand rock. Determine the total base shear and lateral loads at each floor level for 5% of damping using seismic coefficient method	5	5	4	4.2.2

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GR22

GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

Department of Civil Engineering

I M.Tech. II Semester (Mid- II Examination)

STRUCTURAL DYNAMICS

(Objective)

Time: 15 Minutes **MODEL PAPER** *Max. Marks: 5 Marks*

Name: ___________________________ Roll No. | | | | | | D | | | | |

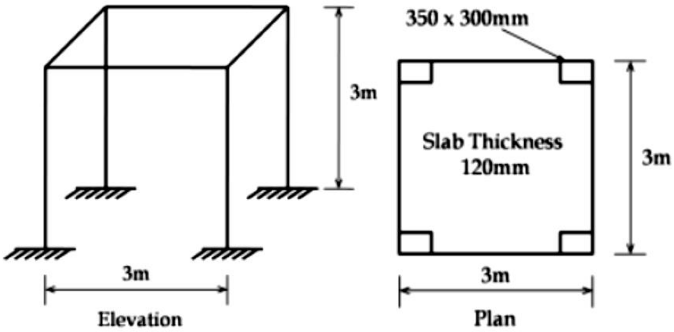
	M	CO	BL	PI
1. IS Code used for seismic analysis? a) IS 1893 b) IS 456 c) IS 1983 d) IS 875	1	5	1	2.1.1
2. Which one is linear dynamic analysis? a) Equivalent static method b) Push over analysis c) Response spectrum method d) Time history method	1	5	2	2.1.1
3. IS Code for wind analysis? a) IS 875-Part III b) IS 875-Part II c) IS 875-Part I d) IS 875-Part IV	1	5	1	2.1.1
4.Wind Load Explanatory Hand Book a)SP 34 b) SP 64 c) SP 16 d) SP 40	1	5	1	2.1.1
5.Phenomenon which affects the design of a tall building is a) Vortex shedding b) Gust c) Galloping d) Flutter	1	5	2	1.2.1
6.Single high pressure impulses acting directly on the exterior envelope over milliseconds causing localized damage is called a) Blast loads b) Seismic loads c) Impact loads d) Live loads	1	5	2	4.1.1
7.Design of machine foundations can be categorized into three approaches a) Static Analysis with Rule of Thumb b) Natural Frequency Analysis c) Forced Vibration Analysis d) Time history analysis	1	5	1	3.3.1
8.Only fundamental natural frequency and model vector of vibration are found in a) Stodola method b) Holzer method c) Wilson method d) Newmark method	1	4	2	2.1.1
9.A pattern of motion in which all parts of the system move sinusoidally with the same frequency and with a fixed phase relation is called a) Normal mode b) eigen value c) eigen vector d) General mode	1	3	2	3.3.1
10.The most general motion of a system is a superposition of its normal modes. a) True b) False c) if not orthogonal d) not always	1	3	2	2.1.1

***

ASSIGNMENT -1

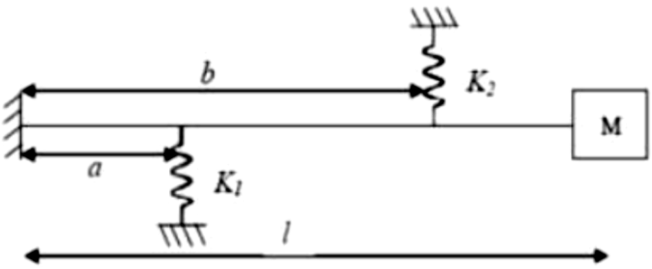
1.

A building consists of 4 columns of size *350 x 300mm*, with bay width as *3m*. The slab thickness is *120mm*, and floor to floor height is *3m*. Considering the grade of concrete as *M25*, find the natural frequency and time period of building in two orthogonal directions.



2.

Write the equation of motion for the system below



3.

One of the construction companies hires you to determine the dynamic properties of a frame system for which it has lost the original blue prints. Being a civil engineer, you were assigned to do a free vibration test of the frame system. Supplied with a hydraulic jack, you were able to apply a jacking force to displace the frame. With a jacking force of *134 kN*, you noted down that the frame has displaced *0.76 cm*. On the first return swing after release, the frame did not come back to the release point but rather it stopped at *0.64 cm* towards it. You recorded time between the release and the first return as 2 sec. Determine the following

a. Weight of the frame
b. Natural frequency
c. Logarithmic decrement
d. Damping ratio
e. Damping frequency
f. Amplitude of the frame after 6 cycles

ASSIGNMENT -1

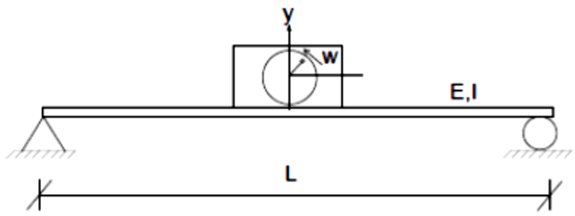
4.

The vertical suspension system of an automobile is idealized as a viscously damped SDG system. Under the *1360 kg* weight of the car the suspension system deflects *5.08 cm*. The suspension is designed to be critically damped.

a. Calculate the damping and stiffness coefficients of the suspension
b. With four *64 kg* passengers in the car, what is the effective damping ratio?
c. Calculate the natural vibration frequency for case (b).

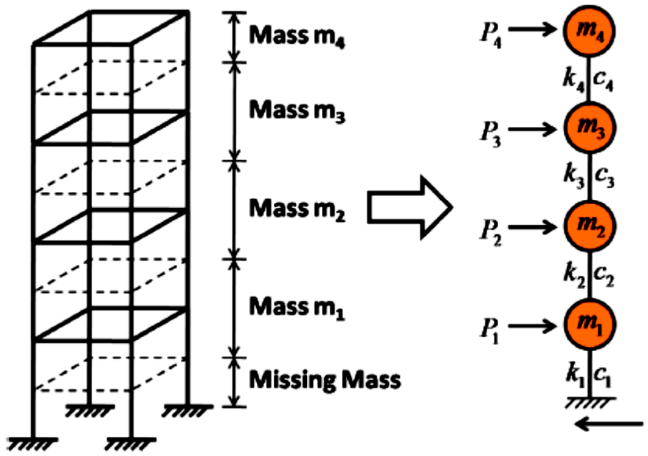
5.

A simple beam supports at its centre a machine having a weight w= 7300 kg. The beam is made up of two standard S 8 x 23 sections with a clear span L=3.67m and total cross sectional moment of inertia I=5345cm^4. The motor runs at 300 rpm, and its rotor is out of balance to the extent of W = 18.15kg at a radius of e_o=25.4cm. What will be the amplitude of the steady state response if the equivalent viscous damping for the system is assumed 10% of the critical?



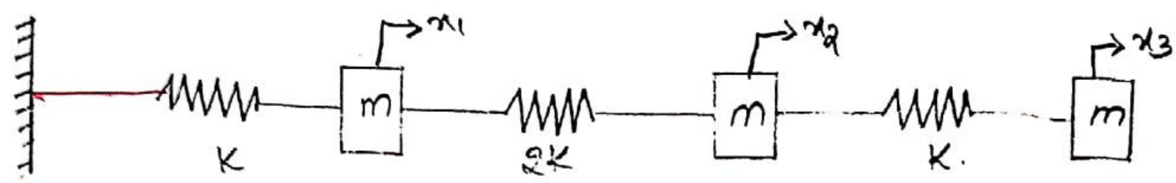
6.

Derive the equation of motion for the multi degree of freedom system shown in the figure 1.



STRUCTURAL DYNAMICS
Final Assignment

Q.No.		Marks
1. a)	Classify vibrations	(2)
b)	Define 'Phase angle'	(2)
c)	Explain the significance of the D'Alembert's principle	(2)
d)	Explain the importance of Mode shape in structural dynamics?	(2)
e)	Express the equation of motion for SDOF system subjected to the seismic excitation.	(2)
f)	Illustrate the simple vibratory SDOF system with damper	(2)
g)	Represent SHM vectorially	(2)

2	(a)	Formulate equations of motion for SDOF systems using 1) D'Alembert's Principle 2) Newton's Law and 3) Energy method	(7)
	(b)	A structure is modelled as a viscously damped oscillator with a spring constant K=5900 kN/m and un-damped natural frequency ω_n =25 rad/s. Experimentally, it was found that a force of 0.5 kN produced a relative velocity of 50 mm/s in the damping element. Determine: (i) the damping ratio ξ; (ii) the damped period T_d; (iii) the logarithmic decrement of damping δ; and (iv) the ratio between two consecutive amplitudes	(7)
3	(a)	Derive the expressions for about Critical damped, Over damped and under damped SDOF systems	(7)
	(b)	Derive an expression for undamped forced vibration of SDOF subjected to harmonic excitation.	(7)
4	(a)	State and prove the orthogonality property of mode shapes	(7)
	(b)	Model multi degrees of freedom discrete parameter system and formulate its equation of motion.	(7)
5	(a)	Develop Lagrange's equations of motion	(7)
	(b)	Determine the Natural Frequencies of the system shown in figure below. Use Holzer's Method	(7)
6	(a)	Develop the governing differential equation for transverse flexural vibration of a continuous system subjected to dynamic loading. Apply the above equation for a beam with simply supported end condition	(7)
	(b)	Explain the procedural steps for Response Spectrum Method of Seismic analysis	(7)
7	(a)	Examine whether the log – decrement is also given by the equation δ = 1/n log (U_0/ U_n) represents the amplitude after n cycles have elapsed.	(5)
	(b)	Calculate the natural frequencies and draw mode shapes for the given system 	(5)
	(c)	Explain the procedure involve in the Equivalent Static Analysis	(4)

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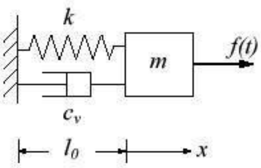
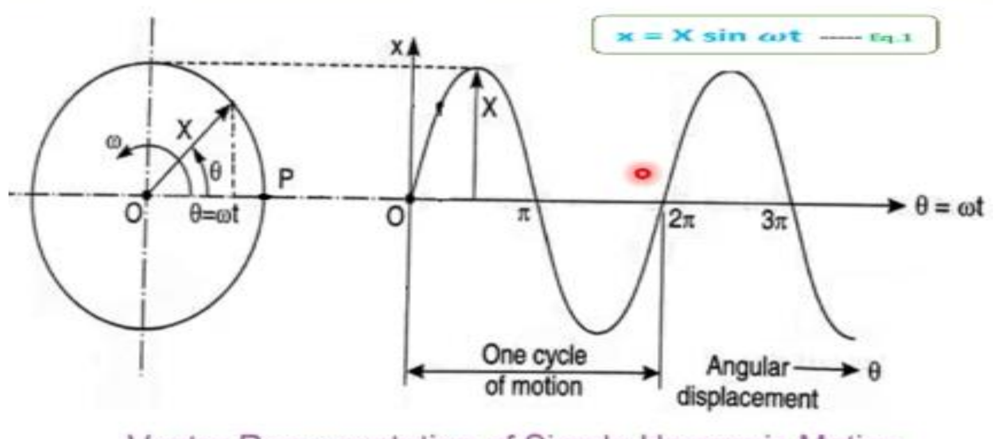
Assignment Final
Roll No21241D2007
Name: MARIYALA VAISHNAVI

Q.No.		Marks	CO	BT
1. a)	CLASSIFICATION OF VIBRATIONS:- Vibrations are broadly classified into 2 cases. i) Free vibrations ii) Forced vibrations. FREE VIBRATIONS:- vibrations of structure is started by the application of external loads, which is subsequently removed then the resulting oscillation where no external load is present is called free vibration. FORCED VIBRATIONS:- This is to and fro motion of the structure due to external excitation. Depending on nature of loading they are classified as periodic & non-periodic / deterministic & stochastic loading	(2)	1	2
b)	PHASE ANGLE:- The phase of an oscillation is defined as the amount of oscillation lags behind or leads infront of a reference oscillations.	(2)	2	2

c)	D' Alemberts principle :- According to newtons second law of motion $f=ma$. where f is the external force acting on the body whose mass is 'm' & acceleration 'a' $\Sigma F \rightarrow$ resultant or Net force $ma \rightarrow$ inertia force $\Sigma f - ma = 0$ which means that by applying force $(-ma)$ all on the body the body will be in equilibrium as the sum of all the forces acting on body is zero. Such equilibrium is called dynamic equilibrium. The force '$-ma$' is called "inertia force" or "D'Alemberts force". The body will be in dynamic equilibrium under the action of external force 'f' & internal force '$-ma$'. this is known as D'Alembert's principle.	(2)	3	2
d)	A mode shape is the deformation that the component would show when vibrating at the natural frequency. It is an abstract mathematical parameter which defines a deflection pattern as if that mode existed in isolation from all others in the structure. A mode shape is a deflection pattern related to a particular natural frequency and represents the relative displacement of all parts of a structure for that particular mode.	(2)	4	2
e)	$m\ddot{x} + c\dot{x} + Kx = -m\ddot{y}$	(2)	5	1
f)		(2)	2	2

	g)	 Vector Representation of Simple Harmonic Motion	(2)	4	2

2	(a)	**BASED ON NEWTON'S SECOND LAW:-** According newtons second law rate of change of linear momentum is proportional to force applied on it $\frac{d}{dt}(mv) \propto F$ — (1) $\frac{dv}{dt} = a = \ddot{x}$ — (2) $a = \ddot{x}$ $m\ddot{x} \propto F$ — (3) $m\ddot{x} = cF$ (for ideal system) ($\because c = 1$) $m\ddot{x} = \sum F$ ΣF = Sum of all forces acting on the body. → Net force	(7)	1	2

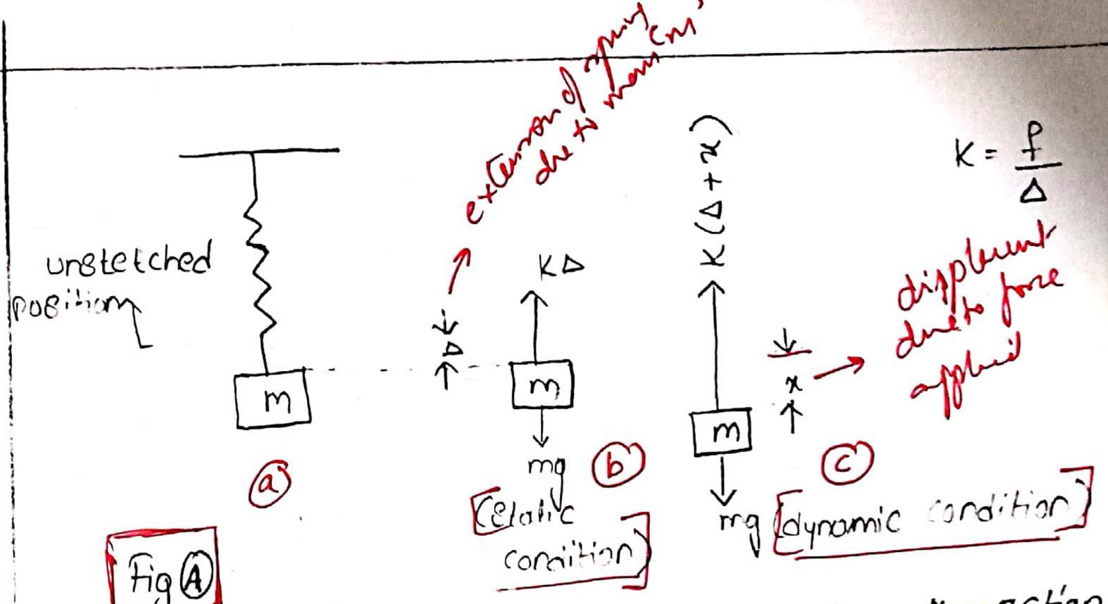


Fig Ⓐ

The body 'm' is in equilibrium under the action of 2 forces

Δ → is extention of spring after mass is Suspended

If body is moving down with acceleration '$\ddot{x}$' in downward direction then

$$m\ddot{x} = \Sigma F \quad \text{— ①} \quad (\Sigma F \rightarrow \text{net force})$$

$$mg = K\Delta \quad \text{— ②} \quad \text{fig ⓑ}$$

$$m\ddot{x} = [mg - K(\Delta + x)] \quad \text{— ③} \quad ([\ ] \rightarrow \Sigma F)$$

$$K\Delta = mg$$

$$m\ddot{x} = mg - K\Delta - Kx$$

$$m\ddot{x} = -Kx$$

$$\boxed{m\ddot{x} + Kx = 0}$$

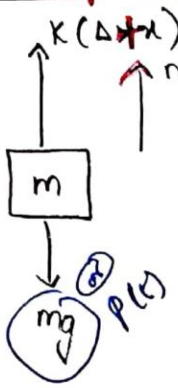
Undamped free vibration

* Equation of motion for SDOF free undamped Vibration

② **BASED ON D'ALEMBERT'S PRINCIPLE :-**

As per D'Alemberts principle to maintain dynamic equilibrium the forces acting on body are

(Spring force)



due to mass 'm'

$\ddot{x}$ - acceleration with which is moves downwards

$$mg = m\ddot{x} + k(\Delta + x) \quad \text{— (1) dynamic condition}$$

$$mg = k\Delta \quad \text{— (2) elastic condition}$$

$$mg = m\ddot{x} + mg + kx \quad \text{— (3)}$$

$$m\ddot{x} + kx = 0$$

3) Energy method :-

This method applicable only to conservative system (where there is no loss of energy) (Since there is no loss of energy, total energy remains constant). When a structure is vibrated the total energy in system is partly Kinatic energy, partly P.E (elastic strain energy). *

The kinetic energy is due to mass 'm' & velocity ($\dot{x}$) & P.E is due to spring constant or/ spring stiffness & displacement *

$$E = K.E + P.E$$

Total energy remains constant, (w.r.t time)

$E = C$

$$\frac{dE}{dt} = 0$$

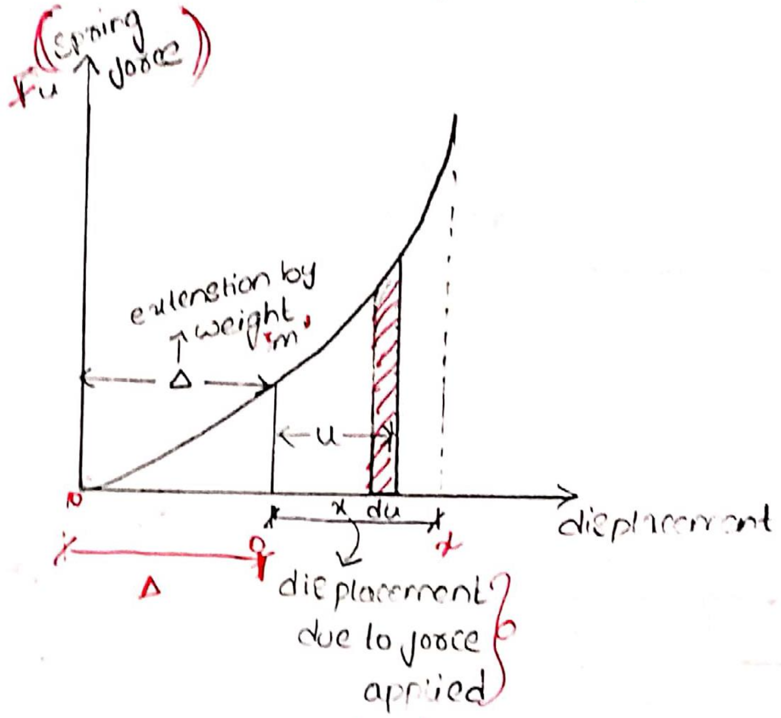
$$\frac{d}{dt}[KE + PE] = 0$$

$$K.E = \frac{1}{2}mv^2 \quad (\because v = \dot{x})$$

$$= \frac{1}{2}m\dot{x}^2 \rightarrow (1)$$

P.E of System consists of 2 points.

1. loss or gain in P.E of mass (energy lost or gained)
2. Strain energy of Spring (energy stored)



Consider element 'du' at $x = u$

SE or Spring force $F_u = K(u+\Delta)$

SE or Workdone by Spring = force × displacement

$$dw = K(u+\Delta)\,du$$

Potential energy

PE = Strain energy of Spring − loss of P due to force applied

(As loss in P.E is gained.)

$$= \int_0^x K(u+\Delta)\,du - mg \cdot x$$

$$\left[\frac{Ku^2}{2}\right]_0^x + \left[K\Delta u\right]_0^x - mg \cdot x$$

$$= \frac{Kx^2}{2} + mgx - mgx \quad (K\Delta = mg)$$

$$P.E = \frac{Kx^2}{2} \rightarrow (2)$$

& $K.E = \frac{1}{2}m\dot{x}^2 \rightarrow (1)$

		from ① & ② $\frac{d}{dt}\left[\frac{1}{2}m\dot{x}^2 + \frac{1}{2}Kx^2\right] = 0$ $\frac{dE}{dt} = 0$ $\frac{1}{2}m\ddot{x} \times 2\dot{x} + \frac{1}{2}K \times 2x \cdot \dot{x} = 0$ $m\ddot{x} + Kx = 0$			
	(b)	damping factor $\zeta = \frac{c}{C_c}$ $C_c = 2\sqrt{Km}$ natural frequency of damped oscillation $\omega_d = \omega_n\sqrt{1-\zeta^2}$ ✓ $\omega_n = \sqrt{\frac{K}{m}}$ $\frac{1}{T_d} = \frac{\omega_d}{2\pi}$ $\ln\left[\frac{x_0}{x_1}\right] = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$	(7)	1	4

3	(a)	(see below)	(7)	2	3

$$m\ddot{x} + c\dot{x} + Kx = 0 \rightarrow (1)$$

This is differential equation for SDOF system - damped - free vibration.

The solution for this 2nd order Equation can be expressed as

exponential solution →

$$x = X e^{st}$$ (X → amplitude, s → constant)

$$\dot{x} = s X e^{st}$$

$$\ddot{x} = s^2 X e^{st}$$

Sub all above 3 in eqn (1)

$$m \times s^2 X e^{st} + c s X e^{st} + K X e^{st} = 0$$

$$ms^2 + cs + K = 0$$

$$(1)s^2 + \left(\frac{c}{m}\right)s + \left(\frac{K}{m}\right) = 0$$

($ax^2 + bx + c = 0$: $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ or $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$)

The roots of Equation are

$$s_{1,2} = \frac{-c}{2m} \pm \frac{1}{2}\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{K}{m}\right)}$$

The Solution for eqn (1) is given by $x = x_1(t) + x_2(t)$

$$x = X_1 e^{\left[\frac{-c}{2m} + \frac{1}{2}\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{K}{m}\right)}\right]t} + X_2 e^{\left[\frac{-c}{2m} - \frac{1}{2}\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{K}{m}\right)}\right]t}$$

($\because e^{a+b} = e^a \cdot e^b$)

$x = Xe^{st}$

$$x = e^{-\left(\frac{c}{2m}\right)t}\left[X_1 e^{\frac{1}{2}\left[\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{K}{m}\right)}\right]t} + X_2 e^{-\frac{1}{2}\left[\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{K}{m}\right)}\right]t}\right]$$

There are 3 possible cases of Solution based on the term in the determiner

*Case (i) :- Over damped Case

$$\left(\frac{c}{m}\right)^2 > 4\left(\frac{k}{m}\right)$$

Case (ii) :- Critically damped Case

$$\left(\frac{c}{m}\right)^2 = 4\left(\frac{k}{m}\right)$$

Case (iii) :- under damped Case

$$\left(\frac{c}{m}\right)^2 < 4\left(\frac{k}{m}\right)$$

C = damping constant

C_c = critical damping constant

Let 'C_c' be the critical damping co-efficient

critical damped case :-

$$\left(\frac{C_c}{m}\right)^2 = 4\left(\frac{k}{m}\right)$$

$$C_c = 2\sqrt{km}$$

$$= 2\sqrt{\frac{k}{m} \times m^2}$$

$\omega_n^2 = \frac{k}{m}$

$\omega_n = \sqrt{\frac{k}{m}}$

$$C_c = 2\omega_n m$$

$$\boxed{C_c = 2\omega_n m = 2\sqrt{km}}$$ Critical Damping coefficient

CASE(i) :- under damped

usually practically all systems are under damped

$$\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{k}{m}\right)} = i\sqrt{4\left(\frac{k}{m}\right) - \left(\frac{c}{m}\right)^2}$$

$\sqrt{-x} = i\sqrt{x}$

$i = \sqrt{-1}$

imaginary part

$\sqrt{-x} = \sqrt{-1}\sqrt{x}$

$= i\sqrt{x}$

$$i\sqrt{4\omega_n^2 - \left(\frac{c}{c_c} \times \frac{c_c}{m}\right)^2}$$

Zeta $$\boxed{\zeta = \frac{c}{c_c}}$$

* Damping factor 'ζ' is the ratio of damping co-efficient 'c' to the critical damping coefficient (c_c)

$$i\sqrt{4\omega_n^2 - \zeta^2 \times \left(\frac{2m\omega_n}{m}\right)^2}$$

$$\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{k}{m}\right)} = i\left[2\omega_n\sqrt{1-\zeta^2}\right]$$

we are considering only inside part 'i' remain same.

$$x = e^{-\left(\frac{c}{2m}\right)t}\left[x_1 e^{\frac{1}{2}\left[i2\omega_n\sqrt{1-\zeta^2}\right]t} + x_2 e^{-\frac{1}{2}\left[i2\omega_n\sqrt{1-\zeta^2}\right]}\right]$$

$$x = e^{-\left(\frac{c}{2m}\right)t}\left[x_1 e^{\left(i\omega_n\sqrt{1-\zeta^2}\right)t} + x_2 e^{-\left(i\omega_n\sqrt{1-\zeta^2}\right)}\right] \quad (i)$$

icient

ase.

Let us assume

$$\omega_n\sqrt{1-\zeta^2} = \omega_d \qquad \zeta = \frac{c}{c_c}$$

natural frequency of undamped free vibrations

where,

ω_d = damped natural angular frequency

ω_d is natural frequency of damped free vibration.

∴ For under damped case the solution is

$$x = e^{-\left(\frac{c}{2m}\right)t}\left[x_1 e^{i\omega_d t} + x_2 e^{-i\omega_d t}\right] \rightarrow (a)$$

Case (ii):- Critically damped

For critically damped case since two roots are equal the general solution can be expressed as follows:-

(one independent solution is)

$$x = x_1(t) + x_2(t)$$

ficient (c_c)

$x_1(t)$ is one independent solution which is expressed as follows

For critically Damped Case, $\zeta = 1$

$$x_1(t) = X_1 e^{-\left(\frac{c}{2m}\right)t}$$

another independent solution $x_2(t)$ may be found using

$$x_2(t) = X_2 t e^{-\left(\frac{c}{2m}\right)t}$$

and these 2 equations satisfy eqn (1) $m\ddot{x} + c\dot{x} + kx = 0$ *

The General Solution for critically damped System is obtained by Superimposing this 2 independent Solution

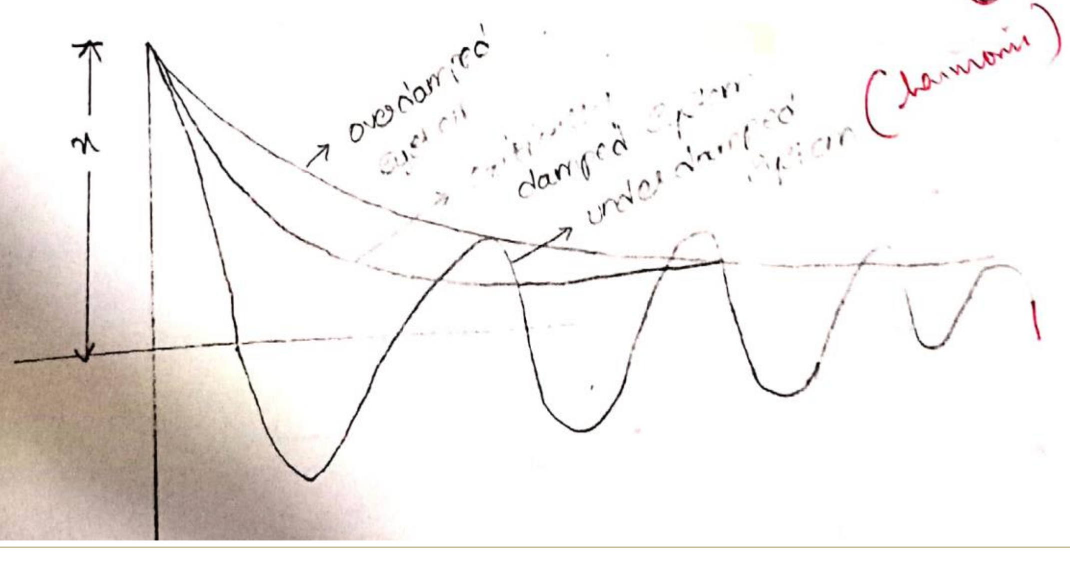
$$x = (X_1 + X_2 t) e^{-\left(\frac{c}{2m}\right)t} \quad \text{— (b)}$$

Case (iii) :- Over damped

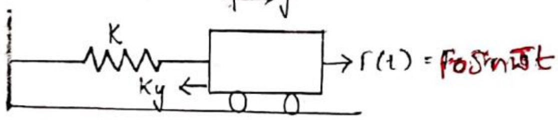
$$x = e^{-\frac{c}{2m}t}\left[X_1 e^{\omega n\left(\sqrt{\zeta^2-1}\right)t} + X_2 e^{-\omega n\left(\sqrt{\zeta^2-1}\right)t}\right]$$

$$\because \frac{c}{2m} = \frac{c}{c_c} \times \frac{c_c}{2m} = \zeta \times \frac{2m\omega n}{2m} = \zeta \omega n$$

$$x = e^{-\zeta \omega n t}\left[X_1 e^{\omega n\left(\sqrt{\zeta^2-1}\right)t} + X_2 e^{-\omega n\left(\sqrt{\zeta^2-1}\right)t}\right] \quad \text{(c)}$$



(b) (7) 2 3



ω = velocity for forced vibration

The differential equation can be written in the form of

force imposed sinusoidally

$$m\ddot{y} + ky = F_0 \sin \bar{\omega} t \rightarrow (1)$$

$m\ddot{y} + c\dot{y} + ky = F(t)$

$c = 0$

F_0 – Peak amplitude

$\bar{\omega}$ – Angular velocity for forced vibration

The solution for this equation can be expressed in the form of

$$y(t) = y_c(t) + y_p(t) \rightarrow 1(a)$$

$y_c(t)$ – complimentary solution (real part) – belongs to free vibration

$y_p(t)$ – particular solution or (Imaginary part) – belongs to forced vibrat

Complimentary solution – belongs to free vibration

$$y_c(t) = A\cos\omega t + B\sin\omega t \quad - (2)$$

$y_p(t) = Y \sin \bar{\omega} t$

$\dot{y}_p(t) = \bar{\omega} \cdot Y \cos \bar{\omega} t$

$\ddot{y}_p(t) = -\bar{\omega}^2 Y \sin \bar{\omega} t$

eqn (1)

$$y(t) = A\cos\omega t + B\sin\omega t + \frac{f_0/k}{1-r^2}\sin\bar{\omega} t \rightarrow (5)$$

This term corresponds to undamped free vibration → This term corresponds undamped free vibration

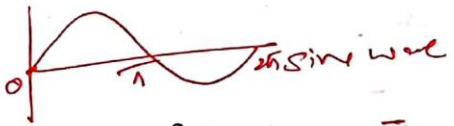
For sinewave at origin: $t = 0$, $y_0 = 0$, $v_0 = 0$ } Initial BC conditions

{ Time measured from mean → sine wave; from extreme → cosine wave }

Sub in eqn (5)

$$0 = A + 0 + 0$$

$$A = 0$$



$$\dot{y}(t) = -\omega A\sin\omega t + \omega B\cos\omega t + \frac{f_0/k}{1-r^2}\bar{\omega}\cos\bar{\omega} t \rightarrow 5(a)$$

$\dot{y}(t) \rightarrow v_0$

$$0 = 0 + \omega B + \frac{f_0/k}{1-r^2}\bar{\omega}$$

$$-\frac{f_0/k}{1-r^2}\times\left(\frac{\bar{\omega}}{\omega}\right) = B \quad \left(\frac{\bar{\omega}}{\omega} \rightarrow r\right)$$

$$\left\{ A = 0, \quad B = -r\frac{(f_0/k)}{1-r^2} \right.$$

$$B = -(r)\frac{f_0/k}{1-r^2}$$

Sub (A) & (B) in eqn (5)

$$y(t) = -(r)\frac{f_0/k}{1-r^2}\sin\omega t + \frac{f_0/k}{1-r^2}\sin\bar{\omega} t$$

$$y(t) = \frac{f_0/k}{1-r^2}\left[\sin\bar{\omega} t - r\sin\omega t\right] \rightarrow (6)$$

$\left(\frac{f_0/k}{1-r^2}\cdot\sin\bar{\omega} t\right)$ {forcing frequency term} {free freq. term} → $\frac{f_0/k}{1-r^2}\, r\cdot\sin\omega t$

---

4 (a) — (7) — 3 — 2

Let ϕi and ϕj are two distinct mode shapes

Condition or 0rthogonality

$\phi_i{}^T \ \phi_j = 1$	$for\ i = j$
$\phi_i{}^T \ \phi_j = 1$	$for\ i \neq j$

Normalization is a process to make non orthogonal modes to an orthogonal modes. The resultant mode shape is called as weighted modes. We can normalize it by using mass matrix or stiffness matrix.

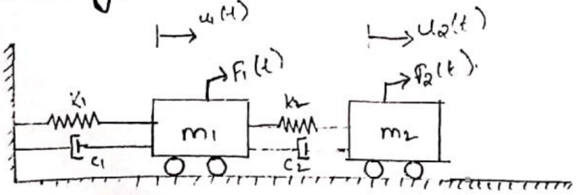
Let us consider two mode shapes ϕi and ϕj

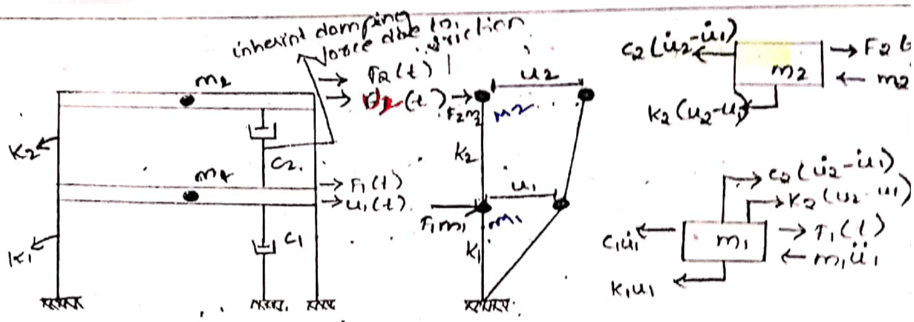
$$(k - \omega i^2 m)\phi i = 0$$

$$[\phi_j]^T k\, \phi_i = \omega_i{}^2 [\phi_j]^T m\, \phi_i$$

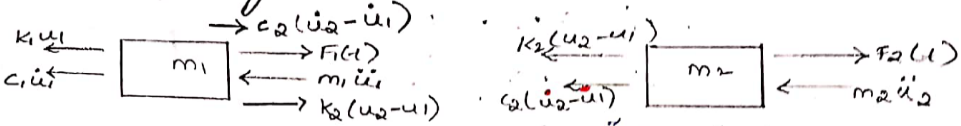
(b)

Step-1: Modelling :-





Free body diagram



For each mass, forces acting horizontally = 0

$$\Sigma F_x = 0$$

$m_1 \rightarrow m_1\ddot{u}_1 + c_1\dot{u}_1 + k_1 u_1 + c_2(\dot{u}_1-\dot{u}_2) + k_2(u_1-u_2) = F_1 \rightarrow$ (1)

$m_2 \rightarrow m_2\ddot{u}_2 + c_2(\dot{u}_2-\dot{u}_1) + k_2(u_2-u_1) = F_2(t)$ — (2)

$$\ddot{u}_1 m_1 + \dot{u}_1(c_1+c_2) + \dot{u}_2(-c_2) + u_1(k_1+k_2) + u_2(-k_2) = F_1(t)$$

$$\ddot{u}_2 m_2 + \dot{u}_1(-c_2) + \dot{u}_2(c_2) + u_1(-k_2) + u_2(k_2) = F_2(t)$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

Matrix ← $[M]\ddot{u} + C\dot{u} + Ku = F$ → (a)   F = load vector

M = mass matrix (diagonal matrix)

$\ddot{U}$ = vector of accelerations for each degree of freedom

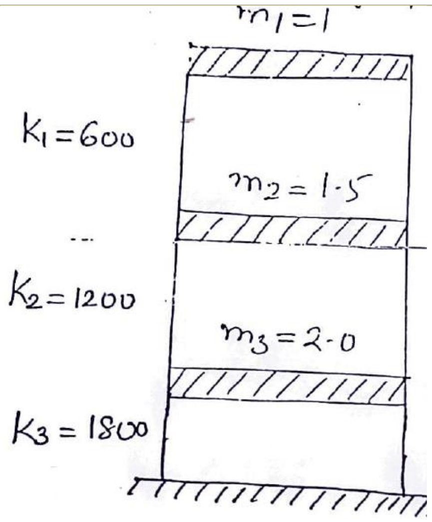
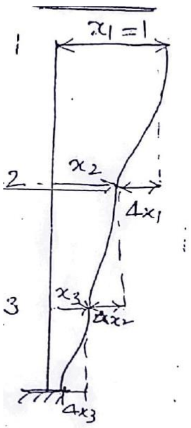
C = damping matrix [Symmetrical matrix]

$\dot{u}$ = vector of velocities for each degree of freedom

K = Stiffness matrix [Symmetrical matrix]

u = vector of displacements for each degree of freedom

(7) 3 4

5	(a)	$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial R}{\partial \dot{q}_i}+\frac{\partial U}{\partial q_i}=Q_i$$ *where*: q_i : independent coordinates necessary to describe system's motion at any instant Q_i : corresponding loading in each coordinate $U = f_1(q_i)$: potential energy in terms of coordinates $T = f_2(\dot{q}_i^2)$: kinetic energy in terms of system masses, mass inertias, linear/angular velocities $R = f_3(\dot{q}_i^2)$: energy dissipation due to viscous friction 1. Select a complete and independent set of coordinates q_i's 2. Identify loading Q_i in each coordinate 3. Derive T, U, R 4. Substitute the results from 1,2, and 3 into the Lagrange's equation.	(7)	4	2
	(b)	  Note:- i) If the assumed force is correct $x_b = 0$ ii) If not correct $x_b \neq 0$ If $x_b = +ve$, increase ω, $x_b = -ve$, decrease " $F = K \cdot x$ $x_b \rightarrow$ base displacement $\omega = \sqrt{\frac{K}{m}} \Rightarrow \omega^2 = \frac{K}{m}$ $\Rightarrow K = \omega^2 m$. $F = \omega^2 m x$	(7)	4	4

1st Trial Assume fundamental frequency

$\omega^2 = 100$

further assume that the top displacement $x_1 = 1$

∴ Numbering is from top to bottom.

Inertia force @ 1st level $F_{I_1} = \omega^2 m_1 x_1 = 100 \times 1 \times 1 = 100$ kN

Top storey shear

$V_1 = F_{I_1} = 100$

dividing this by top storey stiffness, k_1, we get the top storey deformation $\Delta x_1 = \frac{V_1}{k_1} = \frac{100}{600} = 0.167$ cm.

displacement of 2nd storey $= x_2 = x_1 - \Delta x_1 = 0.833$

Inertia force @ 2nd level

$F_{I_2} = \omega^2 m_2 x_2 = 100 \times 1.5 \times 0.833 = 125$

2^{nd} storey shear

$V_2 = FI_1 + FI_2 = 100 + 125 = 225$

$\Delta x_2 = \frac{V_2}{K_2} = \frac{225}{1200} = 0.188$

$\therefore x_3 = x_2 - \Delta x_2 = 0.833 - 0.188 = 0.645$

$FI_3 = \omega^2 m_3 x_3 = 100 \times 2 \times 0.645 = 129$

$V_3 = FI_1 + FI_2 + FI_3 = 100 + 125 + 129 = 354$

$\therefore \Delta x_3 = \frac{V_3}{K_3} = \frac{354}{1800} = 0.197$

$\therefore x_B = x_3 - \Delta x_3 = 0.645 - 0.197 = 0.448$ +ve value,

As we are more away from the result in the 2^{nd} trial

Let us assume $\omega^2 = 200$ and repeat the calculations

trial

$FI_1 = \omega^2 m_1 x_1 = 200 \times 1 \times 1 = 200$

$V_1 = \frac{FI_1}{} = 200$

$\Delta x_1 = \frac{V_1}{K_1} = \frac{200}{600} = 0.33$

$x_2 = x_1 - \Delta x_1 = 1.0 - 0.33 = 0.67$

$FI_2 = \omega^2 m_2 x_2 = 200 \times 1.5 \times 0.67 = 201$

$V_2 = FI_1 + FI_2 = 200 + 201 = 401$

$\therefore \Delta x_2 = \frac{V_2}{K_2} = \frac{401}{1200} = 0.334$

$x_3 = x_2 - \Delta x_2 = 0.67 - 0.334 = 0.336$

$FI_3 = 200 \times 2 \times 0.336 = 134.4$

$V_3 = FI_1 + FI_2 + FI_3$
$= 200 + 201 + 134.4 = 535.4$

$\therefore \Delta x_3 = \frac{V_3}{K_3} = \frac{535.4}{1800} = 0.297$

$x_B = x_3 - \Delta x_3$
$= 0.336 - 0.297$
$= 0.039 \neq 0$

but close to zero

III Trial Let $\omega^2 = 210$; $x_1 = 1$

$FI_1 = \omega^2 m_1 x_1 = 210 \times 1 \times 1 = 210$

$\therefore V_1 = 210$

$\Delta x_1 = \frac{V_1}{K_1} = \frac{210}{600} = 0.35$

$\therefore x_2 = x_1 - \Delta x_1 = 1 - 0.35 = 0.65$

$FI_2 = \omega^2 m_2 x_2 = 210 \times 1.5 \times 0.65 = 205$

$V_2 = FI_1 + FI_2 = 210 + 205 = 415$

$\Delta x_2 = \frac{V_2}{K_2} = \frac{415}{1200} = 0.346.$

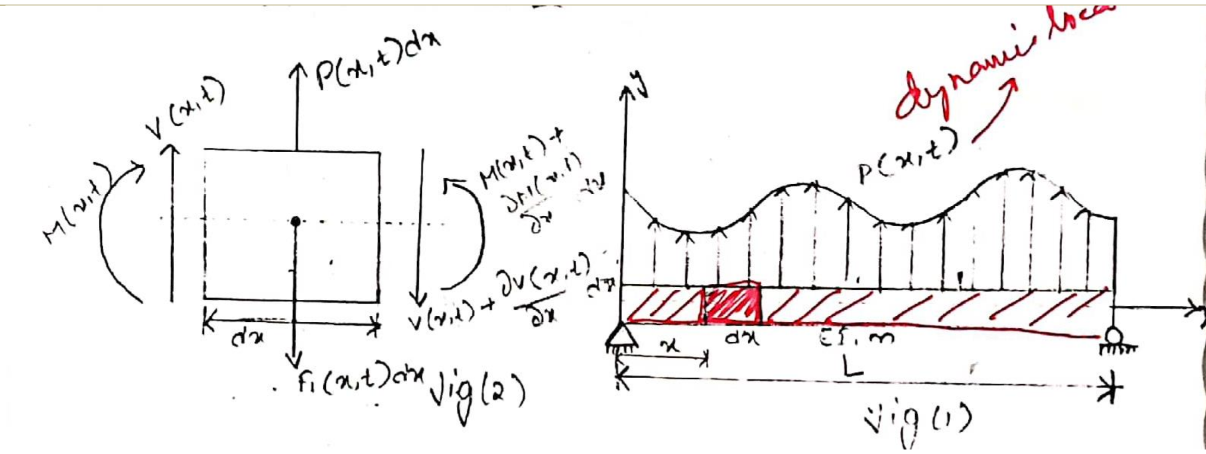
$x_3 = x_2 - \Delta x_2 = 0.65 - 0.346 = 0.304$

$FI_3 = \omega^2 m_3 x_3 = 210 \times 2 \times 0.304 = 128$

$V_3 = FI_1 + FI_2 + FI_3 = 210 + 205 + 128 = 543$

$\Delta x_3 = \frac{V_3}{K_3} = \frac{543}{1800} = 0.302$

$\therefore x_b = x_3 - \Delta x_3 = 0.304 - 0.302 = 0.002$ $\omega^2 = 209$

6	(a)	 $$EI \frac{\partial^4 y}{\partial x^4} + m \cdot \frac{\partial^2 y}{\partial t^2} = P.$$ This is governing diff. Eqn for beam subjected to flexural vibration. (Damped vibration) The general solution for this Equation is given in the form of $y(x,t) = \phi(x) Y(t)$ — (2) where, $\phi(x)$ = deformed shape of the beam $Y(t)$ = amplitude of the vibration	(7)	5	3

$$\ddot{y}(t) + \omega^2 y(t) = 0 \rightarrow 3(b)$$

Eqn 3(b) is similar to Equation of motion for SDOF System so the solution for Eqn 3(b) is in the form of

Y_0 = Y at time t

$$y(t) = Y_0 \cos\omega t + \left(\frac{Y_0}{\omega}\right)\sin\omega t \rightarrow (4)$$

EOM →

In order to evaluate ω we use Eqn 3(a) & introduce α term

$$\omega = \sqrt{\frac{\alpha^4 EI}{m}}$$

frequency parameter

$$\alpha^4 = \frac{\omega^2 m}{EI} \rightarrow (5)$$

Assume a solution of form

$$s = \frac{\partial}{\partial x}$$

beam property

$$\phi(x) = Ce^{sx}$$

Sub this in Eqn 3(a)

$$EI \cdot \frac{\partial^4 [Ce^{sx}]}{\partial x^4} = m \cdot C e^{sx} \omega^2 \Rightarrow \frac{\partial^4 [Ce^{sx}]}{\partial x^4} = \left(\frac{m\omega^2}{EI}\right) Ce^{sx}$$

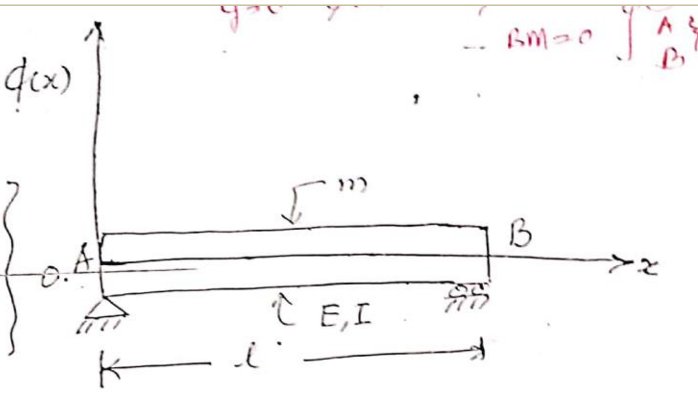
α^4 [

$$(s^4 - \alpha^4) Ce^{sx} = 0 \rightarrow (6) \quad \left(s = \frac{\partial}{\partial x}\right)$$

$$\phi(x) = A_1 \sin a_n x + A_2 \cos a_n x + A_3 \sinh a_n x + A_4 \cosh a_n x$$

The deflection is

$y = 0$ means $\phi(x) = 0$ deflection zero $\left(\because y(x,t) = \phi(x) \cdot Y(t) \right)$



B.M = 0

$M = EI \frac{\partial^2 y}{\partial x^2}$ i.e. $\phi''(x) = 0$

$$\therefore \omega_n = \sqrt{\frac{EI}{\bar{m}}}\, a_n^2$$

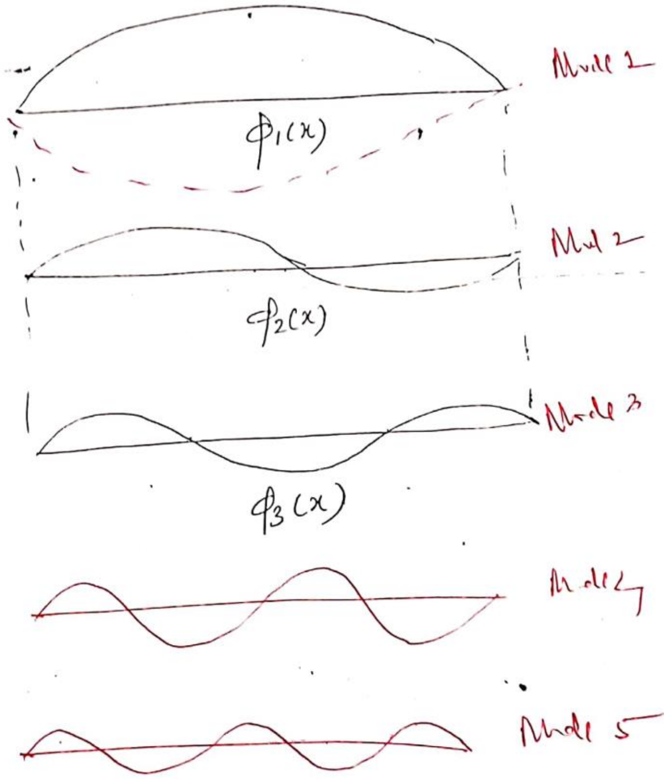
substitute $a_n = \frac{n\pi}{l}$

$$\therefore \omega_n = \frac{n^2\pi^2}{l^2}\sqrt{\frac{EI}{\bar{m}}}$$

$$\therefore \omega_1 = \frac{\pi^2}{l^2}\sqrt{\frac{EI}{\bar{m}}}$$

$$\omega_2 = \frac{2^2\pi^2}{l^2}\sqrt{\frac{EI}{\bar{m}}}$$

$$\omega_3 = \frac{3^2\pi^2}{l^2}\sqrt{\frac{EI}{\bar{m}}}$$



	(b)	**The response spectrum method of analysis is developed using the following steps.** A modal analysis of the structure is carried out to obtain mode shapes, frequencies & modal participation factors. 1. First Eigenvalues and Eigenvectors are determined. 2. Then modal participation factors are determined. 3. Modal masses are determined. 4. Lateral force at each floor in each mode is determined. 5. Storey shear forces in each mode is determined. 6. Storey shear force due to all modes are determined. 7. At last Lateral forces at each storey is determined by square root of sum of squares (SRSS) and complete quadratic combination (CQC).	(7)	5	2

7	(a)	$\zeta\omega_n = \frac{c}{2m}$ $x = X e^{-\zeta\omega_n t}\cos(\omega_d t + \phi)$ $x(t) = e^{-c/2m\,t}(A\cos\omega_d t + B\sin\omega_d t)$, $\omega_d = \omega_n\sqrt{1-\zeta^2}$ where X & ϕ are constants X is amplitude & ϕ is phase angle Introduce boundary conditions :- x_0 (displacement) & v_0 (velocity) at time period $(t) = t$ displacement $(x) = x_0$ th Solution for under damped case is $x_0 = X e^{-\zeta\omega_n t}\cos(\omega_d t + \phi) \rightarrow (d)$ after one time period the boundary conditions are $t = t + t_p$, $x = x_1$ } B.C's Then the solution is $x_1 = X e^{-\zeta\omega_n(t+t_p)}\cos(\omega_d(t+t_p) + \phi)$ (e) divide Eqn(d) / Eqn(e) $$\frac{x_0}{x_1} = \frac{X e^{-\zeta\omega_n t}\cos(\omega_d t + \phi)}{X e^{-\zeta\omega_n(t+t_p)}\cos(\omega_d(t+t_p) + \phi)}$$ t_p = time period $= \frac{1}{f_p}$ frequency $= \frac{2\pi}{\omega_d}$ $(\omega_d t_p = 2\pi)$ $\frac{e^a}{e^b} = e^{a-b}$ $$\frac{x_0}{x_1} = \frac{X e^{-\zeta\omega_n t}\cos(\omega_d t + \phi)}{X e^{-\zeta\omega_n t}\cdot e^{-\zeta\omega_n t_p}\cos(\omega_d(t+t_p) + \phi)}$$ $e^{-\zeta\omega_n t - (-\zeta\omega_n t \cdot e^{\zeta\omega_n t_p})} = e^{\zeta\omega_n t_p}$ 15	(5)	1	2

$$\frac{x_0}{x_1} = e^{-\zeta \omega_n t_p} \cdot \frac{\cos(\omega_d t + \phi)}{\cos(\omega_d t + 2\pi + \phi)}$$

$\omega_d t_p = 2\pi$

$\because \cos\theta = \cos(2\pi + \theta)$

$$\frac{x_0}{x_1} = e^{\zeta \omega_n t_p}$$

$a = e^b$

$b = \ln(a)$

$$\ln\left(\frac{x_0}{x_1}\right) = \zeta \omega_n t_p = \zeta \omega_n \times \frac{2\pi}{\omega_d}$$

$$= \zeta \omega_n \times \frac{2\pi}{\omega_n \sqrt{1-\zeta^2}}$$

ln → to base e

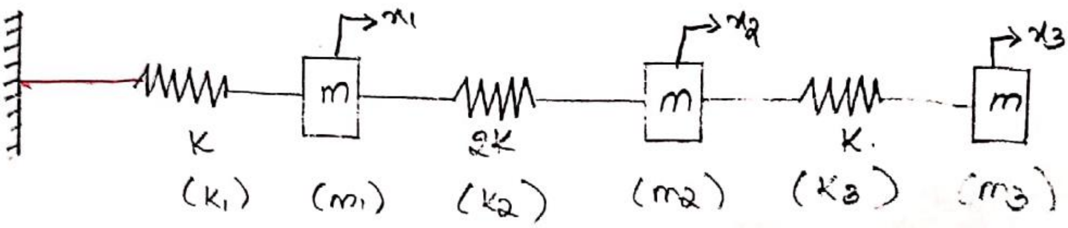
log → to base 10

$$\ln\left(\frac{x_0}{x_1}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

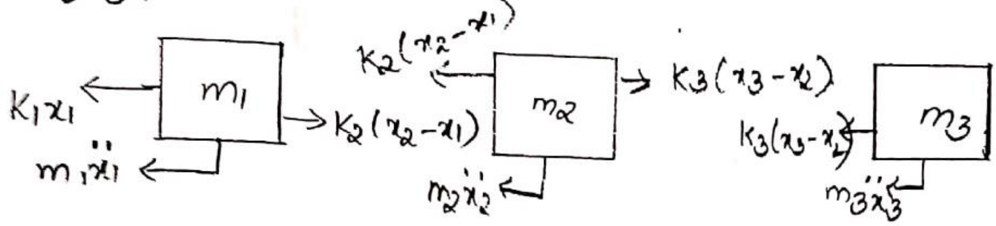
rate at which amplitude of free damped vibration

This equation is known as logarithmic decrement

(b) (5) 3 4



F.B.D:-



Apply D'Alemberts Principle.

For m_1: $m_1\ddot{x}_1 + Kx_1 + K_2x_1 - K_2x_2 = 0$

$m = m_1$, $K_2 = 2K$.

$m\ddot{x}_1 + 3Kx_1 - 2Kx_2 = 0 \rightarrow (1)$

For m_2

$m_2\ddot{x}_2 + K_2x_2 - K_2x_1 - K_3x_3 + K_3x_2 = 0$

$m\ddot{x}_2 - 2Kx_1 + 3K2 - Kx_3 = 0 \rightarrow (2)$

For m_3: $m_3\ddot{x}_3 + K_3(x_3 - x_2) = 0$

$m\ddot{x}_3 - Kx_2 + Kx_3 = 0 \rightarrow (3)$

Matrix form:-

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} 3K & -2K & 0 \\ -2K & 3K & -K \\ 0 & -K & K \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This can be expressed as eigen value proble

$$\left[[K] - \omega_n^2[m]\right][\phi] = 0$$

Sub $\omega_n^2 = \lambda$

$$\left[\begin{bmatrix} 3K & -2K & 0 \\ -2K & 3K & -K \\ 0 & -K & K \end{bmatrix} - \begin{bmatrix} m\omega_n^2 & 0 & 0 \\ 0 & m\omega_n^2 & 0 \\ 0 & 0 & m\omega_n^2 \end{bmatrix}\right]\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = 0$$

$$(3K - \omega_n^2)\phi_1 - 2K\phi_2 = 0$$

$$-2K\phi_1 + (3K - m\omega_n^2)\phi_2 - K\phi_3 = 0$$

$$-K\phi_2 + (K - m\omega_n^2)\phi_3 = 0$$

$$\begin{vmatrix} 3K - \omega_n^2 & -2K & 0 \\ -2K & (3K - m\omega_n^2) & -K \\ 0 & -K & (K - m\omega_n^2) \end{vmatrix} = 0$$

$$\lambda^3 - 7\left(\frac{K}{m}\right)\lambda^2 + 10\left(\frac{K}{m}\right)^2\lambda - 2\left(\frac{K}{m}\right)^3 = 0$$

$$\lambda_1 = 5.13\left(\frac{K}{m}\right) \Rightarrow \omega_{n1} = 2.26\sqrt{\frac{K}{m}}$$

$$\lambda_2 = 0.238\left(\frac{K}{m}\right) \Rightarrow \omega_{n2} = 0.488\sqrt{\frac{K}{m}}$$

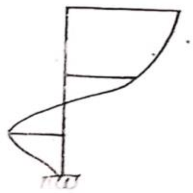
$$\lambda_3 = 1.62\left(\frac{K}{m}\right) \Rightarrow \omega_{n3} = 1.279\sqrt{\frac{K}{m}}$$

Node Shape 1:-

$$\phi_1 = \begin{bmatrix} 1.00 \\ 1.38 \\ 1.81 \end{bmatrix}$$

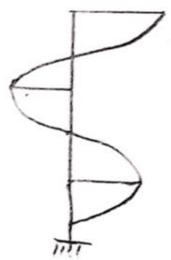
Node Shape 2:-

$$\phi_2 = \begin{bmatrix} 1.00 \\ 0.685 \\ -1.06 \end{bmatrix}$$



Node Shape 3:-

$$\phi_3 = \begin{bmatrix} 1.00 \\ -1.015 \\ 0.268 \end{bmatrix}$$



	(c)	1. Firstly, the calculation of lumped masses into various floor levels is done. This involves calculation of masses on roofs and other floors taking into consideration the weight imposed by walls, columns, beams, floors, infills and slabs. 2. Then fundamental natural period is determined using the formula $T_a = 0.075 \times h^{0.75}$ Where, T_a is the fundamental natural period of vibration in seconds and h is the height of building in meters. This is as per Cl 7.6.2 of IS 1893 Part 1: 2016. 3. As per Cl 7.6.1. of IS 1893 Part 1: 2016, we calculate Base Shear, $V_B = A_h$ 4. Then the design Base Shear is distributed along the height of the building as per expression: $$Q_i = V_B \frac{W_i h_i^2}{\sum_{i=1}^{n} W_i h_i^2}$$ Q_i = Design lateral forces at level i, W_i = Seismic weights of the floor i h_i = Height of the floor i n = Number of stories	(4)	5	2

Assignment - Week 1

The due date for submitting this assignment has passed.

Due on 2017-02-08, 23:00 IST.

Submitted assignment

- All the questions are compulsory.
- All questions are of multiple choice type.
- There is no negative marking for wrong answer.
- Maximum marks of the assignment is 25.

Q.01. The amplitude for a S.H.M. given by the equation $x = 3sin(3pt) + 4cos(3pt)$ is

1 point

- 3
- 4
- 5
- 7

Q.02. The two harmonic motions given by $x_1(t) = 5sin(\omega t + \pi/3)$ and $x_2(t) = 10sin(\omega t + \pi/4)$. Determine their resultant.

2 points

- $x(t) = 14.88sin(\omega t + 0.277\pi)$
- $x(t) = 12.5sin(\omega t + 0.277\pi)$
- $x(t) = 14.88sin(\omega t + 0.872\pi)$
- $x(t) = 12.5sin(\omega t + 0.872\pi)$

Q.03. A body describes simultaneously two motions, $x_1 = 5sin(50t)$ cm and $x_2 = 6sin(51t)$ cm. What are the maximum and minimum amplitudes of the combined motion and what is the beat frequency?

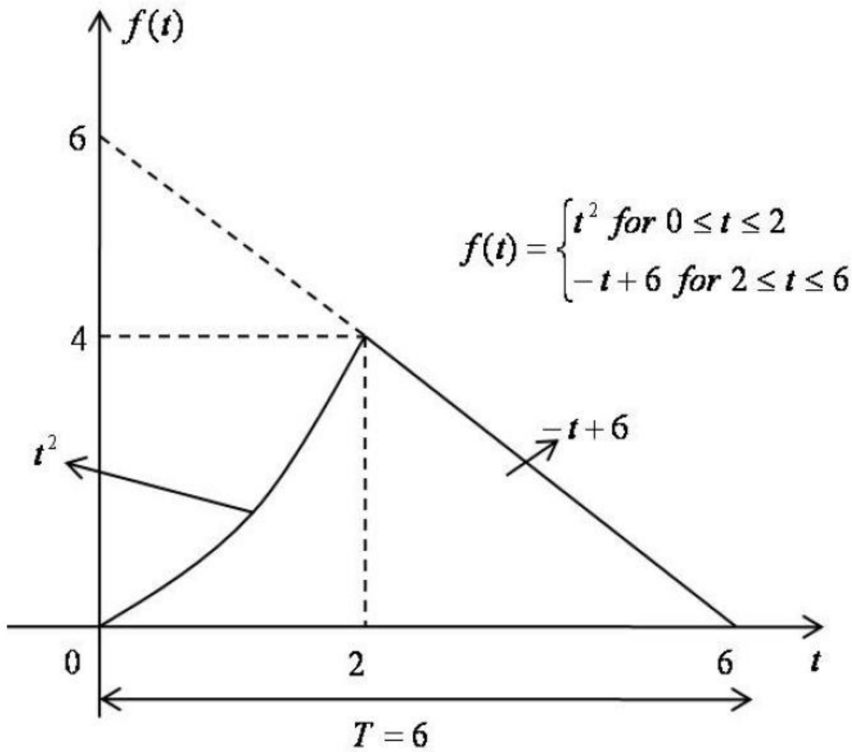
2 points

- 7.8 cm, 1 cm and 1 rad/s
- 7.8 cm, 1 cm and 1.414 rad/s
- 11 cm, 1 cm and 1 rad/s
- 11 cm, 1 cm and 1.414 rad/s

Q.04. Given the following periodic function, *f(t)*

3 points

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(n\omega t) + b_n \sin(n\omega t)\right)$$



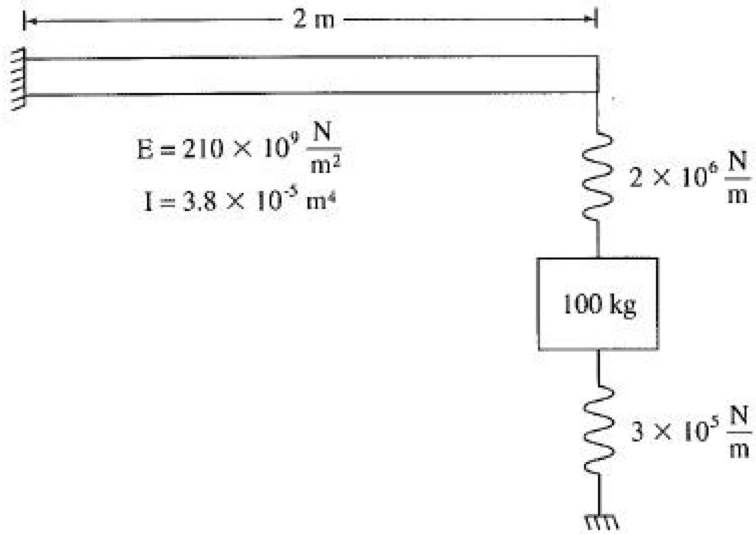
The coefficient a_0 of the continuous Fourier series associated with the above given function f(t) can be computed as

- 8/9
- 16/9
- 24/9
- 32/9

Q.05. Determine the torsional stiffness of the shaft (G = 210 GPa) of length 1.5 m having internal and external radius of the shaft 15 mm and 30 mm respectively. **3 points**

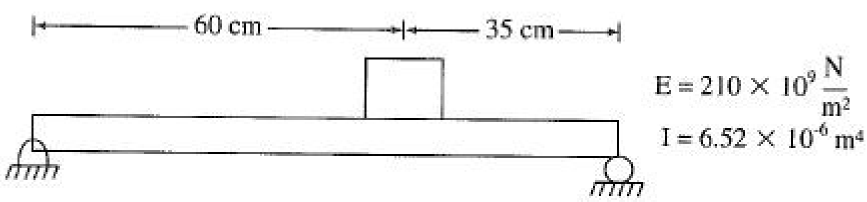
- 134 KN-m/rad
- 89 KN-m/rad
- 60 KN-m/rad
- 167 KN-m/rad

Q.06. Determine the equivalent stiffness (in MN/m) of the system: **3 points**



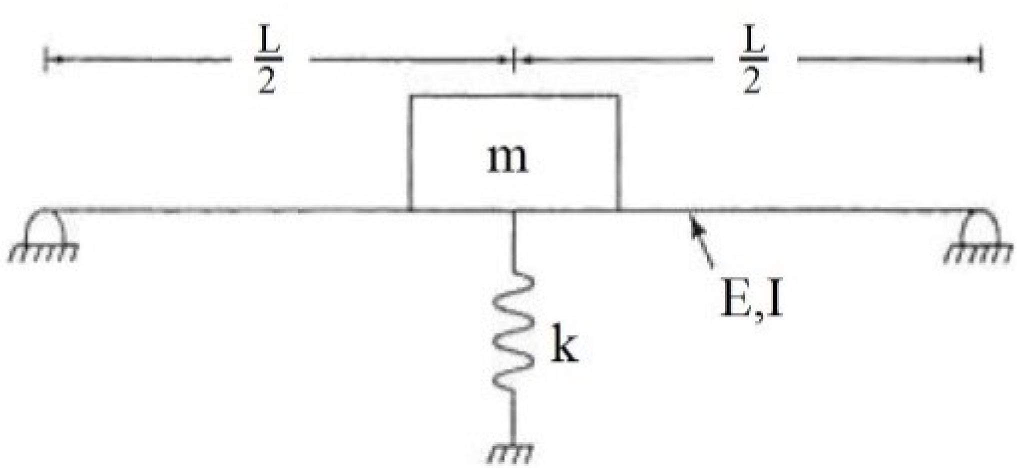
- 1
- 1.5
- 2
- 2.5

Q.07. Determine the equivalent stiffness (MN/m) of the system: **3 points**



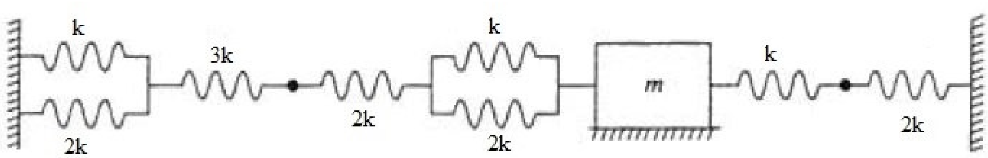
- 78.5
- 82.7
- 88.5
- 95.3

Q.08. Calculate the equivalent stiffness (MN/m) of the system of figure when the deflection of the machine is used as the generalized coordinate. Where, the values E= 210 GPa, I= 200 cm^4, K= 1.2 MN/m, L= 2 **3 points** m are



- 1.25
- 1.41
- 2.23
- 3.72

Q.09. Model the system shown in the figure by a block attached to a single spring of an equivalent stiffness (N/m). The value of each of spring stiffness (k= 6000 N/m) is **3 points**



- 9000
- 8000
- 7000
- 4000

Q.10. There are two dampers which are connected in parallel combination and placed between two moving parts of machine having maximum relative velocity of 2 m/s. Find the maximum damping force exerted by the combination if damping coefficient of the dampers are 7 Ns/m and 14 Ns/m. **2 points**

- 4.67 N
- 10.5 N
- 21 N
- 42 N

Assignment - Week 2

The due date for submitting this assignment has passed.

Due on 2017-02-08, 23:00 IST.

Submitted assignment

- All the questions are compulsory.
- All questions are of multiple choice type.
- There is no negative marking for wrong answer.
- Maximum marks of the assignment is 25.

Q.01. The equation of free vibrations as a system is $\ddot{x} + 36\pi^2 x = 0$. Its natural frequency is *1 point*

- 6 Hz
- 3π Hz
- 3 Hz
- 6π Hz

Q.02. A reciprocating engine, running at 80 rad/s, is supported on springs. The static deflection of the spring is 1 mm. Take g =10 m/s^2. When the engine runs, what will be the natural frequency of the system? *1 point*

- 80 rad/s
- 90 rad/s
- 100 rad/s
- 160 rad/s

Q.03. In case of free vibrations with viscous damping, the damping force is proportional to *1 point*

- Displacement
- Velocity
- Acceleration
- Natural frequency

Q.04. Determine logarithmic decrement, if the amplitude of a vibrating body reduces to 1/6th in two cycles. *2 points*

- 0.223
- 0.8958
- 0.3890
- None of the above

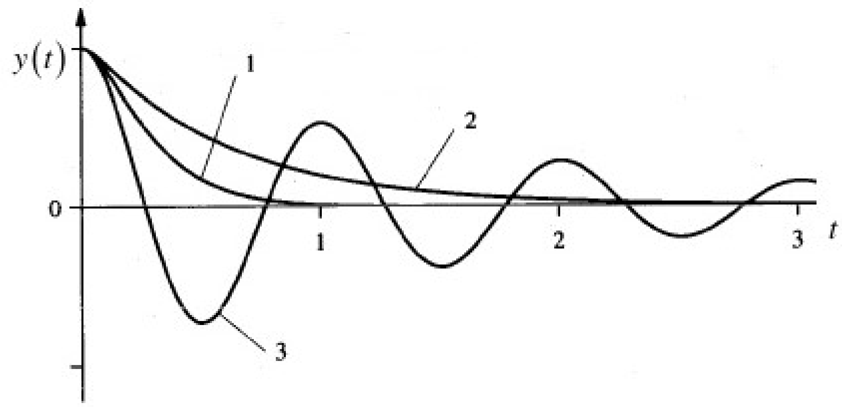
Q.05. Calculate coefficient of viscous damper, if the system is critically damped. Consider the following data: *1 point*

- Mass of spring mass damper system = 350 kg
- Static deflection = 2 mm
- Natural frequency of the system = 60 rad/sec

- 100 KNs/m

- 80 KNs/m
- 42 KNs/m
- None of the above

Q.06. Vibration responses of a system are shown and numbered in figure for various damping conditions. Arrange them in the order of underdamped, critically damped and over damped responses. ***1 point***



- 1,2,3
- 2,3,1
- 3,1,2
- 3,2,1

Q.07. Fill the correct words in the paragraph from the options given below: ***2 points***

Envelope of viscous damping is ……… and it is ………, in case of coulomb damping. Vibrating frequency of system for viscous damping is ……… its natural frequency where as in case of coulomb damping, it is ……… its natural frequency.

(1) less than (2) greater than (3) equal to (4) straight line (5) exponential curve (6) parabolic curve

Choose the correct sequence:

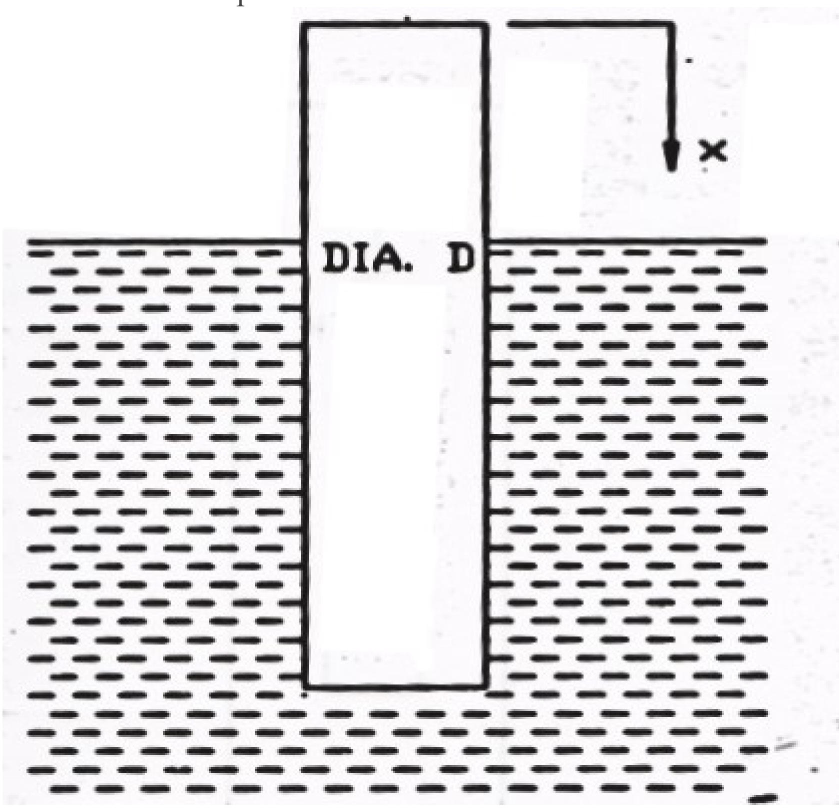
- 4,5,2,1
- 5,4,1,3
- 6,4,2,3
- 5,6,1,2

Q.08. Select the correct arrangement from the following information. ***2 points***

(P) Viscous damping	(1) Dry friction
(Q) Coulomb damping	(2) Microscopic slip
(R) Structural damping	(3) Flow through orifice
(S) Interfacial damping	(4) Internal friction of molecules

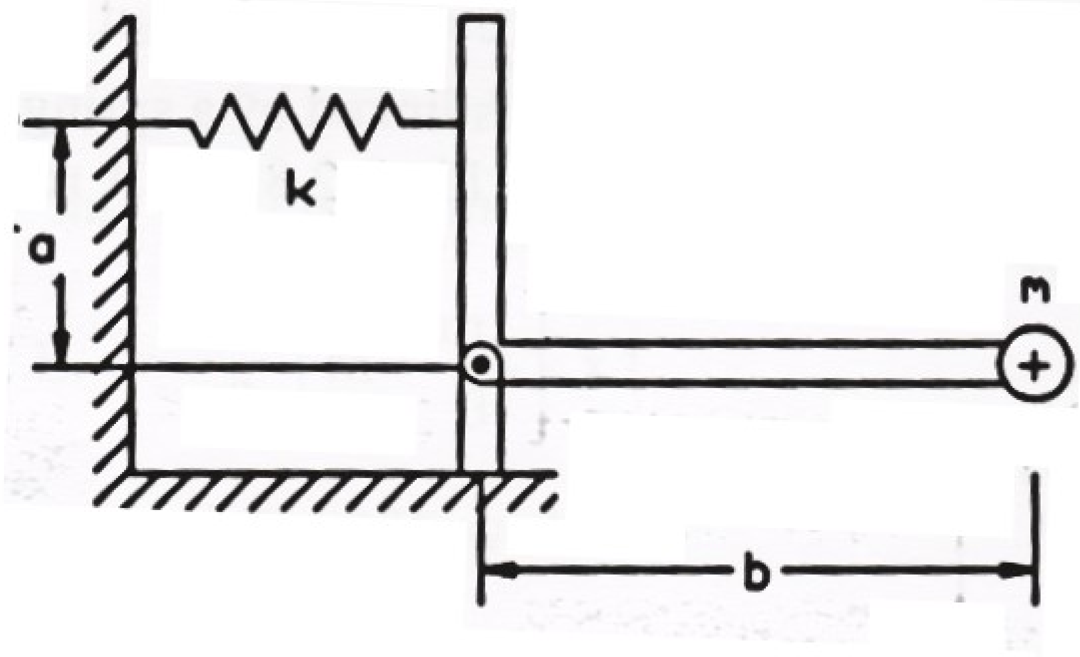
- P3, Q2, R4, S1
- P1, Q2, R3, S4
- P3, Q1, R4, S2
- P4, Q2, R1, S3

Q.09. A cylinder of diameter D and mass M floats vertically in a liquid of mass density ρ as shown in figure. It is depressed slightly and released. Find the period of its oscillation. ***2 points***



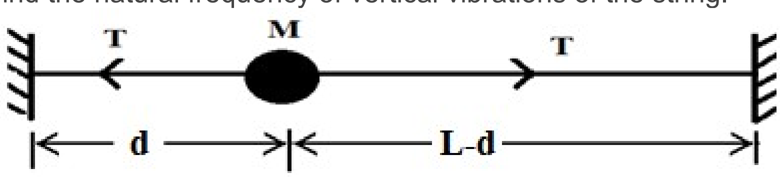
- $2\pi\sqrt{(4M/(\pi\rho g D^2))}$
- $2\pi\sqrt{(M/(4\pi\rho g D^2))}$
- $2\pi\sqrt{(M/(\pi\rho g D^2))}$
- $2\pi\sqrt{(M/\pi\rho g D)}$

Q.10. Find the natural frequency of oscillation for the system shown in figure. Assuming the bell crank lever to be light and stiff and the mass m to be ***2 points*** concentrated.



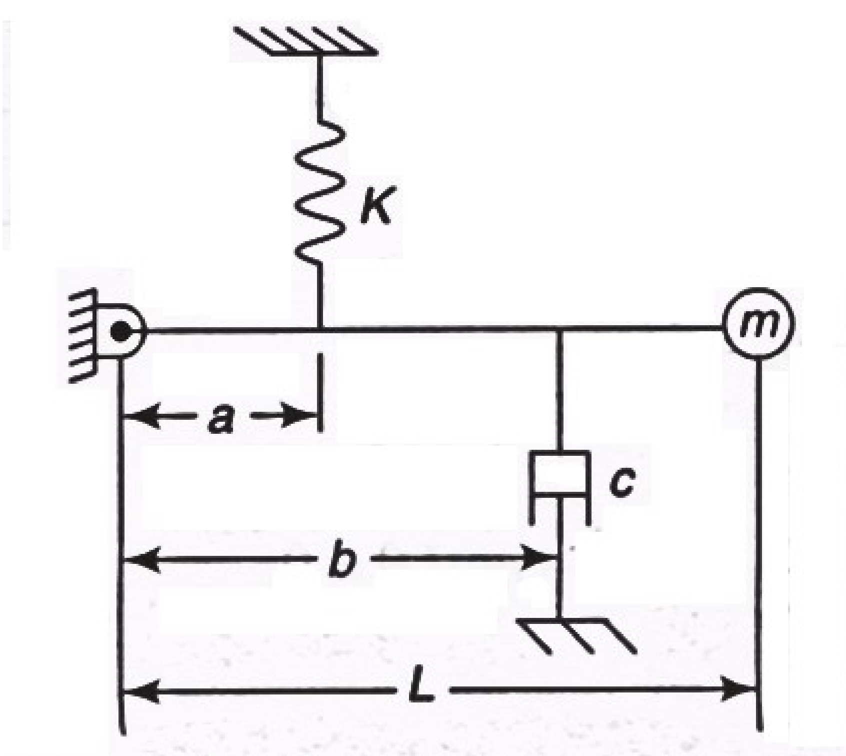
- $\sqrt{(kb^2)/(ma^2)}$
- $\sqrt{(kb/ma)}$
- $\sqrt{(ka/mb)}$
- $\sqrt{(ka^2)/(mb^2)}$

Q.11. A string of length L, under tension T which can be assumed to remain constant for small displacement. A particle of mass M is attached to string at distance d from the left side as shown in figure. For small oscillations, find the natural frequency of vertical vibrations of the string. ***2 points***



- $\sqrt{(T/(m(l-d)))}$
- $\sqrt{(T/(md(l-d)))}$
- $\sqrt{(Tl/(md(l-d)))}$
- $\sqrt{(T/mdl)}$

Q.12. A system shown in figure has c= 20 Ns/m, k= 500 N/m, L= 1 m, b= 0.5 m, a= 0.25 m and m= 10 kg. Find the damping ratio for the system. ***2 points***



- 0.1414
- 0.4135
- 0.5612
- 0.6521

Q.13. A spring mass system having 10 kg mass and 220 N/m spring stiffness. The mass is given an initial displacement of 2 cm and released from rest. In how many cycles the amplitude will be reduced to half of its initial value, Assume system to be viscously damped with damping ratio equal to 0.06. ***2 points***

- 1
- 2
- 3
- 5

Q.14. A spring mass damper system has mass, m=2 kg and spring stiffness, k=500 N/m. An initial amplitude of 1 cm is given to the mass and it is released from rest. After 5 complete cycles its amplitude is found to be 0.5 cm. Determine the friction force, assuming the damping to be purely Coulomb. ***2 points***

- 0.125
- 0.250
- 1.125
- 3.125

Q.15. A free vibration test is conducted on a cantilever beam. For this, a cable attached to the end of beam applies a lateral force of 1000N and pulls the beam horizontally by 2 cm. The cable is suddenly cut and the resulting free vibration is recorded. At the end of four complete cycles the time is 2 sec and the amplitude is 1 cm. Find damping ratio. ***2 points***

- 0.1000
- 0.0775

- 0.0550
- 0.0275

Assignment - Week 3

The due date for submitting this assignment has passed.

Due on 2017-02-15, 23:00 IST.

Submitted assignment

- All the questions are compulsory.
- All questions are of multiple choice type.
- There is no negative marking for wrong answer.
- Maximum marks of the assignment is 25.

Q.01. In forced vibration problem, ………. represents the steady state response and ………. represents the transient response. The frequency of forced vibrating body is equal to the ………. .
Opt the order of option to correctly fill the above lines.
(1) Complementary function (2) Particular integral (3) Applied frequency (4) Natural frequency

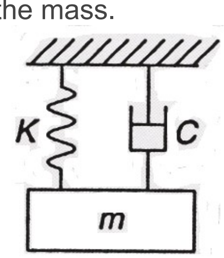
1 point

- 1,2,4
- 1,3,2
- 2,1,3
- 2,1,4

Q.02. Calculate the steady state response of the mass shown in figure, if an exciting force, F(t)=30 sin(15t) N, acts on the mass.

2 points

k = 9000 N/m
c = 12 Ns/m
m = 10 kg



- $4.44 sin(15t - 0.027)$ mm
- $4.44 sin(15t - 0.027\pi)$ mm
- $14.11 sin(15t - 0.027)$ mm
- $14.11 sin(15t - 0.027\pi)$ mm

Q.03. Magnification factor is maximum when $r = (\omega/\omega_n)$ is

2 points

- $\sqrt{(1-2\xi^2)}$
- $\sqrt{(1-\xi^2)}$
- $\xi\sqrt{(1-\xi^2)}$
- 2ξ

Q.04. In frequency response curve, the maximum value of the magnification factor is **2 points**

- $\xi/\sqrt{(1-\xi^2)}$
- $1/(2\xi\sqrt{(1-\xi^2)})$
- $1/(\xi\sqrt{(1-\xi^2)})$
- $1/(1-\xi^2)$

Q.05. A 50 kg machine is mounted on four parallel springs each of stiffness 0.25 MN/m. When the machine operates at 40 Hz, the machine's steady state amplitude is measured as 2 mm. What is the magnitude of the excitation force provided to the machine at this speed? **2 points**

- 4336.7 N
- 5336.2 N
- 1542.7 N
- 6823.5 N

Q.06. A single cylinder vertical petrol engine of total mass 350 kg is mounted upon a steel chassis frame and causes a vertical static deflection of 0.25 cm. The reciprocating parts of the engine have a mass of 28 kg and move through a vertical stroke of 15 cm with S.H.M. A dashpot is provided, the damping resistance of which is directly proportional to the velocity and amounts to 520 N at 0.4 m/s. Determine the amplitude of steady state forced vibrations when the driving shaft of the engine rotates at 500 rpm. **2 points**

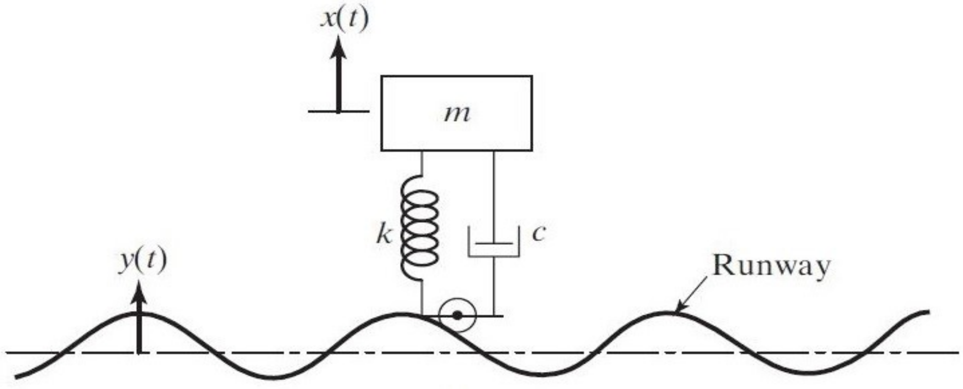
- 6.2 mm
- 10.3 mm
- 13.8 mm
- 16.2 mm

Q.07. A motor of mass 100 kg is mounted on a simple beam that has a stiffness of 160000 N/m at that point. The rotor of the motor has a mass of 10 kg and has an eccentricity of 1 mm. What will be the amplitude of vibration of the motor when it runs at 1500 rpm? Neglect the damping and weight of the beam and the deflection of the motor shaft. **2 points**

- 0.11 mm
- 1.1 mm
- 2.3 mm
- 3.6 mm

Q.08. The landing gear of an airplane can be idealized as the spring-mass-damper system shown in figure. If the runway surface is described y(t) = y_0 cos ωt, determine the value of c that limits the amplitude of vibration of the airplane (x) to 0.1 m. Assume m = 2000 kg, y_0 = 0.2 m, k = 5 MN/m and ω = **3 points**

157.08 rad/s.



- 32 KNs/m
- 56 KNs/m
- 253 KNs/m
- 159 KNs/m

Q.09. A spring mass system has a natural frequency of 5 Hz. When the mass is at rest, the support is made to move up with displacement y = 25 sin (3πt), where t is in second and y in millimeters, measured from the beginning of the motion. Determine the distance through which the mass moves in the first 0.1 second. ***3 points***

- 15.50 mm
- 18.25 mm
- 22.25 mm
- 26.20 mm

Q.10. Determine the power required to run a motor carrying an eccentric at 2400 rpm when the motor is mounted on a concrete block and the whole assembly is supported by felt pads placed on the foundation. The damping coefficient of the system is 36 KNs/m and the amplitude of vibration is 1.5 mm. ***3 points***

- 1560 watt
- 1930 watt
- 2270 watt
- 2560 watt

Q.11. What is the effect of damping on phase angle at resonance frequency? ***1 point***

- Phase angle increases as damping increases
- Damping has no effect on phase angle
- Phase angle increases as damping decreases
- None of the above

Q.12. A single degree of freedom system having mass 1 kg and stiffness 10 KN/m initially at rest is subjected to an impulse force of magnitude 5 KN for 10^{-4} seconds. The amplitude in mm of the resulting free vibration is ***2 points***

- 0.5
- 1.0
- 5.0

○ 10.0

ASSIGNMENT - 1

WEEK - 1

'Fundamental of Vibrations'

25 Marks

Solution →

(1.) $x = 3\sin(3pt) + 4\cos(3pt)$

let, $x = A\sin(3pt + \phi)$

$\Rightarrow A\sin(3pt+\phi) = 3\sin(3pt) + 4\cos(3pt)$

$\Rightarrow \begin{pmatrix} A\sin 3pt \cdot \cos\phi \\ + A\cos 3pt \cdot \sin\phi \end{pmatrix} = 3 \cdot \sin 3pt + 4\cos 3pt$

By comparing on Both sides.

$A\cos\phi = 3 \quad \Rightarrow \quad A = \sqrt{3^2 + 4^2} = 5$

$A\sin\phi = 4$

Amplitude = 5

(2.) $x_1 = 5\sin(\omega t + \pi/3)$,

$x_2 = 10\sin(\omega t + \pi/4)$

$\therefore x = x_1 + x_2 \quad \& \quad x = A\sin(\omega t + \phi)$

$\Rightarrow A\sin(\omega t + \phi) = 5\sin(\omega t + \pi/3) + 10\sin(\omega t + \pi/4)$

$\Rightarrow \begin{bmatrix} A\sin\omega t \cdot \cos\phi \\ + A\cos\omega t \cdot \sin\phi \end{bmatrix} = 5\left[\sin\omega t \cdot \cos\frac{\pi}{3} + \cos\omega t \cdot \sin\pi/3\right] + 10\left[\sin\omega t \cdot \cos\frac{\pi}{4} + \cos\omega t \cdot \sin\pi/4\right]$

$\Rightarrow \begin{bmatrix} (A\cdot\cos\phi)\cdot\sin\omega t \\ + (A\sin\phi)\cdot\cos\omega t \end{bmatrix} = \begin{bmatrix} \left(5\cdot\frac{1}{2} + 10\cdot\frac{1}{\sqrt{2}}\right)\sin\omega t \\ + \left(5\cdot\frac{\sqrt{3}}{2} + 10\cdot\frac{1}{\sqrt{2}}\right)\cdot\cos\omega t \end{bmatrix}$

By comparing on both sides.

$$A \cos\phi = \frac{5+10\sqrt{2}}{2}$$

$$A \sin\phi = \frac{5\sqrt{3}+10\sqrt{2}}{2}$$

$$\Rightarrow \quad A = \sqrt{\left(\frac{5+10\sqrt{2}}{2}\right)^2 + \left(\frac{5\sqrt{3}+10\sqrt{2}}{2}\right)^2} = 14.886$$

$$\Rightarrow \quad \tan\phi = \frac{5\sqrt{3}+10\sqrt{2}}{5+10\sqrt{2}} = 1.191$$

$$\therefore \quad \phi = \tan^{-1}(1.191) = 49.98^\circ = (0.277\pi)\text{rad.}$$

$$\Rightarrow \quad A = 14.886 \quad \& \quad \phi = (0.277\pi)\text{rad.}$$

$$\therefore \quad x = 14.88 \sin(\omega t + 0.277\pi)$$

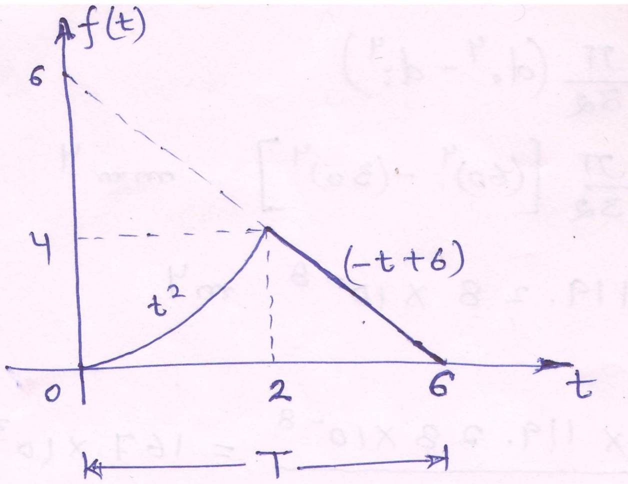
(3.) $x_1 = 5\sin(50t)$ cm, $x_2 = 6\sin(51t)$ cm

max. amplitude $= a + b = 5 + 6 = 11$ cm

min. amplitude $= |a - b| = |5 - 6| = 1$ cm

Beat frequency $= \omega_2 - \omega_1 = 51 - 50 = 1$ rad/s

4.



$$f(t) = \begin{cases} t^2 & , 0 \le t \le 2 \\ -t+6 & , 2 \le t \le 6 \end{cases}$$

$$\therefore \quad T = \frac{2\pi}{\omega} = 6$$

$$\Rightarrow \quad \frac{\omega}{\pi} = \frac{1}{3}$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t \right)$$

$$\therefore \quad a_0 = \frac{\omega}{\pi} \int_t^{t+\frac{2\pi}{\omega}} f(t) \cdot dt$$

$$= \frac{1}{3} \int_0^6 f(t)\, dt = \frac{1}{3}\left[\int_0^2 t^2 dt + \int_2^6 (-t+6)\, dt \right]$$

$$= \frac{1}{3} \times \left[\left| \frac{t^3}{3} \right|_0^2 - \left| \frac{t^2}{2} \right|_2^6 + 6 \left| t \right|_2^6 \right]$$

$$= \frac{1}{3} \times \left[\frac{1}{3} \times 8 - \frac{1}{2}(36-4) + 6(6-2) \right]$$

$$= \frac{1}{3} \left[\frac{8}{3} - 16 + 24 \right]$$

$$\boxed{a_0 = \frac{32}{9}}$$

(5.)

$G = 210\ GPa$

$L = 1.5\ m$

$r_i = 15\ mm$

$r_o = 30\ mm$

$$J = \frac{\pi}{32}\left(d_o^4 - d_i^4\right)$$

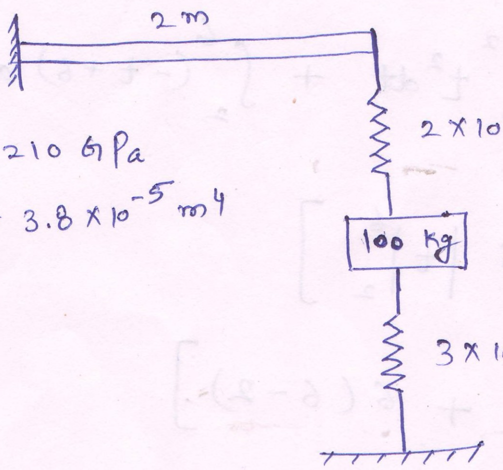
$$J = \frac{\pi}{32}\left[(60)^4 - (30)^4\right]\ mm^4$$

$$J = 119.28 \times 10^{-8}\ m^4$$

$$K_t = \frac{T}{\theta} = \frac{GJ}{l} = \frac{210 \times 10^9 \times 119.28 \times 10^{-8}}{1.5} = 167 \times 10^3\ \frac{Nm}{rad}$$

$$K_t = 167\ KNm/rad$$

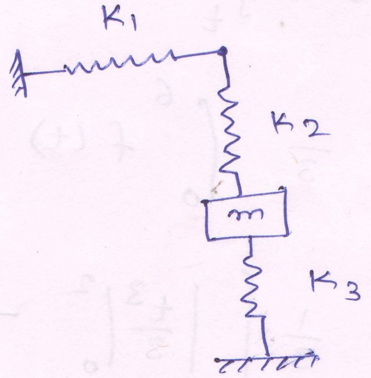
(6.)



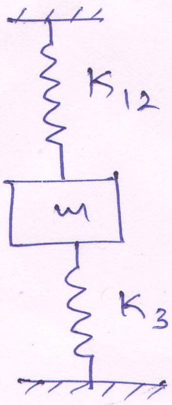
$E = 210\ GPa$

$I = 3.8 \times 10^{-5}\ m^4$

$\equiv$



$$K_1 = \frac{W}{\delta} = \frac{3EI}{L^3} = \frac{3 \times 210 \times 10^9 \times 3.8 \times 10^{-5}}{(2)^3} = 30 \times 10^5\ N/m$$



$$\therefore \frac{1}{K_{12}} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$= \frac{1}{30 \times 10^5} + \frac{1}{2 \times 10^6}$$

$$= \left[\frac{1}{3} + \frac{1}{2}\right] \times \frac{1}{10^6} = \frac{5}{6 \times 10^6}$$

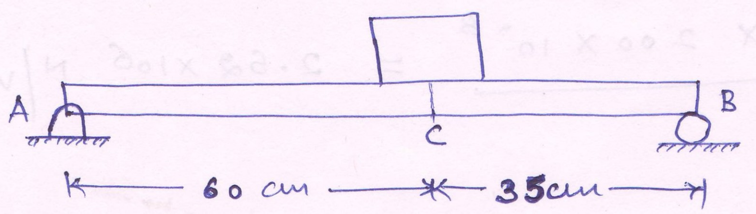
$$\therefore K_{12} = 12 \times 10^5\ N/m$$

$K_{eq} = K_{12} + K_3 = 12 \times 10^5 + 3 \times 10^5 = 15 \times 10^5 \ \frac{N}{m}$

$K_{eq.} = 1.5 \times 10^6 \ N/m$

$K_{eq.} = 1.5 \ MN/m$

7.

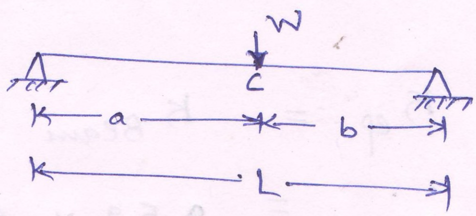


$E = 210 \times 10^9 \ N/m^2$

$I = 6.52 \times 10^{-6} \ m^4$

for the case of simply supported beam;

$$\therefore \ \delta_c = \frac{W a^2 b^2}{3EIL}$$



$$\Rightarrow \ K = \frac{W}{\delta_c} = \frac{3EIL}{a^2 b^2}$$

$$K = \frac{3 \times 210 \times 10^9 \times 6.52 \times 10^{-6} \times 0.95}{(0.6)^2 (0.35)^2}$$

$$K = 88.485 \times 10^6 \ N/m$$

$$\Rightarrow \ K = 88.5 \ MN/m$$

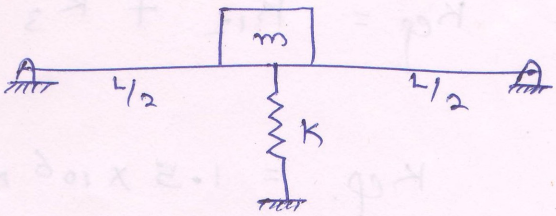
(8.)

$E = 210\ GPa,$

$I = 200\ cm^4,$

$K = 1.2\ MN/m,$

$L = 2\ m$



$$K_{Beam} = \frac{W}{\delta} = \frac{W}{\frac{Wl^3}{48EI}} = \frac{48EI}{l^3}$$

$$K_{Beam} = \frac{48 \times 210 \times 10^9 \times 200 \times 10^{-8}}{(2)^3} = 2.52 \times 10^6\ N/m$$

AS, Beam & spring are in parallel combination because deflection will be same in Both.

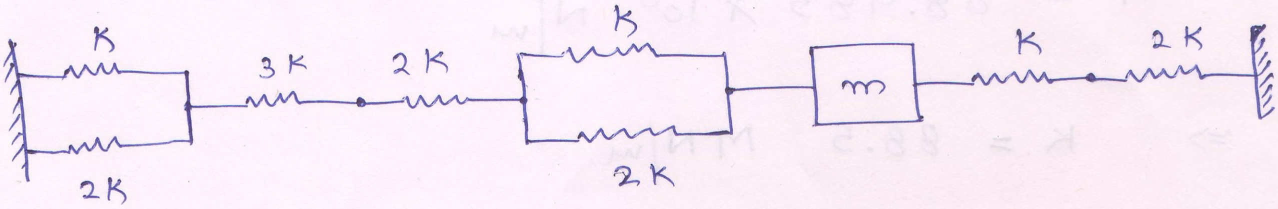
$$\Rightarrow \quad K_{eq.} = K_{Beam} + K$$

$$= 2.52 \times 10^6 + 1.2 \times 10^6$$

$$= 3.72 \times 10^6\ N/m$$

$$K_{eq.} = 3.72\ MN/m$$

(9.)



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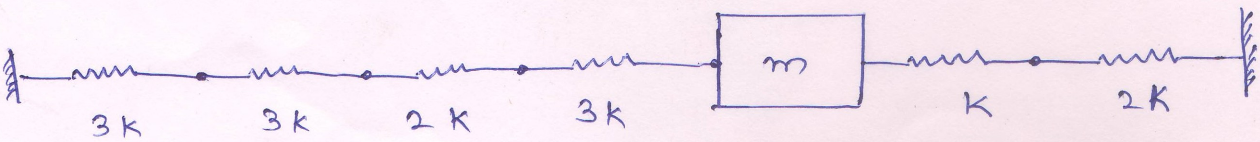
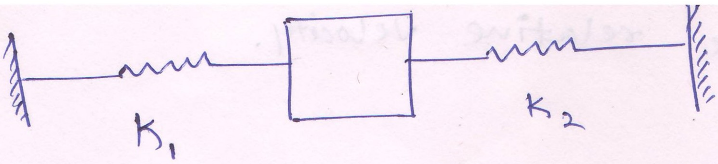




$$\Rightarrow \frac{1}{K_1} = \frac{1}{3K} + \frac{1}{3K} + \frac{1}{2K} + \frac{1}{3K} = \frac{1}{K}\left[\frac{2+2+3+2}{6}\right] = \frac{9}{6K}$$

$$\therefore \quad K_1 = \frac{2K}{3}$$

$$\Rightarrow \frac{1}{K_2} = \frac{1}{K} + \frac{1}{2K} = \frac{3}{2K}$$

$$\therefore \quad K_2 = \frac{2K}{3}$$

$$K_{eq.} = K_1 + K_2 = \frac{2K}{3} + \frac{2K}{3} = \frac{4K}{3}$$

$$K_{eq.} = \frac{4 \times 6000}{3} = 8000$$

$$\Rightarrow \quad K_{eq.} = 8000 \text{ N/m}$$

(10.) Given: max. relative velocity, $(\dot{x}_2 - \dot{x}_1)_{max} = 2$ m/s

Two dampers are in parallel combination.

$$C_1 = 7 \text{ Ns/m}, \quad C_2 = 14 \text{ Ns/m}$$

$$C_{eq.} = C_1 + C_2$$

$$= 7 + 14$$

$$= 21 \text{ Ns/m}$$

$$\text{Force} = C_{eq} \times \text{relative velocity.}$$

$$\therefore \quad F_{max} = C_{eq} \times (\dot{x}_2 - \dot{x}_1)_{max.}$$

$$= 21 \times 2 = 42 \text{ N}$$

$$\Rightarrow \quad F_{max.} = 42 \text{ N}$$

ASSIGNMENT - 2

Solution "Free vibration of SDOF System"

(1.)

$$\ddot{x} + 36\pi^2 x = 0$$

$$\ddot{x} + \omega_n^2 x = 0$$

$$\Rightarrow \omega_n = \sqrt{36\pi^2} = 6\pi \text{ rad/s}$$

$$\Rightarrow f_n = \frac{1}{2\pi}\omega_n = 3 \text{ Hz} \qquad \Rightarrow \boxed{f_n = 3 \text{ Hz}}$$

(2.)

$\omega = 80$ rad/s

$\delta_{static} = 1\text{ mm} = 10^{-3}\text{ m}$

$g = 10\text{ m/s}^2$

$$\omega_n = \sqrt{\frac{g}{\delta_{static}}} = \sqrt{\frac{10}{10^{-3}}} = \sqrt{10^4} = 100 \text{ rad/s}$$

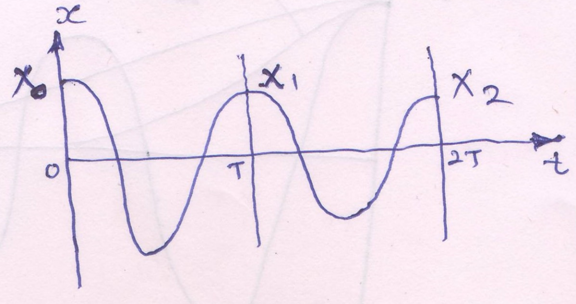
$$\boxed{\omega_n = 100 \text{ rad/s}}$$

(3.) In viscous damping;

$$F = c\dot{x}$$

Force α velocity

(4.)



$$X_2 = \frac{1}{6}^{th} \text{ of } X_0 = \frac{X_0}{6}$$

$$\frac{X_0}{X_1} = \frac{X_1}{X_2} = e^{\delta}$$

$$\frac{x_0}{x_2} = e^{2\delta} \qquad \& \qquad \frac{x_0}{x_2} = 6$$

$$\Rightarrow \quad e^{2\delta} = 6$$

$$\delta = \frac{1}{2} \ln 6 = 0.8958$$

$$\boxed{\delta = 0.8958}$$

(5.)

$m = 350$ kg

$\delta_{static} = 2 \times 10^{-3}$ m

$\omega_n = 60$ rad/s

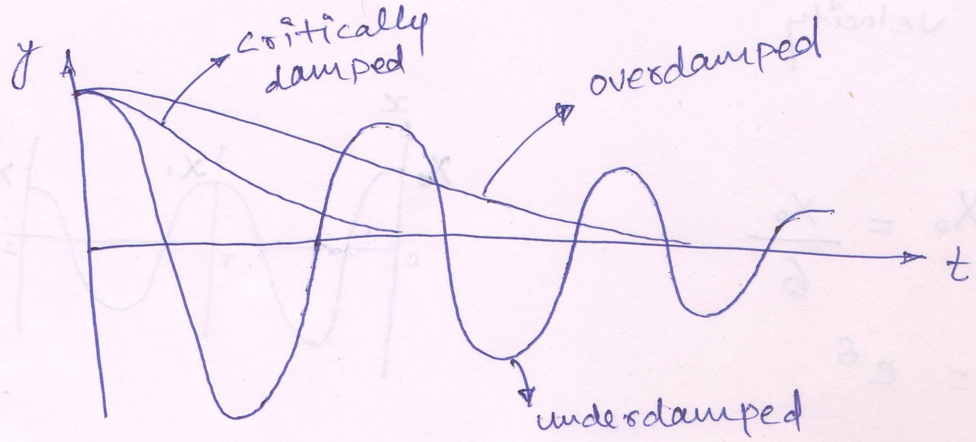
$$\frac{c}{m} = 2 \xi \omega_n$$

if $\xi = 1$, $c = c_c$ [critical damping]

$$c_c = 2 m \omega_n = 2 \times 350 \times 60 = 42000$$

$$c_c = 42 \times 10^3 \ \text{Ns/m}$$

(6.)



(7.) Envelope of viscous damping is exponential curve and it is straight line in case of coulomb damping.

Vibrating frequency of viscous damping is less than its natural frequency where as in case of coulomb damping, it is equal to its natural frequency.

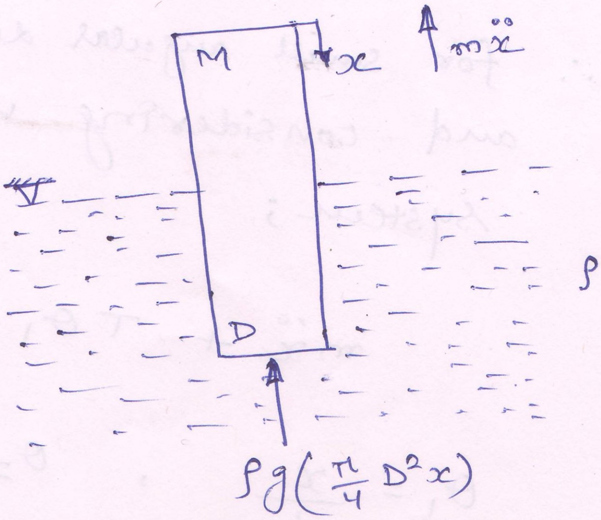
(8.) viscous damping — flow through orifice

coulomb damping — Dry friction

Structural damping — Internal friction of molecules

Interfacial damping — Microscopic slip.

(9.)



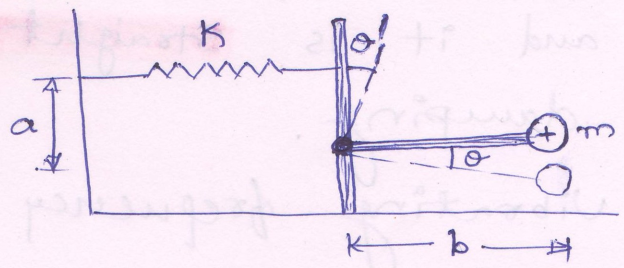
$$M\ddot{x} + \left(\frac{\pi}{4}D^2 x\right)\rho g = 0$$

$$\ddot{x} + \frac{\pi D^2 \rho g}{4M} x = 0$$

$$\Rightarrow \omega_n = \sqrt{\frac{\pi D^2 \rho g}{4M}}$$

$$\Rightarrow T = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{4M}{\pi D^2 \rho g}}$$

(10.) By torque method;

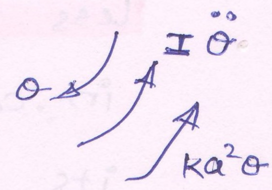


$$I\ddot{\theta} + Ka^2\theta = 0$$

$$\ddot{\theta} + \frac{Ka^2}{I}\theta = 0$$

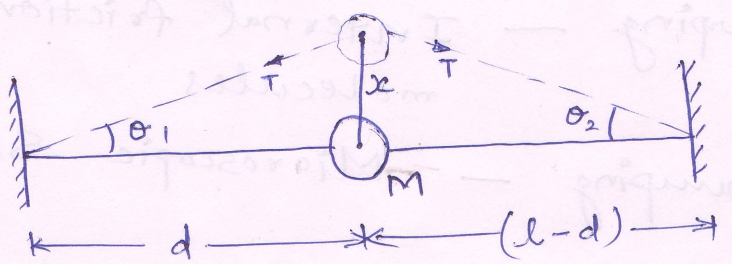
$$\omega_n = \sqrt{\frac{Ka^2}{I}}$$

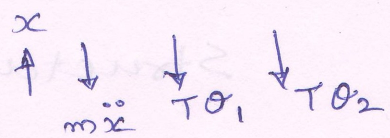
$$\boxed{I = mb^2}$$



$$\omega_n = \sqrt{\frac{Ka^2}{mb^2}}$$

(11.)





∴ for small angular displacement; $\sin\theta \simeq \theta$, $\cos\theta \simeq 1$

and considering vertical vibration for the system;

$$m\ddot{x} + T\theta_1 + T\theta_2 = 0$$

$$\theta_1 = \frac{x}{d}, \quad \theta_2 = \frac{x}{l-d}$$

$$\Rightarrow m\ddot{x} + T\frac{x}{d} + \frac{Tx}{l-d}$$

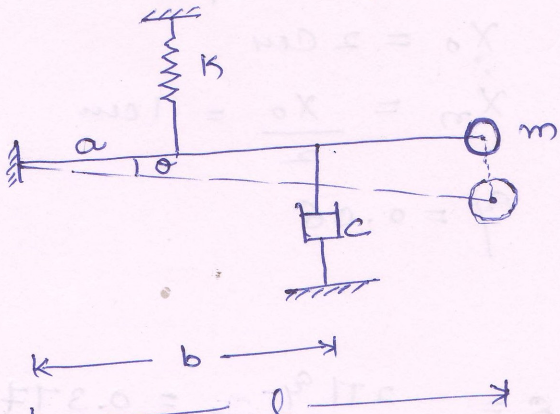
$$\Rightarrow m\ddot{x} + T\frac{(l-d+d)}{d(l-d)}x = 0$$

$$\Rightarrow m\ddot{x} + \frac{Tl\,x}{d(l-d)} = 0$$

$$\ddot{x} + \frac{Tl}{md(l-d)} x = 0$$

$$\Rightarrow \omega_n = \sqrt{\frac{Tl}{md(l-d)}} \text{ rad/s.}$$

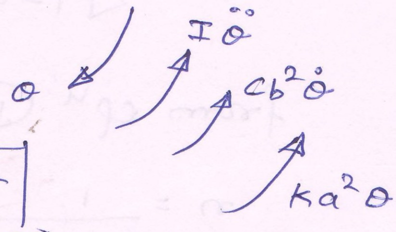
(12.)



$$I\ddot{\theta} + Ka^2\theta + cb^2\dot{\theta} = 0$$

$$ml^2\ddot{\theta} + cb^2\dot{\theta} + Ka^2\theta = 0$$

$$\Rightarrow \ddot{\theta} + \frac{cb^2}{ml^2}\dot{\theta} + \frac{Ka^2}{ml^2}\theta = 0$$



$$\therefore \ddot{\theta} + \frac{c}{m}\dot{\theta} + \frac{k}{m}\theta = 0$$

$$\boxed{I = ml^2}$$

$$\therefore \ddot{\theta} + 2\xi\omega_n\dot{\theta} + \omega_n^2 = 0$$

$$\Rightarrow 2\xi\omega_n = \frac{cb^2}{ml^2}$$

$$\omega_n^2 = \frac{Ka^2}{ml^2}$$

$$\omega_n = \sqrt{\frac{Ka^2}{ml^2}}$$

$$\Rightarrow \xi = \frac{cb^2}{2ml^2} \times \frac{1}{\omega_n}$$

$$\xi = \frac{cb^2}{2ml^2}\sqrt{\frac{ml^2}{Ka^2}}$$

$$\therefore \xi = \frac{cb^2}{2al}\sqrt{\frac{ml^2}{Ka^2}}$$

Given: $c = 20$ Ns/m, $b = \frac{1}{2}$ m, $a = \frac{1}{4}$ m, $l = 1$ m, $m = 10$ Kg

$K = 500$ N/m

$\Rightarrow \quad \xi = 0.1414$

(13.)

$m = 10 kg$

$K = 220 N/m$

$x_0 = 2 cm$

$x_n = \frac{x_0}{2} = 1 cm$

$\xi = 0.06$

$\frac{x_0}{x_n} = e^{n\delta}$

$2 = e^{n\delta}$

$n = \frac{1}{\delta} \ln 2 \quad — (1)$

$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = 0.377$

from eqn (1)

$n = \frac{1}{0.377} \ln 2 = 1.83 \simeq 2$ cycle

$n = 2$ cycle

(14.)

$x_0, x_1, x_2, x_3, x_4, x_5, - - - - -$

$$\boxed{x_0 - x_1 = \frac{4f}{K}}$$ → for coulomb damping

Given

$x_0 = 1 cm, \quad x_5 = 0.5 cm.$

$m = 2 kg, \quad K = 500 N/m$

$\therefore \quad x_0 - x_1 = x_1 - x_2 = x_2 - x_3 = x_3 - x_4 = x_4 - x_5$

$\Rightarrow \quad x_0 - x_5 = 5 \times \left(\frac{4f}{K}\right)$

$(1 - 0.5) \times 10^{-2} = 5 \times \frac{4f}{500}$

$$F = \frac{500 \times 0.5 \times 10^{-2}}{5 \times 4} = \frac{1}{8} \text{ N}$$

$$F = 0.125 \text{ N}$$

(15.) $x_0 = 2$ cm

$x_4 = 1$ cm

$$\Rightarrow \frac{x_0}{x_4} = e^{4\delta}$$

$$\Rightarrow 2 = e^{4\delta} \Rightarrow \boxed{\delta = \frac{1}{4} \ln 2}$$

$$\Rightarrow \delta = 0.1732$$

And $\Rightarrow \delta = \dfrac{2\pi\xi}{\sqrt{1-\xi^2}} = 0.1732$

$$\Rightarrow \xi = 0.0275$$

Assignment : 03

Force vibration of SDOF systems

Q1. In forced vibration problem, Particular integral represents the steady state response and Complementary function represents the transient response. The frequency of forced vibrating body is equal to the applied frequency.

Q2. given $F_0 = 30$ N

$\omega = 15$ rad/s

$K = 9000$ N/m

$c = 12$ Ns/m

$m = 10$ kg

$\omega_n = \sqrt{K/m} = \sqrt{\frac{9000}{10}} = 30$ rad/s , $\frac{\omega}{\omega_n} = \frac{15}{30} = \frac{1}{2}$

$\xi = \frac{c}{2\sqrt{Km}} = \frac{12}{2\sqrt{90000}} = \frac{12}{2\times 300} = 0.02$

Steady state respone $x(t) = \frac{F_0/K}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} \sin(\omega t - \phi)$

$\phi = \tan^{-1}\frac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2} = \tan^{-1}\frac{2\times 0.02\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} = \tan^{-1}\frac{0.02}{\frac{3}{4}}$

$= 0.027$ rad

$A = \frac{F_0/K}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} = \frac{30/9000}{\sqrt{\left(1-\frac{1}{4}\right)^2 + \left(2\times.02\times\frac{1}{2}\right)^2}} = \frac{1/300}{\sqrt{\frac{9}{16}+.0004}}$

$= \frac{1}{300\times\sqrt{.5629}} = \frac{1}{300\times.75027} = 4.44\times10^{-3}$ m

$= 4.44$ mm

$x(t) = 4.44\sin(15t - 0.027)$ mm

Q3

$MF = \frac{1}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$

$\frac{d(MF)}{dr} = \frac{2(1-r^2)(-2r) + 2(2\xi r)\cdot 2\xi}{-2\left[(1-r^2)^2+(2\xi r)^2\right]^{3/2}} = 0$

$$\Rightarrow -4r(1-r^2) + 8\xi^2 r = 0$$

$$\Rightarrow 2\xi^2 = 1-r^2$$

$$\Rightarrow r = \sqrt{1-2\xi^2}$$

Q4 $$MF = \frac{1}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$MF_{\max \text{ at } r=\sqrt{1-2\xi^2}} = \frac{1}{\sqrt{(1-1+2\xi^2)^2+4\xi^2(1-2\xi^2)}}$$

$$= \frac{1}{\sqrt{4\xi^4+4\xi^2-8\xi^4}}$$

$$= \frac{1}{4\xi^2-4\xi^4}$$

$$MF\Big|_{\max \text{ at } r=\sqrt{1-2\xi^2}} = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

Q5 $K = 0.25$ MN/m , $x = 2$ mm

$K_{eq} = 4 \times 0.25 = 1$ MN/m

$m = 50$

$\omega = 40$ Hz $= 40 \times 2\pi = 80\pi$ rad/s

$$\omega_n = \sqrt{\frac{K_{eq}}{m}} = \sqrt{\frac{1000000}{50}} = 141.42 \text{ rad/s}$$

$$r = \frac{\omega}{\omega_n} = 1.78$$

$$M = \frac{1}{r^2-1} \quad \text{for } r>1$$

$$= 0.461169$$

$$F_0 = \frac{m\omega_n^2 x}{M} = \frac{50 \times (141.42)^2 \times 0.002}{.461169} = 4.3367 \text{ KN}$$

$=$

Q6 $m = 350$ kg

$\Delta st = 0.0025$ m

$\omega = 500$ rpm $= \frac{500 \times 2\pi}{60} = 52.3598$ rad/s

$m_0 = 28$ kg

$e = \frac{0.15}{2}$ m $= 0.075$ m

$c = 520/0.4 = 1300$ Ns/m

$\omega_n = \sqrt{g/\delta} = \sqrt{\frac{9.8}{.0025}} = 62.6099$ rad/s

$r = \frac{\omega}{\omega_n} = \frac{52.3598}{62.6099} = 0.83628$

$\xi = \frac{c}{2m\omega_n} = 0.02966$

$\frac{m_0}{m} = \frac{28}{350} = 0.08$

$$\frac{X}{\frac{m_0}{m}e} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\Rightarrow \frac{X}{.08 \times .075} = \frac{(.83628)^2}{\sqrt{(1-(.83628)^2)^2 + (2 \times 0.02966 \times .83628)^2}}$$

$\Rightarrow X = 13.77$ mm

Q7 $m = 100$ kg

$K = 16,0000$ N/m

$m_0 = 10$ kg

$e = 1$ mm

$\omega = 1500$ rpm $= \frac{1500 \times 2\pi}{60} = 157.0796$ rad/s

$\omega_n = \sqrt{\frac{160000}{100}} = 40$ rad/s

$r = \frac{\omega}{\omega_n} = 3.92699$

$$\frac{X}{\frac{m_0}{m}e} = \frac{r^2}{r^2-1} \quad \text{for } r > 1$$

$$X = \frac{(3.92699)^2 \times \frac{1}{10} \times 10^{-3}}{3.92699^2 - 1} = 0.1069 \text{ mm}$$

$$= 0.11 \text{ mm}$$

Q8

$m = 2000$ kg

$K = 5$ MN/m

$\omega = 157.08$ rad/s

$y_0 = 0.2$ m

$X_0 = 0.1$ m

$$\omega_n = \sqrt{\frac{5000000}{2000}} = 50 \text{ rad/s}$$

$$r = \frac{\omega}{\omega_n} = 3.1416$$

Amplitude of vibration under base excitation.

$$\frac{X}{Y} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$\Rightarrow \frac{0.1}{0.2} = \frac{\sqrt{1+(2\xi \times 3.1416)^2}}{\sqrt{(1-r^2)^2+(2\xi \times 3.1416)^2}}$$

$$\Rightarrow \frac{1}{4} = \frac{1+39.4786\xi^2}{78.6707+39.4786\xi^2}$$

$$\Rightarrow 78.6707 + 39.4786\xi^2 = 4 + 4 \times 39.4786\xi^2$$

$$\Rightarrow \xi^2 = 0.63047$$

$$\Rightarrow \xi = 0.79402$$

$$C = \xi C_c = 0.79402 \times 2 \times m\omega_n$$

$$= 0.79402 \times 2 \times 2000 \times 50$$

$$= 158804 \text{ Ns/m}$$

Q9

$$m\ddot{x} + K(x-y) = 0$$

$$\Rightarrow m\ddot{x} + Kx = Ky$$

$$m\ddot{x} + Kx = KY \sin\omega t$$

$\omega_n = 5$ Hz $= 10\pi$ rad/s

$Y = 25$ mm

$\omega = 3\pi$ rad/s

$t = 0.1$

$r = \frac{\omega}{\omega_n} = \frac{3\pi}{10\pi} = 0.3$

$x(t) = X_c + X_p = A\cos\omega_n t + B\sin\omega_n t + X\sin(\omega t + \alpha - \phi)$

$X = \frac{Y}{1-r^2} = \frac{25}{1-.3^2} = 27.4725 = 27.5$

$\phi - \alpha = 0$ if $r<1$

$= 180°$ if $r>1$

$X_p = 27.5 \sin 3\pi t$

$x(t) = A\cos\omega_n t + B\sin\omega_n t + 27.5\sin\omega t$

At $t = 0$, $x = 0$

$\dot{x} = 0$

$0 = A + B \times 0 \Rightarrow A = 0$

$0 = A\omega_n \cdot 0 + B\omega_n \cdot 1 + 27.5\,\omega \cdot 1$

$\Rightarrow B = -27.5\left(\frac{\omega}{\omega_n}\right)$

$x(t) = -8.25\sin\omega_n t + 27.5\sin\omega t$

$x(0.1) = -8.25\sin(10\pi \times 0.1) + 27.5\sin(3\pi \times 0.1)$

$= 22.24796$

$= \underline{22.25}$ mm

Q 10 $\omega = 2400$ rpm $= \frac{2400 \times 2\pi}{60} = 80\pi$ rad/s

$C = 36000$ Ns/m

$x = 1.5 \times 10^{-3}$ m

WD/cycle $= \pi C \omega x^2 = \pi \times 36000 \times (80\pi) \times (1.5 \times 10^{-3})^2$

WD/sec $= \pi C \omega x^2 \times \frac{\omega}{2\pi} = \pi \times 3600 \times 80\pi \times (1.5\times10^{-3})^2 \times 40$

$= 2558.20$ J/s

Q11 Damping has no effect on phase angle at resonance frequency. phase angle at this condition is always $\pi/2$.

Q12 $m = 1$ kg

$K = 10$ KN/m $= 10000$ N/m

$F = 5000$ N

$t = 10^{-4}$

$\omega_n = \sqrt{K/m} = \sqrt{\frac{10000}{1}} = 100$ rad/s

Impulse force provide displacement and velocity to the system at $t = 10^{-4}$ sec.

$a = F/m = 5000$ m/s^2

$v = u + at = 0 + 5000 \times 10^{-4} = .5$ m/s

~~$s = ut + \frac{1}{2}at^2 = \frac{1}{2} \times 5000 \times 10^{-8} = 25 \times 10^{-6} = 25 \times 10^{-3}$ mm $= .025$ mm~~

$\frac{1}{2} K x^2 = \frac{1}{2} m v^2$

$\frac{1}{2} \times 10000\, x^2 = \frac{1}{2} \times 1 \times (.5)^2$

$\Rightarrow x = 5$ mm

2nd Method

$x(t) = x_0 \cos\omega_n t + \frac{\dot{x}_0}{\omega_n} \sin\omega_n t$

$= \left[x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2\right]^{1/2} \sin(\omega_n t + \phi)$

Amplitude $= \left[x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2\right]^{1/2}$

$x_0 = ut + \frac{1}{2}at^2 = 25 \times 10^{-6}$

$\dot{x}_0 = v = 0.5$ m/s

Amplitude $= \left[(25 \times 10^{-6})^2 + \left(\frac{.5}{100}\right)^2\right]^{1/2} \approx 5 \times 10^{-3}$

≈ 5 mm

ASSIGNMENT - 4

'Forced vibration of SDoF systems'

25 Marks

Solution →

1. $X = 2\text{ cm}$, $m = 2\text{ kg}$

$f = 150\text{ Hz}$

$\omega = 2\pi f = 300\pi \text{ rad/s}$

$\xi = 0.06$

$f_d = 30\text{ Hz}$

$\omega_d = 2\pi f_d = 60\pi \text{ rad/s}$

As ξ is very small; $\omega_d \simeq \omega_n$

$\Rightarrow \omega_d = \sqrt{1-\xi^2}\,\omega_n$

$$\boxed{\omega_d \simeq \omega_n}$$

$\omega_n = 60\pi \text{ rad/s}$

$c = 2m\omega_n\xi = 2\times 2\times 60\pi \times 0.06 = 45.24$

$c = 45.24\text{ Ns/m}$

$\therefore$ Energy Dissipated per cycle $= \pi c \omega X^2$

$= \pi \times 45.24 \times 300\pi \times (0.02)^2$

$= 53.58\text{ J/cycle}$

$\therefore$ Energy Dissipated per sec. $\Rightarrow$

$\Rightarrow$ **53.58** J/cycle $\times f$ cycle/s

$\Rightarrow 53.58 \times 150$ J/s

$\Rightarrow 8037.01$ W

$\Rightarrow 8.037$ KW

(2) $m = 75\ kg$, stroke = 0.08

$K = 11.76 \times 10^5\ N/m$

$\xi = 0.2$

$\therefore \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{11.76 \times 10^5}{75}} = 125\ rad/s$

$N = 3000\ rpm$

$\therefore \omega = \frac{2\pi N}{60} = \frac{2\pi \times 3000}{60} = 100\pi\ rad/s$

$\Rightarrow r = \frac{\omega}{\omega_n} = \frac{100\pi}{125} = 2.51$

& $m_o = 2\ kg$, $e = \frac{0.08}{2} = 0.04\ m$

$\therefore F_o = m_o e \omega^2 = 2 \times 0.04 \times (100\pi)^2 = 7900\ N$

$\therefore$ Transmissibility ratio;

$$T = \frac{F_T}{F_o} = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\frac{F_T}{7900} = \frac{\sqrt{1 + (2 \times 0.2 \times 2.51)^2}}{\sqrt{(1-(2.51)^2)^2 + (2 \times 0.2 \times 2.51)^2}}$$

$\Rightarrow F_T = 2078\ N$

(3.) $m = 5\,kg$, $F = \mu mg = 12.26\,N$.

$K = 8000\,N/m$

$\mu = 0.25$

$F_0 = 25\,N$

$f = 6\,Hz$

$\omega = 2\pi f = 12\pi\ rad/s$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{8000}{5}} = 40\ rad/s$$

$$r = \frac{\omega}{\omega_n} = \frac{12\pi}{40} = 0.9424$$

$$\therefore \frac{X}{F_0/K} = \frac{\sqrt{1-\left(\frac{4F}{\pi F_0}\right)^2}}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

$$\Rightarrow \frac{X}{25/8000} = \frac{\sqrt{1-\left(\frac{4\times 12.26}{\pi\times 25}\right)^2}}{1-(0.9424)^2}$$

$$\Rightarrow X = 0.0218$$

$$\therefore C_{eq} = \frac{4F}{\pi\omega X} = \frac{4\times 12.26}{\pi\times 12\pi\times 0.0218} = 19\ N\text{-}s/m$$

(4.) Load $= 5000\,N$

$X_{static} = 0.05\,m$

$F_0 = 1000\,N$

$X = 0.1\,m$

At resonace,

$$X = \frac{X_{static}}{2\xi}$$

$$\Rightarrow 0.1 = \frac{0.05}{2\xi}$$

$$\Rightarrow \xi = 0.25$$

f $c = \frac{F_0}{X\omega}$

$\omega_n = \sqrt{\frac{g}{X_{static}}} = \sqrt{\frac{9.812}{0.05}} = 14.007 \text{ rad/s}$

for resonance, $\omega = \omega_n$

$c = \frac{F_0}{X\omega} = \frac{1000}{0.1 \times 14.007} = 714 \text{ Ns/m}$

Energy Dissipated / cycle $= \pi c \omega X^2$

$= \pi \times 714 \times 14.007 \times (0.1)^2$

$= 314 \text{ J/cycle.}$

(5.)

$K = 60000 \text{ N/m}$

$X_1 = 40 \text{ mm}$, $\Delta w_1 = 3.8 \text{ Nm}$

$X_2 = ?$, $\Delta w_2 = 9.5 \text{ Nm}$

$\Delta w \longrightarrow$ change in energy.

$\Delta w = \pi \beta K X^2$

$\therefore \frac{\Delta w_1}{\Delta w_2} = \frac{X_1^2}{X_2^2}$

$\Rightarrow X_2 = \sqrt{\frac{\Delta w_2}{\Delta w_1}} \times X_1$

$\Rightarrow X_2 = \sqrt{\frac{9.5}{3.8}} \times 40 = 63.25 \text{ mm}$

$X_2 = 63.25 \text{ mm}$

(6.) $m = 60\,Kg$, $\xi = 0.1$

$\omega^2 Y$ = floor's acceleration amplitude = $1.5\, m/s^2$

$\omega^2 X$ = equipments acc. amplitude = $0.75\, m/s^2$

$$\therefore \quad T = \frac{\omega^2 X}{\omega^2 Y} = \frac{0.75}{1.5} = 0.5$$

$f = 90\, Hz$

$\Rightarrow \omega = 180\pi$ rad/s

$$\therefore \quad T = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \Rightarrow \underline{r = 1.76}$$

$$\therefore \quad r = \frac{\omega}{\omega_n} \Rightarrow \omega_n = \frac{\omega}{r} = \frac{180\pi}{1.76} = 321.3 \text{ rad/s}$$

$\Rightarrow K = m\,\omega_n^2$

$K = 6.19$ MN/m

(7.) $m = 10\, Kg$

$Y = 4\, mm$

$N = 2500$ rpm

Acceleration amplitude of device = 5g

$\xi = 0.05$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2500}{60} = 261.8 \text{ rad/s}$$

$\therefore \omega^2 Y = 274.15\, m/s^2$

Maximum allowable transmissibility ratio;

$$\therefore \quad T_{max} = \frac{\omega^2 X}{\omega^2 Y} = \frac{5g}{274.15} = 0.1789$$

$$T_{max} = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \Rightarrow \underline{r_{min} = 2.60}$$

Since the isolator is placed b/w the floor & flow-monitoring device, its deformation is equal to relative displacement b/w floor & device.

$$\frac{Z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\frac{Z}{4\text{ mm}} = \frac{(2.6)^2}{\sqrt{(1-(2.6)^2)^2 + (2 \times 0.05 \times 2.6)^2}}$$

$$\Rightarrow Z = 4.69 \text{ mm}$$

(8.) $\Delta_{static} = 0.1\text{ m}$, $Y = 0.08$ m

$\lambda = 14$ m

$v = 60$ Km/hr $= \frac{60 \times 1000}{3600} = 16.67$ m/s

$\therefore f = \frac{v}{\lambda} = \frac{16.67}{14} = 1.19$ Hz

$\omega = 2\pi f = 7.48$ rad/s

$\therefore \omega_n = \sqrt{\frac{g}{\Delta_{static}}} = \sqrt{\frac{9.81}{0.1}} = 9.9$ rad/s

$\Rightarrow r = \frac{\omega}{\omega_n} = \frac{7.48}{9.9} = 0.755$

for $\xi = 0$,

$$\frac{X}{Y} = \left| \frac{1}{1-r^2} \right|$$

$$\frac{X}{0.08} = \left| \frac{1}{1-(0.755)^2} \right|$$

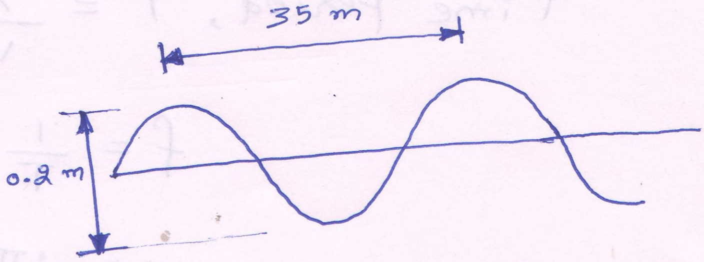
$$\Rightarrow \underline{X = 0.186 \text{ m}}$$

(9.) $f_n = 3$ Hz

$\omega_n = 6\pi$ rad/s

$\xi = 0.2$

$v = 60$ km/hr $= \frac{100}{6}$ m/s



$$Y = \frac{0.2}{2} = 0.1$$

$\therefore$ Time period, $T = \frac{\lambda}{v} = \frac{35}{100/6}$

$$T = 2.1 \text{ sec.}$$

$$\Rightarrow f = \frac{1}{T} = \frac{10}{21} \text{ Hz}$$

$$\therefore r = \frac{\omega}{\omega_n} = \frac{f}{f_n} = \frac{10/21}{3} = 0.1587$$

$$\therefore \frac{X}{Y} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$\Rightarrow \frac{X}{0.1} = \frac{\sqrt{1+(2\times0.2\times0.1587)^2}}{\sqrt{(1-(0.1587)^2)^2+(2\times0.2\times0.1587)^2}}$$

$$\Rightarrow X = 0.10257 \text{ m}$$

$$\Rightarrow \underline{X = 10.25 \text{ cm}}$$

(10.) $m = 1500\ kg$

$v = 90\ km/h = \frac{90 \times 1000}{3600} = 25\ m/s$

$\lambda = 3.7\ m$

$Y = 0.1\ m$

Time period, $T = \frac{\lambda}{v} = \frac{3.7}{25} = 0.148$ sec

$f = \frac{1}{T} = 6.756$ Hz

$\omega = 2\pi f = 42.454$ rad/s

$K = 450 \times 10^3$ N/m

$\xi = 0.2$

$\therefore \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{450 \times 10^3}{1500}} = 17.32$ rad/s

$\therefore r = \frac{\omega}{\omega_n} = 2.45$

$$\therefore \frac{X}{Y} = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\Rightarrow \frac{X}{0.1} = \frac{\sqrt{1 + (2 \times 0.2 \times 2.45)^2}}{\sqrt{(1-(2.45)^2)^2 + (2 \times 0.2 \times 2.45)^2}}$$

$\Rightarrow X = 0.0274$ m

$\Rightarrow$ $X = 2.74$ cm

(11.) at, $r = \frac{\omega}{\omega_n} = \sqrt{2}$

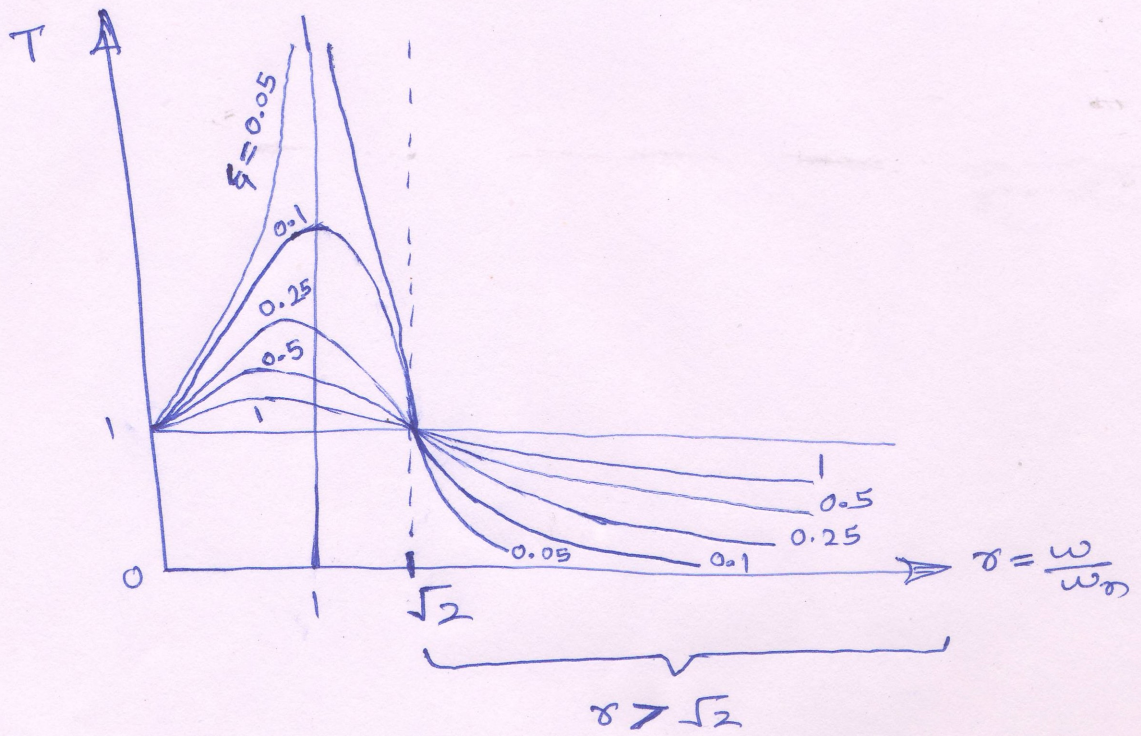
$$\therefore \quad T = \frac{F_T}{F_0} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$\Rightarrow \quad T = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-(\sqrt{2})^2)^2+(2\xi r)^2}}$$

$$\Rightarrow \quad T = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{1+(2\xi r)^2}} = 1$$

$$\Rightarrow \quad \boxed{T = 1}$$

(12.)



when, $\left(r = \frac{\omega}{\omega_n}\right) > \sqrt{2}$;

In this region, if we go for increasing the damping, i.e. increasing the value of ξ, Transmissibility ratio increases.

$\Rightarrow$ Transmitted force tends to increase.

$\therefore$ Hence increasing the damping in this region proves to be detrimental.

'Forced vibration of SDoF systems' 25 Marks

Solution →

(1.) $X = 2\ cm$, $m = 2\ kg$

$f = 150\ Hz$

$\omega = 2\pi f = 300\pi\ rad/s$

$\xi = 0.06$

$f_d = 30\ Hz$

$\omega_d = 2\pi f_d = 60\pi\ rad/s$

As ξ is very small; $\omega_d \simeq \omega_n$

$\Rightarrow \omega_d = \sqrt{1-\xi^2}\ \omega_n$

$\boxed{\omega_d \simeq \omega_n}$

$\omega_n = 60\pi\ rad/s$

$c = 2m\omega_n\xi = 2 \times 2 \times 60\pi \times 0.06 = 45.24$

$c = 45.24\ Ns/m$

$\therefore$ Energy Dissipated per cycle $= \pi c \omega X^2$

$= \pi \times 45.24 \times 300\pi \times (0.02)^2$

$= 53.58\ J/cycle$

$\therefore$ Energy Dissipated per sec. $\Rightarrow$

$\Rightarrow$ **53.58** $J/cycle \times f\ cycle/s$

$\Rightarrow 53.58 \times 150\ J/s$

$\Rightarrow 8037.01\ W$

$\Rightarrow 8.037\ KW$

(2) $m = 75$ kg , stroke $= 0.08$

$K = 11.76 \times 10^5$ N/m

$\xi = 0.2$

$$\therefore \quad \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{11.76 \times 10^5}{75}} = 125 \text{ rad/s}$$

$N = 3000$ rpm

$$\therefore \quad \omega = \frac{2\pi N}{60} = \frac{2\pi \times 3000}{60} = 100\pi \text{ rad/s}$$

$$\Rightarrow \quad r = \frac{\omega}{\omega_n} = \frac{100\pi}{125} = 2.51$$

& $m_o = 2$ kg , $e = \frac{0.08}{2} = 0.04$ m

$$\therefore \quad F_o = m_o e \omega^2 = 2 \times 0.04 \times (100\pi)^2 = 7900 \text{ N}$$

$\therefore$ Transmissibility ratio;

$$T = \frac{F_T}{F_o} = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\frac{F_T}{7900} = \frac{\sqrt{1 + (2 \times 0.2 \times 2.51)^2}}{\sqrt{(1-(2.51)^2)^2 + (2 \times 0.2 \times 2.51)^2}}$$

$$\Rightarrow \quad F_T = 2078 \text{ N}$$

(3.) $m = 5\ Kg$, $F = \mu mg = 12.26\ N$.

$K = 8000\ N/m$

$\mu = 0.25$

$F_0 = 25\ N$

$f = 6\ Hz$

$\omega = 2\pi f = 12\pi\ rad/s$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{8000}{5}} = 40\ rad/s$$

$$r = \frac{\omega}{\omega_n} = \frac{12\pi}{40} = 0.9424$$

$$\therefore \frac{X}{F_0/K} = \frac{\sqrt{1-\left(\frac{4F}{\pi F_0}\right)^2}}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

$$\Rightarrow \frac{X}{25/8000} = \frac{\sqrt{1-\left(\frac{4\times 12.26}{\pi\times 25}\right)^2}}{1-(0.9424)^2}$$

$$\Rightarrow X = 0.0218$$

$$\therefore C_{eq} = \frac{4F}{\pi\omega X} = \frac{4\times 12.26}{\pi\times 12\pi\times 0.0218} = 19\ N\text{-}s/m$$

(4.) Load $= 5000\ N$

$X_{static} = 0.05\ m$

$F_0 = 1000\ N$

$X = 0.1\ m$

At resonace,

$$X = \frac{X_{static}}{2\xi}$$

$$\Rightarrow 0.1 = \frac{0.05}{2\xi}$$

$$\Rightarrow \xi = 0.25$$

f $c = \frac{F_0}{X\omega}$

$\omega_n = \sqrt{\frac{g}{X_{static}}} = \sqrt{\frac{9.812}{0.05}} = 14.007$ rad/s

for resonance, $\omega = \omega_n$

$c = \frac{F_0}{X\omega} = \frac{1000}{0.1 \times 14.007} = 714$ Ns/m

Energy Dissipated / cycle $= \pi c \omega X^2$

$= \pi \times 714 \times 14.007 \times (0.1)^2$

$= 314$ J/cycle.

(5.)

$K = 60000$ N/m

$X_1 = 40$ mm , $\Delta W_1 = 3.8$ Nm

$X_2 = ?$, $\Delta W_2 = 9.5$ Nm

$\Delta W \longrightarrow$ change in energy.

$\Delta W = \pi \beta K X^2$

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$\Rightarrow X_2 = \sqrt{\frac{\Delta W_2}{\Delta W_1}} \times X_1$

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(6.) $m = 60\ Kg$, $\xi = 0.1$

$\omega^2 Y$ = floor's acceleration amplitude = $1.5\ m/s^2$

$\omega^2 X$ = equipments acc. amplitude = $0.75\ m/s^2$

$$\therefore\ T = \frac{\omega^2 X}{\omega^2 Y} = \frac{0.75}{1.5} = 0.5$$

$f = 90\ Hz$

$\Rightarrow \omega = 180\pi\ rad/s$

$$\therefore\ T = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}} \Rightarrow \underline{r = 1.76}$$

$$\therefore\ r = \frac{\omega}{\omega_n} \Rightarrow \omega_n = \frac{\omega}{r} = \frac{180\pi}{1.76} = 321.3\ rad/s$$

$\Rightarrow K = m\,\omega_n^2$

$K = 6.19\ MN/m$

(7.) $m = 10\ Kg$

$Y = 4\ mm$

$N = 2500\ rpm$

Acceleration amplitude of device = ~~26.18~~ $5g$

$\xi = 0.05$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2500}{60} = 261.8\ rad/s$$

$\therefore\ \omega^2 Y = 274.15\ m/s^2$

Maximum allowable transmissibility ratio;

$$\therefore\ T_{max} = \frac{\omega^2 X}{\omega^2 Y} = \frac{5g}{274.15} = 0.1789$$

$$\hookrightarrow T_{max} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}} \Rightarrow \underline{r_{min} = 2.60}$$

since the isolator is placed b/w the floor & flow-monitoring device, its deformation is equal to relative displacement b/w floor & device.

$$\frac{Z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\frac{Z}{4\text{ mm}} = \frac{(2.6)^2}{\sqrt{(1-(2.6)^2)^2 + (2 \times 0.05 \times 2.6)^2}}$$

$$\Rightarrow Z = 4.69 \text{ mm}$$

(8.) $\Delta_{static} = 0.1\text{ m}$, $Y = 0.08\text{ m}$

$\lambda = 14\text{ m}$

$v = 60\text{ Km/hr} = \frac{60 \times 1000}{3600} = 16.67\text{ m/s}$

$\therefore f = \frac{v}{\lambda} = \frac{16.67}{14} = 1.19\text{ Hz}$

$\omega = 2\pi f = 7.48\text{ rad/s}$

$\therefore \omega_n = \sqrt{\frac{g}{\Delta_{static}}} = \sqrt{\frac{9.81}{0.1}} = 9.9\text{ rad/s}$

$\Rightarrow r = \frac{\omega}{\omega_n} = \frac{7.48}{9.9} = 0.755$

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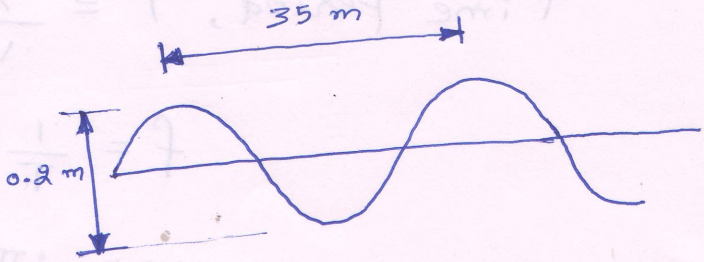
$$\Rightarrow \quad X = 0.186 \text{ m}$$

(9.) $f_n = 3$ Hz

$\omega_n = 6\pi$ rad/s

$\xi = 0.2$

$v = 60$ km/hr $= \frac{100}{6}$ m/s



$Y = \frac{0.2}{2} = 0.1$

$\therefore$ Time period, $T = \frac{\lambda}{v} = \frac{35}{100/6}$

$T = 2.1$ sec.

$\Rightarrow f = \frac{1}{T} = \frac{10}{21}$ Hz

$\therefore r = \frac{\omega}{\omega_n} = \frac{f}{f_n} = \frac{10/21}{3} = 0.1587$

$$\therefore \frac{X}{Y} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$\Rightarrow \frac{X}{0.1} = \frac{\sqrt{1+(2\times0.2\times0.1587)^2}}{\sqrt{(1-(0.1587)^2)^2+(2\times0.2\times0.1587)^2}}$$

$$\Rightarrow X = 0.10257 \text{ m}$$

$$\Rightarrow X = 10.25 \text{ cm}$$

(10.) $m = 1500\ kg$

$v = 90\ km/h = \frac{90 \times 1000}{3600} = 25\ m/s$

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$\omega = 2\pi f = 42.454$ rad/s

$K = 450 \times 10^3$ N/m

$\xi = 0.2$

$\therefore \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{450 \times 10^3}{1500}} = 17.32$ rad/s

$\therefore r = \frac{\omega}{\omega_n} = 2.45$

$$\therefore \frac{X}{Y} = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\Rightarrow \frac{X}{0.1} = \frac{\sqrt{1 + (2 \times 0.2 \times 2.45)^2}}{\sqrt{(1-(2.45)^2)^2 + (2 \times 0.2 \times 2.45)^2}}$$

$\Rightarrow X = 0.0274\ m$

$\Rightarrow \underline{X = 2.74\ cm}$

(11.) at, $r = \frac{\omega}{\omega_n} = \sqrt{2}$

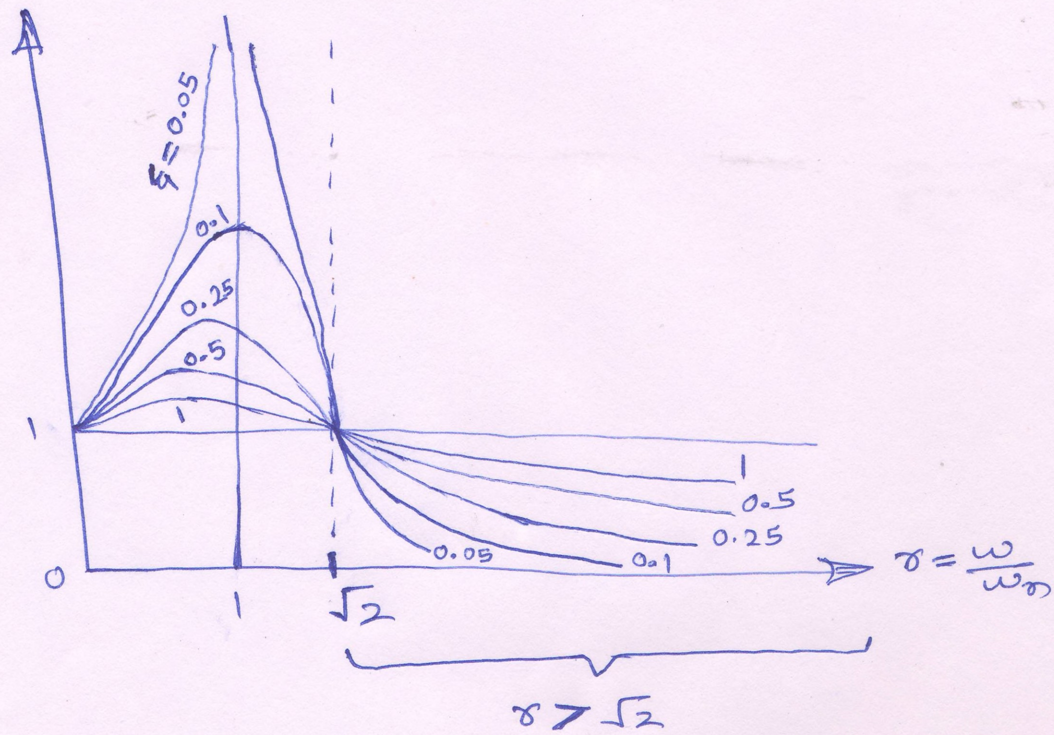
$$\therefore \quad T = \frac{F_T}{F_0} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$\Rightarrow \quad T = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-(\sqrt{2})^2)^2+(2\xi r)^2}}$$

$$\Rightarrow \quad T = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{1+(2\xi r)^2}} = 1$$

$$\Rightarrow \quad \boxed{T = 1}$$

(12.)



when, $\left(r = \frac{\omega}{\omega_n}\right) > \sqrt{2}$;

In this region, if we go for increasing the damping, i.e. increasing the value of ξ, Transmissibility ratio increases.

$\Rightarrow$ Transmitted force tends to increase.

$\therefore$ Hence increasing the damping in this region proves to be detrimental.

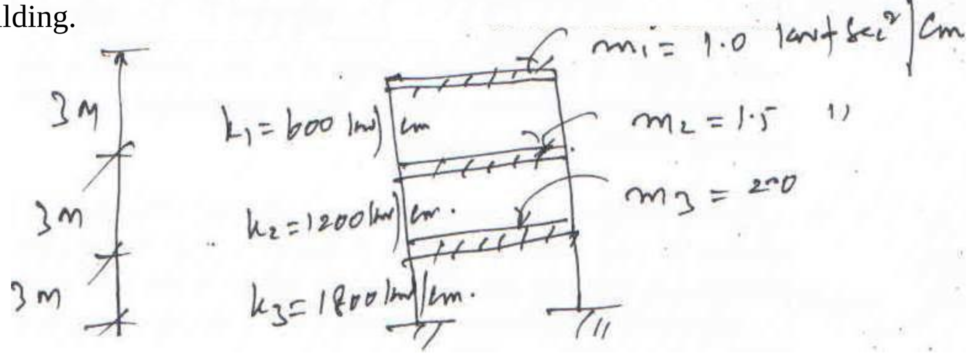
STRUCTURAL DYNAMICS – QUESTION BANK

Two marks Questions

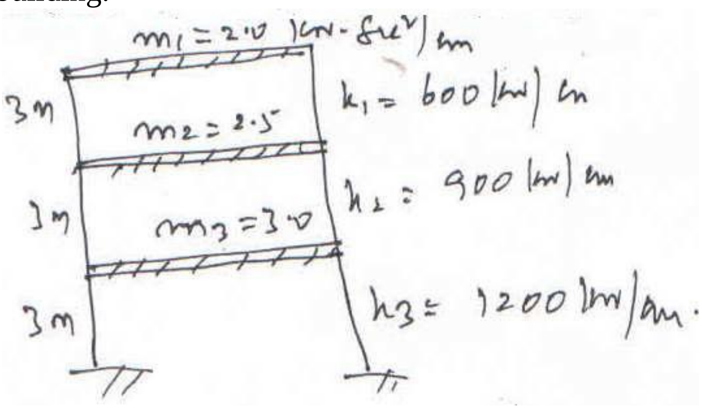
1. What is D' Alemberts principle?
2. What is Degree of freedom
3. Explain Hamilton's principle
4. Explain Coulomb damping
5. Explain logarithmic decrement and obtain the expression for logarithmic decrement.
6. Explain Duhamel's integral for undamped system.
7. Explain mode superposition method
8. Represent SHM vectorially.
9. What are the elements of vibratory system?\
10. Represent SHM vectorially
11. State the Principle of virtual work
12. What is Phase angle?
13. Differentiate between Normal coordinates and Geometric coordinates
14. Discuss about Uncoupling of the MDOF equations of motion
15. SDOF and MDOF systems
16. IS code methods of analysis
17. Assumptions and limitations of Stodola and Holzer method
18. Free and forced vibrations
19. Need for dynamic analysis

Ten marks Questions

1. Obtain the expression for dynamic magnification factor for damped harmonic excitation.
2. Obtain the governing D.E of motion for an undamped free vibration of a beam in flexure and the mode shapes for a propped cantilever beam.
3. A vibrating system consisting of a weight of W=10kg. And a spring stiffness K=20kg/m .in a viscously damped so that the ratio of the consecutives amplitudes in 1.0 to 8.5.Determine.
   a) The notional frequency of the undamped system.
   b) The logarithmic decrement.
   c) Damping ratio
   d) Damping coefficient
   e) Damped natural frequency
4. Find the natural frequency of vibration and mode shapes for the 3 storey idealized shear building.



5. A 3 storey RC building is situated in Delhi The D.L and L.L is lumped at respective floors the soil below the foundation is assumed to be hard rock. Assume the building is used for office purpose. Determine the total base shear and base moment and distribute the base shear along the height of the building.



6. What is the significance of the Fourier series in dynamics and explain it.
7. Discuss about critically damped, over damped and under damped systems
8. Explain and deduce the expressions for free vibrations of SDOF system undamped and damped case.
9. Discuss the response of the undamped oscillator for triangular impulse loading
10. Describe various methods of discretization of the analysis of dynamic system.
11. State and explain orthogonality principle of normal modes.
12. Derive the differential equation of motion for free flexural vibration of the simply supported beam. Sketch the three mode shapes
13. Describe the dynamic analysis procedure as per IS 1893:2002 for a high rise building with 4 stories. Indicate how the mass is lumped and the seismic force is distributed along the height. Distinguish between CQC and SRSS procedures given in the code.
14. Using STODOLA method determine the fundamental frequency of vibration and the corresponding mode shapes for idealized 3 storey shear building for the given data

    $m_1 = 1$; $m_2 - 1.5$; $m_3 = 2$ kN-S/cm$^-$

    $k_1 = 600$; $k_2 = 1200$; $k_3 = 1800$ KN/cm

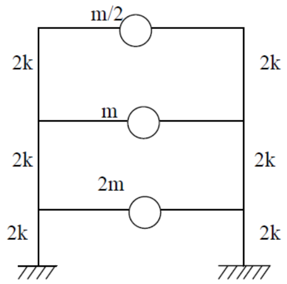
15. For an idealized 5 DOF system subjected to ground acceleration. Formulate the equation of motion.

16. Explain the IS code procedure for obtaining the design seismic forces in multi storey building. [7 M]

17. Derive the governing differential equation for transverse flexural vibration of a continuous system subjected to dynamic loading. Apply the above equation for a beam with one end fixed and other end is free. [15 M]

18. Discuss on the dynamic analysis of structures. What are the basic difference between Response Spectrum analysis and Time History analysis?
19. Explain Duhamel's Integral. Discuss the response of the undamped oscillator for triangular impulse loading.
20. Determine the equivalent stiffness of a typical spring-mass system having springs are in series and parallel combination.

21. Differentiate between discrete and continuous systems?
22. Outline the application of Holzer method to obtain frequencies and modes of vibration.
23. For the multistoried building shown in fig.1. Obtain the frequencies and modes of vibration using Stodola method or Holzer method.. Assume m = 5 x 10^4 kg, k= 5 x 10^4 kN/cm.



24. Detail the procedures of various IS Code methods of seismic analysis for obtaining response of multi-storied buildings. Solve for Natural frequencies, Time periods and Modes of Vibration of the multistory building shown in Fig (a) as an Eigen value problem. Assume m = 5 x 10^4 kg, k= 5 x 10^4 kN/cm.

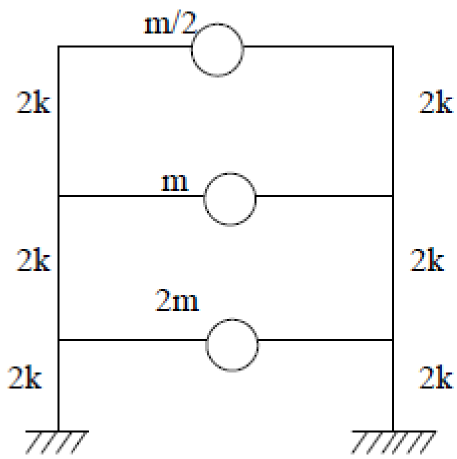


Fig a

25. Describe how the equation of motion can be set up using Newton's second law of motion for the system subjected to earthquake horizontal ground acceleration u_g.
26. Write down the procedural steps for a) Response Spectrum Method and b) Time history methods of seismic analysis.
27. Generate the equation of motion for MDOF system and derive the solution as an Eigen value problem
28. Describe the Modal analysis procedure as per IS 1893:2002 for a high rise building with 4 stories. Indicate how the mass is lumped and the seismic force is distributed along the height. Distinguish between CQC and SRSS procedures given in the code.

27.4.2015

THEORY OF VIBRATIONS

* what do you understand by vibrations.

The machine's having rotating parts are never completely balanced. From the static & dynamic analysis of such machine's or structures it is known that machine's or structures transmit forces to the ground. These forces are periodic in nature

Take an example of simple pendulum where bob starts to & fro motion or we can say oscillations occured when the bob is disturbed from its equilibrium positions. Usually oscillation occurs at natural frequency so it keeps on oscillating till its motion dies out. If such a system is subjected to periodic forces it responds to the impressed frequency which makes system to undergo forced vibration at forcing frequency

If impressed frequency is equal to natural frequency resonance occurs [large oscillations will occurs resulting in excessive dynamic stresses]

All bodies in vibration analysis are treated as elastic bodies instead of rigid bodies

Bodies in dynamic analysis have mass & elasticity because of the mass bodies will have kinetic energy & because of elasticity bodies will have potential energy in the form of elastic strain energy. The change of potential energy to kinetic energy & vice versa will make body vibrate without external excitation.

RIGID BODY DYNAMICS (VS) ELASTIC BODY DYNAMICS (or) deformed :-

The response of any physical object to ~~time~~ dynamic varying loading is important to study in engineering. The physical object may be rigid body or deformable body.

In rigid body dynamics objects are treated as rigid (no deformations) when subjected to dynamic loading *

Ex :- Moments of machinery, flight of an aircraft (or) space vehicle etc.,

In many cases the dynamic response of objects involving deformations is a major concern particularly in civil engineering

Ex:- Design of structural frames in buildings, bridges, aircrafts, ships & offshore platforms

Importance of Vibration Analysis:-

Now-a-days in construction industry their is a tendency to decrease weight & cost of the structure. This decreases the performance of the structure in terms of low energy dissipation (absorption) which makes the structure succeptable to critical vibration. So dynamic analysis of structure in modern construction is important to predict the structural response when subjected to external loading which varies with time. The analysis of structural response is important because,

1. under certain conditions vibration may cause large deformations & severe stresses in the structure particularly when the frequency of existing force coincides with natural frequency of the structure
2. fluctuating stresses
3. Oscillating motion (causes wear & tear and malfunctioning)
4. To find the vibrations in a rotating machines.
5. vibratory motion causes discomfort for humans

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(*) CLASSIFICATION OF VIBRATIONS:-

Vibrations are broadly classified into 2 cases.

i) Free Vibrations
ii) Forced vibrations

(1) FREE VIBRATIONS:-

Vibrations of structure is started by the application of external loads which is subsequently removed then the resulting oscillation where no external load is present is called free vibration.

(2) FORCED VIBRATIONS:-

This is to and fro motion of the structure due to external excitation. Depending on nature of loading they are classified as periodic & non-periodic / deterministic & stochastic loading.

(3) PERIODIC & NON PERIODIC VIBRATIONS:-

Dynamic loads will vary in magnitude direction or position with time.

A special type of dynamic load that varies in magnitude with time repeat itself @ regular intervals is called periodic vibrations or periodic loads.

air around it called viscous damping.

The ~~doing~~ duration of the load that is repeated is called a cycle & time taken to complete the cycle is called period of the load. The loads that donot show periodicity are called non-periodic loads.

DETERMINISTIC & STOCHASTIC VIBRATIONS:-

Loads that can be specified as definite function of time irrespective of whether the time variation is regular or irregular is called deterministic load & corresponding vibration is called deterministic vibration.

magnitude is known but frequency is unpredictable

Certain types of loads cannot be specified as function of time because of their inherent uncertainity in their magnitude & the variation of time such loads can be described using certain statistical parameters such as mean, variance, power spectral density function etc., are called stochastic loads or random loads & their corresponding vibration are random vibrations or stochastic vibrations.

Both magnitude and frequency are unpredictable

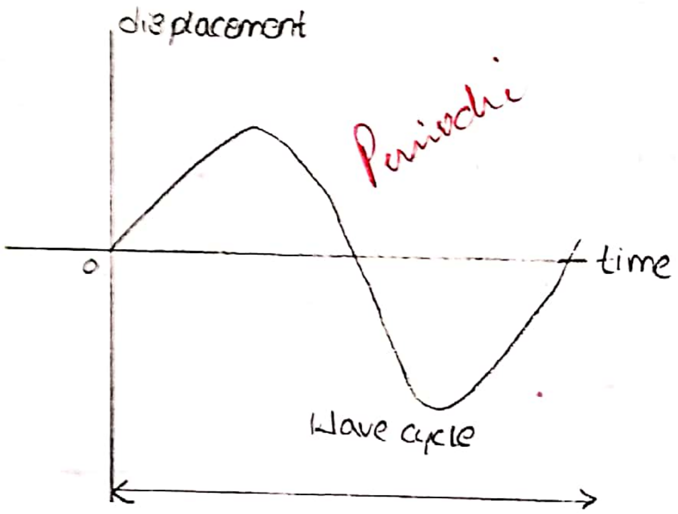
So probably values are obtained

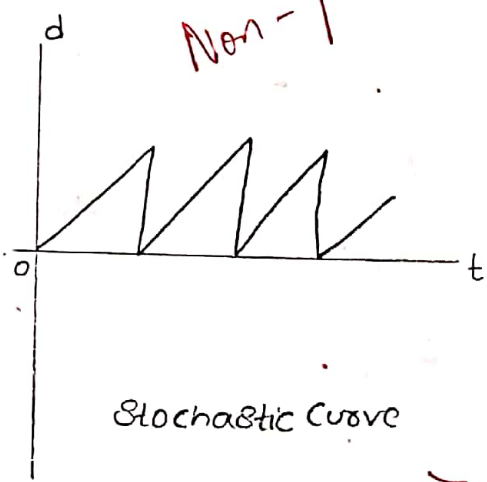
Ex:- Earthquake loads, wind loads, wave load etc are random in nature because the magnitude & frequency distribution cannot be predicted.

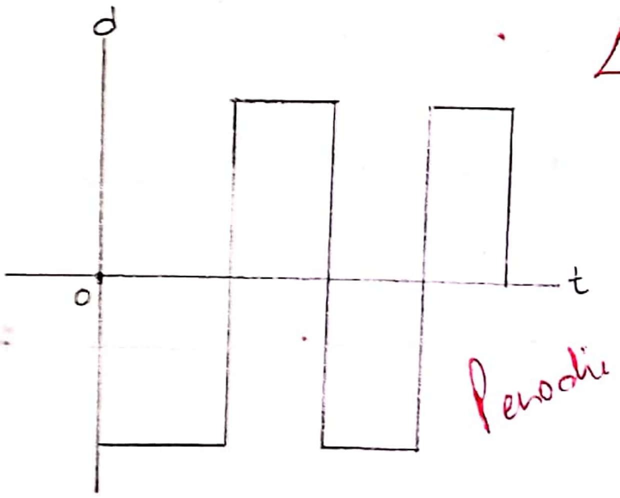
with certainity but can be estimated only as Probable value.

OSCILLATIONS:- *

Oscillations occurs when the system is ~~distributed~~ disturbed from a position of Static Equilibrium. This displacement from Equilibrium changes Periodically with time. Oscillations are said to be periodic & display Periodic motion. *







<Types of Oscillations>
Displacement - time graphs

BASIC PROPERTIES OF OSCILLATING SYSTEM

* **Amplitude**: It is the maximum displacement of the body from its equilibrium position.

It varies with time 'A'

TIME PERIOD : Time taken for the oscillation repeat itself is called Time period

Time taken between successive oscillation of the system. It is expressed in sec. or cycles

FREQUENCY :-

No. of cycles per unit time is called frequency

$f = 1/T$ Hertz or cycles per sec

Hz or CPS

ANGULAR FREQUENCY : (Circular frequency)

Number of rotational oscillations per sec is called angular frequency [radian/sec] rps (cps) or Hz

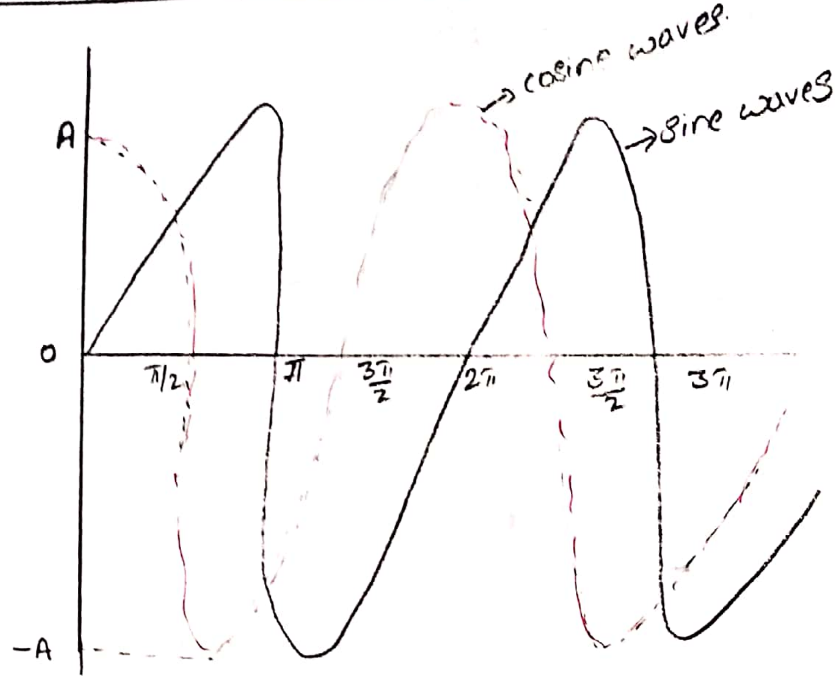
$\omega = \frac{2\pi}{T}$ $[\because f = 1/T]$

$\omega = 2\pi f$

* PHASE ANGLE :-

The phase of an oscillation is defined as the amount of oscillation lags behind or leads infront of a reference oscillations.

For example :- Take a sine oscillation of magnitude A, the angular frequency ω & compare with cosine oscillation & angular frequency (ω)



This diagram shows the phase of oscillations. It can be seen from the diagram that the cosine wave lags behind sine wave by π/2 [or 1/4th of wavelength] So we can say that two waves are out of phase by π/2. or the phase difference is π/2

HARMONIC OSCILLATIONS :- (or) Harmonic oscillation

Harmonic motion are just oscillations that are made up of sine & cos waves. Any oscillations that varies with sine or cos function or both is said to be harmonic oscillations.

For any harmonic oscillations the amplitude at any time is given by

$$y = A\cos\omega t$$
(or)
$$y = A\sin\omega t$$

PERIODIC MOTION:- motion which repeats regular intervals

NATURAL FREQUENCY:-

when a system executes for vibrations which are undamped the frequency of such structure is called Natural frequency.

Frequency of force vibrations is called natural frequency & frequency of forced vibration is called forced frequency on implied frequency.

RESONANCE:-

when frequency of ~~existing~~ exciting force is equal to natural frequency of the structure resonance occur.

DEGREE OF FREEDOM:-

The degree of freedom of vibrating body, the number of independent co-ordinate which are required to define the motion of body @ any given time

SIMPLE HARMONIC MOTION:-

It is a to & fro periodic motion of a particle in which

1.) ~~Oscillation~~ acceleration is proportional to the displacement from mean position

$a \propto x$

2) Acceleration is always directed to the fixed point which is the mean equilibrium position

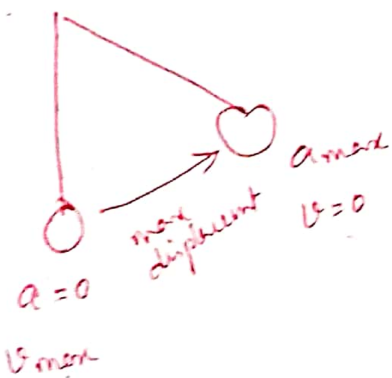
* SHM is represented by the expression

$$x = A\sin\omega t$$
$$x = A\cos\omega t$$

* The example for SHM is attatched a mass @ the end of the spring & set it into motion what happens is SHM

NOTE:-

The acceleration is zero when mass is @ Equilibrium position when velocity is maximum, Acceleration is maximum at max displacement where mass is at rest.



Vibration → occurs if body has mass and elasticity.

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D' Alembert's principle :-

According to newtons second law of motion $f = ma$. where f is the external force acting on the body whose mass is 'm' & acceleration 'a'

$\Sigma F \rightarrow$ resultant or Net force

$ma \rightarrow$ Inertia force

$$\Sigma f - ma = 0$$

which means that by applying force $(-ma)$ on the body the body will be in equilibrium as the sum of all the forces acting on body is zero. Such equilibrium is called dynamic equilibrium. The force '-ma' is called "inertia force" or "D'Alembert's force". The body will be in dynamic equilibrium under the action of external force 'f' & internal force '-ma', this is known as D'Alembert's principle.

Analysis of single degree of freedom system [SDOF] :-

A system which has mass & elasticity will start vibrating if it is disturbed. The natural frequency of the system depends on degree's of freedom of the system. For SDOF system there will be only 1 natural frequency. Similarly for 2 degree of freedom system will have 2 natural frequency for multi DOF system we will have multiple natural frequency

→ A practical system is very complicated. Therefore before proceeding to analyse the system it is desirable to simplify it by modeling the system

SDOF
MDOF

SHM - only one frequency
Harmonic Motion - More than one frequency

1 mass ≃ 1 DOF
1 mass ≃ 1 Mode
1 Mode ≃ 1 natural frequency

→ The modelling of system should be in such a way that the result is within the acceptable accuracy. Instead of considering distributed mass, a lumped mass approach is adopted in the analysis whose dynamic behaviour can be determined by one independent principle co-ordinate [SDOF]

ELEMENTS OF LUMPED PARAMETER VIBRATORY SYSTEM :-

(Lumped mass approach model) (or) (Mass-spring-damper model) (3 element Model) (pg 87)

Lumped parameter vibratory system are made up of following elements

1. Mass (weight of structure) → represents inertial characteristic of the structure
2. Spring (elasticity of structure) → elastic restoring force and potential energy (capacity of the structure)
3. Damper (friction) → frictional characteristics and energy losses (of the structure)
4. Excitation force (load applied)
   ↓ external forces acting on structure at any time 't'. concentrated at a point

1. **MASS** :-

The mass is assumed to be rigid (lumped) & concentrated at centre of gravity

2. **SPRING** :-

The elasticity of structure is represented by helical spring, when deformed it stores energy in the form of potential energy.

The energy stored in spring is given by (SE)

$$\boxed{P.E = \frac{1}{2} k x^2} = SE$$

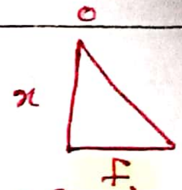
where k - stiffness of spring

x - Amount of displacement due to force 'f'.

$SE = \frac{1}{2} f x \Rightarrow f = kx \Rightarrow \frac{1}{2} k x^2$ $\left(\because k = \frac{f}{x} \right)$

The force on the spring is given by

$f = Kx$



$\left(\frac{1}{2} \times f \times x\right)$

→ Spring is treated as mass less.

→ Workdone by spring is treated as energy restoring element.

3. **DAMPER:-**

In a vibrating system damper is an element which is responsible for loss of energy in the system, it converts energy into heat due to friction [either sliding friction or viscous friction]

* The vibrating system stops vibration because of energy conversion by damper. There are 2 types of damper

i) Viscous damper ✓ (friction) → viscous as fluid, ratio of successive amplitudes are same

ii) Coulomb's damper (sliding) → dry sliding, ratio of difference of successive amplitudes are same

* Viscous damper consists of viscous friction which converts energy into heat. For this damper force is proportional to velocity

F = Damping force

$F \propto v$

(or)

$F \propto \dot{x}$ [$\dot{x}$ - first derivative of x]

* Coulombs damper consists of dry sliding friction. The force direction is always opposite to sliding velocity. In this case force is constant.

4. **EXCITATION FORCE:-**

It is a source of continuous supply of energy to vibratory system [external periodic force] Note:- undamped free vibration SDOF is known as 2 element model – mass-spring, no damping due to free vibration

DIFFERENTIAL EQUATIONS OF MOTION OF SDOF SYSTEMS :- [Undamped free vibrations]

There are several methods to analyse undamped free vibration SDOF system

1. Based on newtons second law
2. Based on D'Alemberts principle
3. Energy method
4. Rayleigh method

(1) BASED ON NEWTON'S SECOND LAW:-

According newtons second law rate of change of linear momentum is proportional to force applied on it

$$\frac{d}{dt}(mv) \propto F \quad \text{—— (1)}$$

$$\frac{dv}{dt} = a = \ddot{x} \quad \text{—— (2)}$$

$a = \ddot{x}$

$$m\ddot{x} \propto F \quad \text{—— (3)}$$

$$m\ddot{x} = CF \quad (\text{for ideal system}) \quad (\because C = 1)$$

$$m\ddot{x} = \Sigma F$$

ΣF = Sum of all forces acting on the body.

Net force

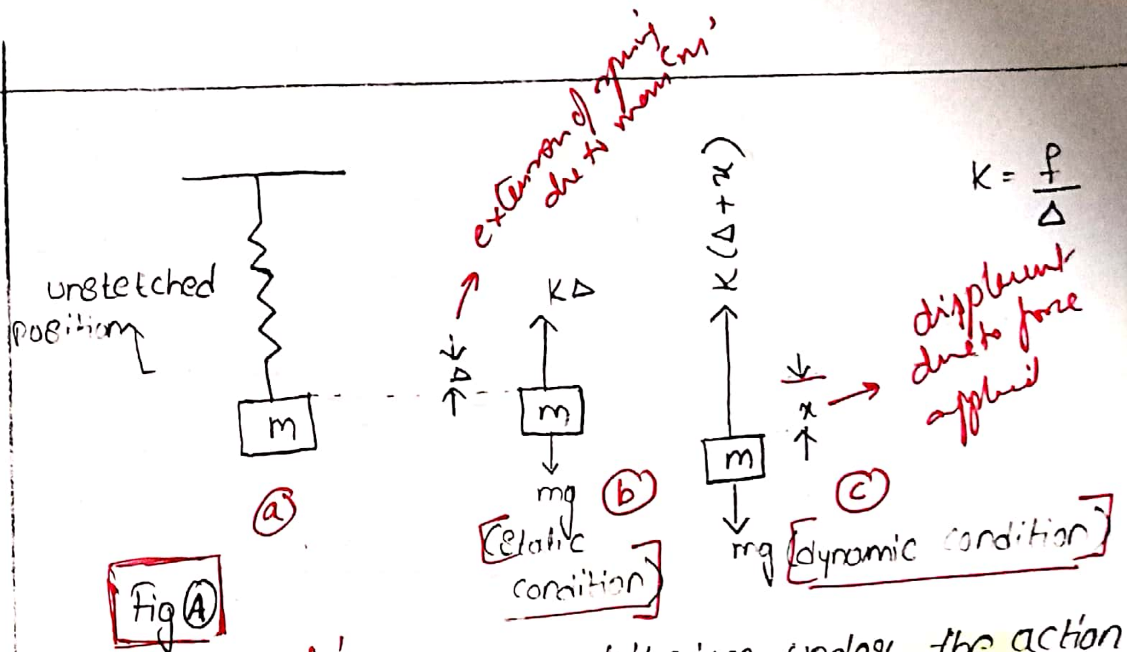


Fig (A)

The body 'm' is in equilibrium under the action of 2 forces

$\Delta \rightarrow$ is extention of spring after mass is suspended

If body is moving down with acceleration '$\ddot{x}$' in downward direction then

$$m\ddot{x} = \Sigma F \quad \text{— (1)} \quad (\rightarrow \text{net force})$$

$$mg = K\Delta \quad \text{— (2)} \quad \text{fig (b)}$$

$$m\ddot{x} = [mg - K(\Delta + x)] \quad \text{— (3)} \quad (\rightarrow \Sigma F)$$

$$m\ddot{x} = mg - K\Delta - Kx \qquad K\Delta = mg$$

$$m\ddot{x} = -Kx$$

$$\boxed{m\ddot{x} + Kx = 0} \quad \text{Undamped free vibration}$$

* Equation of motion for SDOF undamped free vibration

law

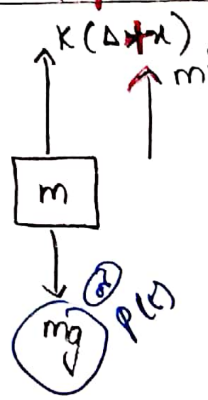
tional

$P = m\ddot{x} + Kx + c\dot{x}$

* (2) BASED ON D'ALEMBERT'S PRINCIPLE :-

As per D'Alemberts principle to maintain dynamic equilibrium the forces acting on body are

(spring force)



$m\ddot{x}$ (Inertia force) — due to mass 'm'

$\ddot{x}$ – acceleration with which is moves downwards

$$mg = m\ddot{x} + K(\Delta + x) \quad \text{— (1) dynamic condition}$$

$$mg = K\Delta \quad \text{— (2) elastic condition}$$

$$mg = m\ddot{x} + mg + Kx \quad \text{— (3)}$$

$$m\ddot{x} + Kx = 0$$

* (3) Energy method :-

This method applicable only to conservative system (where there is no loss of energy) (Since there is no loss of energy, total energy remains constant). When a structure is vibrated the total energy in system is partly kinetic energy, partly P.E (elastic strain energy). *

The kinetic energy is due to mass 'm' & velocity ($\dot{x}$) & P.E is due to spring constant or spring stiffness & displacement } *

$$E = K.E + P.E$$

Total energy remains constant, (w.r.t time)

$$E = C$$

$$\frac{dE}{dt} = 0$$

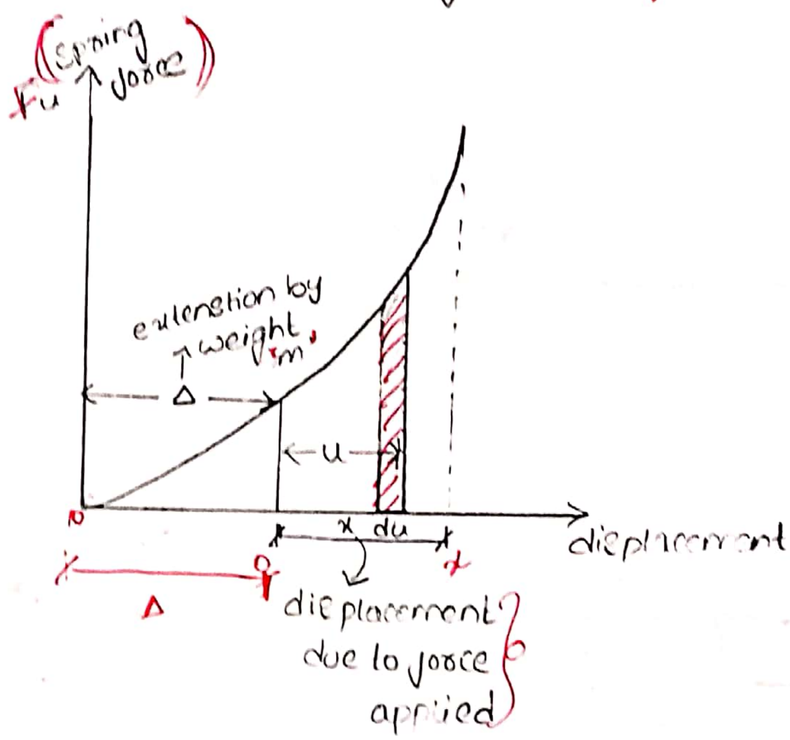
$$\frac{d}{dt}\left[KE + PE\right] = 0$$

$$K.E = \frac{1}{2} m v^2 \quad (\because v = \dot{x})$$

$$= \frac{1}{2} m \dot{x}^2 \rightarrow (1)$$

P.E of System consists of 2 points.

1. loss or gain in P.E of mass (energy lost or gained)
2. Strain energy of Spring (energy stored)



Consider element 'du' at $x=u$

SE or Spring force $F_u = K(u+\Delta)$

SE or Workdone by Spring = force × displacement

$$dw = K(u+\Delta)\,du$$ (du → displacement)

Potential energy

$$PE = \text{Strain energy of Spring} - \text{loss of P.E due to force applied}$$

(with loss in height PE is reduced.)

$$= \int_0^x K(u+\Delta)\,du - mg\cdot x$$

$$\left[\frac{Ku^2}{2}\right]_0^x + \left[K\Delta x\right]_0^x - mg\cdot x$$

$$= \frac{Kx^2}{2} + mgx - mgx \quad (K\Delta = mg)$$

$$\boxed{P.E = \frac{Kx^2}{2}} \rightarrow (2)$$

& $K.E = \frac{1}{2}m\dot{x}^2 \rightarrow (1)$

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from ① & ②

$$\frac{d}{dt}\left[\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right] = 0$$

$$\frac{dE}{dt} = 0$$

$$\frac{1}{2}m\ddot{x} \times 2\dot{x} + \frac{1}{2}K \times 2x \cdot \dot{x} = 0$$

$$m\ddot{x} + Kx = 0$$

4. **Rayliegh method:-** pg 166

Is applicable only to SDOF System

→ It is modified energy method

→ According to this method in a conservative System if P.E is max. K.E is min & vice-versa

@ pg 166

$$\frac{1}{2}m(\dot{x}_{max})^2 = \frac{1}{2}K(x_{max})^2$$

$x(max)$ = Amplitude (A)

$\dot{x}_{max} = A \times \omega$

$x_{max} = A$

$\dot{x}_{max} = -A\omega \Rightarrow v = r\omega$ (re)

$x = a \sin\theta$

$x = -a \sin\omega t$

$x = -a \sin(2\pi - \omega t)$ } phase difference 2π

$$= \frac{1}{2}m(A\omega)^2 = \frac{1}{2}K \cdot A^2$$

$$\frac{1}{2}m\omega^2 = \frac{1}{2}K$$

$$\omega = \sqrt{\frac{K}{m}}$$

relation between linear and circular frequencies.

$$\omega = 2\pi f$$

ω - Angular frequency

1 revolution = 2π radian

pg 16/17 how $\omega = 2\pi f$

$x = a \sin\omega t$

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

for x_{max} a should be max. to a_{max} and $\sin\omega t$ maximum

$\sin\omega t = 1$

$\omega t = \pi/2$

PERIODIC MOTION :- motion which repeats regular intervals

NATURAL FREQUENCY :-

when a system excutes for vibrations which are undamped the frequency of such structure is called Natural frequency.

Frequency of free vibrations is called natural frequency & frequency of forced vibration is called forced frequency or implied frequency.

RESONANCE :-

when frequency of ~~existing~~ exciting force is equal to natural frequency of the structure resonance occur.

DEGREE OF FREEDOM :-

The degree of freedom of vibrating body, the number of independent co-ordinate which are required to define the motion of body @ any given time

amplitude ... wave
of sound
frequency ... pitch

SIMPLE HARMONIC MOTION :-

It is a to & fro periodic motion of a particle in which

1.) ~~Acceleration~~ acceleration is proportional to the displacement from mean position

$a \propto x$

SOLUTION OF DIFFERENTIAL EQUATION (FREE VIBRATION – SOOF

Differential equation of SOOF undamped system is given by

$m\ddot{x}+kx+cx^0=f(t)$

$m\ddot{x}+kx=0$

$x+\frac{k}{m}x=0$

$w_n = \sqrt{\frac{k}{m}}$

w_n=natural angular frequency

natural circular frequency

$\ddot{x}+w_n^2 x=0$

This equation is satisfied by function $Sinw_nt$ & $Cosw_nt$. Then the solution of eqation1 can be written in the form of ----------------------(1)

$X=ASinw_nt+BCosw_nt$ ----------------------------(2)

A& B are constants which can be determined from boundary condition (Initial conditions).

The system can be distributed in two ways :

Case1: By pulling the mass by distance 'X'

Case2: By hitting the mass by means of fast moving object with a velocity of 'V'

Case1 : Putting mass by distance 'X'

T=0 x=X x=0

$x= A\ sinw_nt +B\ cosw_nt$ -----------------(1)

$x^o =A\ sinw_nt- Bw_nsinw_nt$ -------------------(2)

X=Ax Sin0+B Cos0

$X = B$

$X^{o} = A \times w_n \times \cos 0 - B \times w_n \sin 0$

$$\left.\begin{aligned} A &= 0 \\ X &= B \end{aligned}\right\} \text{------------(3)}$$

$x = 0 \times \sin w_n t + X \cos w_n t$

$x = X \cos w_{nt}$

Case2: By hitting the mass by means of fast moving object with a velocity 'V'.

Here; $t=0$; $x=0$; $x^{0}=v$

$x = A\sin w_n t + B\cos w_n t$ -------------------------eqn(1)

$x = A w_n \cos w_n t - B w_n \sin w_n t$

$0 = A\sin 0 + B\cos 0$

$B = 0$

$x^{0} = A \times w_n \cos 0 - B w_n \sin 0$

$V = A\, w_n$

$A = \frac{V}{wn}$

From eqn1 we get

$X = \frac{V}{WN}$

Case1 represents the behavior of undamped system in most practical cases

Solution for this case is

$x = X \cos w_n t$ -----------------------(1)

$x^0 = -Xw_n \sin w_n t$ ----------------------(2)

$= Xw_n \cos(w_n t + \pi/2)$

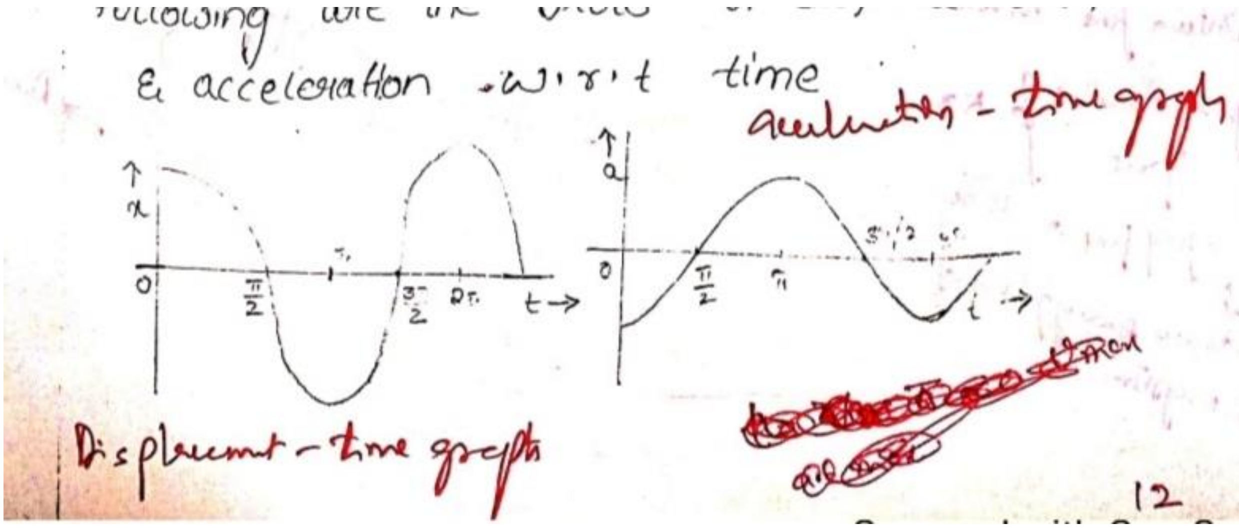
$\ddot{x} = -Xw_n^2 \cos w_n t$ -------------------------(3)

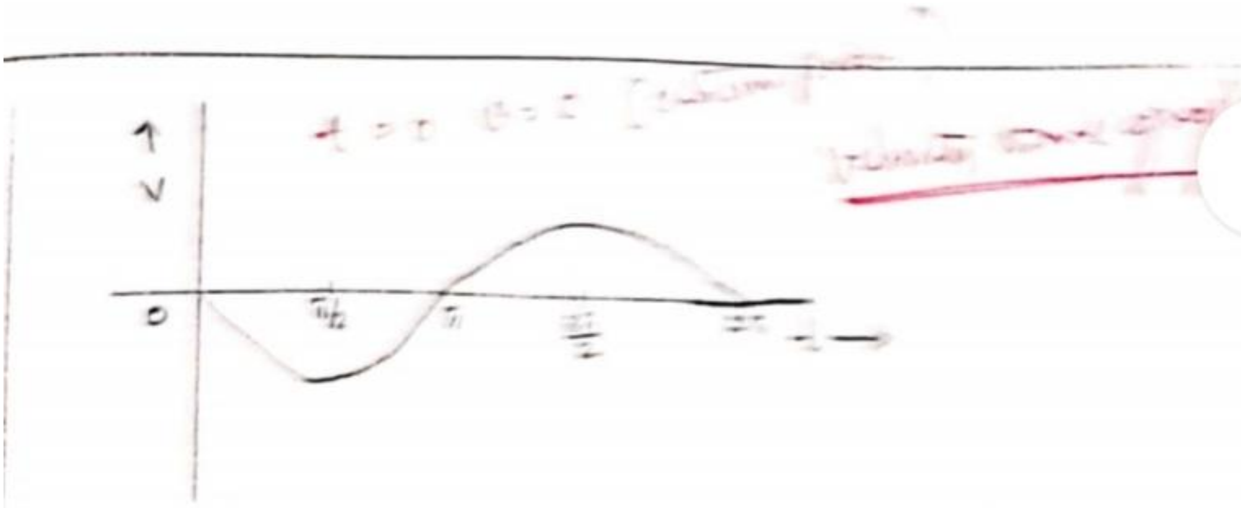
$= Xw_n^2 \cos(w_n t + \pi)$

The expression indicates that velocity vector leads displacement vector by phase angle $\pi/2$. Similarly acceleration leads displacement by phase angle π.

The maximum velocity is $x^0 = Xw_n$, maximum acceleration is $\ddot{x} = xWn^2$.

Following are the plots of displacement, velocity and acceleration w.r.t time





From the plots it can be observed

If a body is disturbed it will start vibrating.

When displacement is max 0 and accerelation is max opposite to displacement

When the displacement is zero velocity at max and accerelation is zero.

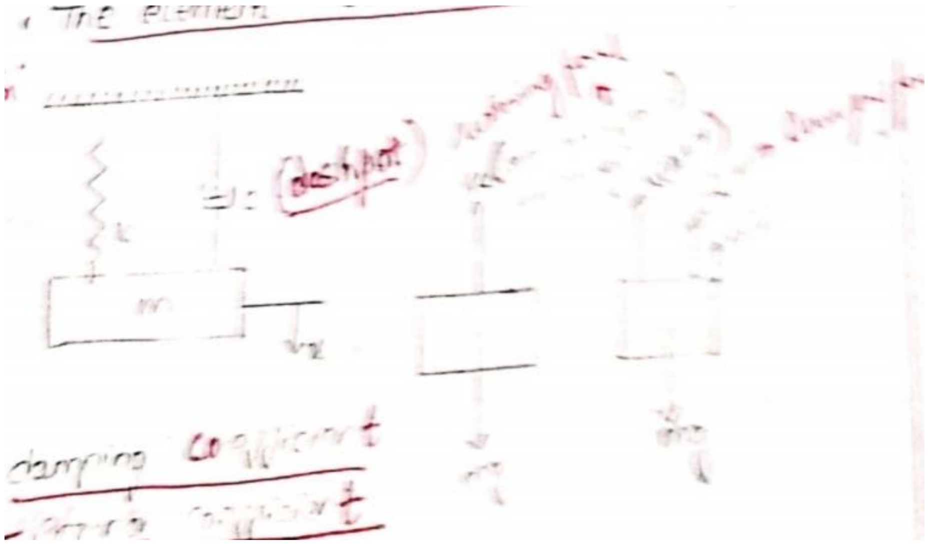
Undamped free vibration zero.

Equation of motion of

Damped free vibration

In damped vibration to mass and spring is used.

The element model



mg=m$\ddot{x}$+k(+x)+cx

$$m\ddot{x}+cx^0+kx=0 \text{ ---------------------(1)}$$

This is differential equation for SOOF system damped-free vibration

The solution for this 2nd order equation can be expressed as

$$x=Xe^{st}$$

$$x^0= S\times e^{st}$$

$\ddot{x}=S^2\times e^{st}$

Sub all 3 in eqn1

$m\times s^2\times e^{st}+ cs\times e^{st} +k \text{ x } e^{st} = 0$

$ms^2 + cs + k =0$

$1s^2 + (c/m)\, s + (k/m) = 0$

$$ax^2 + \text{bx} + \text{c} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

The roots of equation are

$S1, 2= -c/m \pm ½ \sqrt{\frac{c}{m}^2 + 4(\frac{k}{m})^2}$

The solution for eq 1 is given by,

$x = x_1 e^{-\frac{c}{m} + ½\sqrt{\frac{c}{m}^2 + 4(\frac{k}{m})}\ t} + x_2 e^{-\frac{c}{m} - ½\sqrt{\frac{c}{m}^2 + 4(\frac{k}{m})}\ t}$ -----1

$x = e^{-\frac{c}{2m} t}\,[\,x_1 e^{½\sqrt{\frac{c}{m}^2 + 4(\frac{k}{m})}\ t} + x_2 e^{-½\sqrt{\frac{c}{m}^2 + 4(\frac{k}{m})}\ t}\,]$ -----2

this eq 2 is not considered.

There are 3 possible cases of solution based on the term in the indetermined

Case1: over damped case

$$(\frac{c}{m})^2 > 4(\frac{k}{m})$$

Case2: critically damped case

$$(\frac{c}{m})^2 = 4(\frac{k}{m})$$

Case3 : under damped case

$$(\frac{c}{m})^2 < 4(\frac{k}{m})$$

Let ‘C_c’be the critical damping

$$(\frac{Cc}{m})^2 = 4(\frac{k}{m})$$

$$Cc = 2\sqrt{km}$$

$$Cc = 2\sqrt{\frac{k}{m} \times m^2}$$

$C_c = 2w_n m = 2\sqrt{km}$

Case1: under damped

Usually practically all systems are under damped

$$\sqrt{\left(\frac{c}{m}\right)2 - 4(\frac{k}{m})} = i\sqrt{4\left(\frac{k}{m}\right) - (\frac{Cc}{m})^2}$$

$$i\sqrt{\ 4wn2\ \ -(\frac{c}{C}\times\frac{Cc}{m})2}$$

$\zeta=\frac{c}{Cc}$

Damping factor "ζ" is the ratio of damping co-efficient "c" to the critical damping coefficient(Cc)

$$i\sqrt{4wn^2 -\ \zeta\times(\frac{2m\ wn}{m})^2}$$

$$\sqrt{\left(\frac{c}{m}\right)(c/m)\ -\ 4\left(\frac{k}{m}\right)} = [2w_n\sqrt{1\ \ -\zeta^2}\ ]$$

$$X=e^{-(\frac{c}{2m})t}\ [x_1 e^{\frac{1}{2}[i2wn\sqrt{\ 1-\zeta 2}]t}\ \ + x_2\, e^{-\frac{1}{2}\left[i2wn\sqrt{1\ \ -\zeta^2}\right]}\ ]$$

$$X=e^{-(\frac{c}{2m})t}\ [x_1 e^{[iwn\sqrt{1\ \ -\zeta^2}t}\ \ + x_2\, e^{-\left[iwn\sqrt{1\ \ -\zeta^2}\right]}\ ]$$

let us assume

$w_n\sqrt{1-\zeta 2}$ =wd $\qquad\qquad \zeta=\frac{C}{Cc}$

where,

wd= damped natural angular frequency

wd is natural frequency of damped free vibration.

For under damped case the solution is

$$x= e^{-(\frac{c}{2m})t}\ [x_1\, e^{iwdt}+X2e^{-wdt}\ ]$$

Case2: Critically damped

For critically damped case since two roots are equal the general solution can be expressed as follows:

One independent solution is

$$x=x_1(t)+x_2(t)$$

$x_1(t)$ is one independent solution which is expressed as follows:

For critically damped case , ζ =1

$x_1(t)=x_1 e^{-(\frac{c}{2m})t}$

another independent solution $x_2(t)$ may be found using

$x_2(t)=x_2 t e^{-(\frac{c}{2m})t}$

and these 2 equations satisfy eqn1 $m\ddot{x}+cx^0+kx=0$

The general solution for critically damped system is obtained by superimposing this 2 independent solution

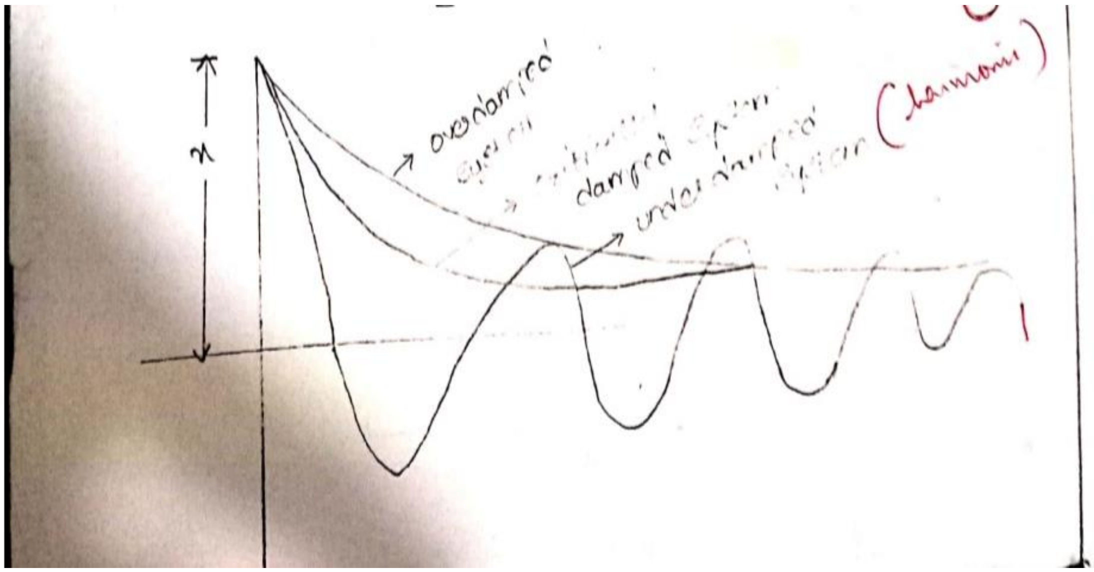
$$x=(x_1+x_2 t)e^{-(\frac{c}{2m})t}$$

Case3: over damped

$x=e^{-\frac{c}{2m}t}\ [x_1 e^{wn(\sqrt{\zeta 2-1})t} +2e^{-wn(\sqrt{\zeta 2-1})t}]$

$\frac{c}{2m}=\frac{C}{Cc}\times\frac{Cc}{2m} = \zeta\times\frac{2mwn}{2m} = \zeta w_n$

$x=e^{-}\zeta w_{nt}\ [x_1 e^{\sqrt{(\zeta-1)}t.wn}\ \sqrt{(\zeta-1)}t + x_2\ e^{-wn\sqrt{(\zeta-1)}t}]$



Boundary conditions

Equation a can be written as $x=Xe^{-\zeta wnt}\cos(wdt+0)$

Where X &0 are constants

X is amplitude &0 is phase angle

Introduce boundary conditions:

X0(displacement) &vo(velocity) at time period (t)=t

displacement(x)=xo

the solution for under damped case is

$xo=xe^{-\zeta wnt}\ ^{\cos(wdt+o)}$

after one time period the boundary conditions are

$t=t+tp$

$x=x_1$

then the solution is

$x_{1=}xe^{-\zeta w_n^{(t+tp)}}\cos(w_d(t+tp)+0$

divide eqnD/eqnE

$\frac{xo}{x1} = \frac{xe - \zeta wnt \ cos(wdt+o).}{xe - \zeta wn(t+tp)\cos(wd(t+tp)+0)}$

tp=time period-$\frac{1}{fpr\ frequency} = \frac{2\pi}{wd}$

$\frac{xo}{x1}$ ={x $e^{-\zeta wnt}$ cos { ($w_d t_p$- 2π)/ (w_d t + θ)}} / x $e^{-\zeta wnt}$ x $e^{-\zeta wnt}$ cos { (w_d(t +t_p)+θ} }

$\frac{xo}{x1}$ = $e\zeta^{wntp}$ $\frac{cos(wdt+0)}{\cos(wdt+2\pi+0)}$

cosθ=cos(2 π + θ)

$\frac{xo}{x1} = e^{-}\zeta^{wntp}$

$\ln(\frac{xo}{x1}) = \zeta^{wntp} = \zeta\text{^}wn \times \frac{2\pi}{wd}$

$= \zeta^{wn} \times \frac{2\pi}{wn\sqrt{1-\zeta\text{^}2}}$

$\ln(\frac{xo}{x1}) = \frac{2\pi\zeta}{\sqrt{1-\zeta\text{^}2}}$

This equation is known as logarithmic decrement

After ‘n’ number of time period is

T=t+ntp n-no. of cycles

The logarithmic decrement

$\ln(\frac{xo}{x1}) = \frac{2\pi\zeta}{\sqrt{1-\zeta\text{^}2}}$

For over damped & critically damped system, mass will return to its original position slowly and there is no vibration.

Vibration is possible only in under damped system because the roots of the solution under damped case are complex & has periodic functions.

Example

A damped system has following elements mass m=4kg , k=1kN/m ,damping constant c=40Nsec/m. Find damping factor, natural frequency of a damped oscillations, logarithmic decrement & no. of cycles after which the original amplitude reduced to 20%.

m=4kg

k=1kN/m

c=40N-sec/m

damping factor $\zeta = \frac{C}{Cc}$

$C_c = 2\sqrt{km}$

$= 2\sqrt{4 \times 1 \times 103}$

$= 2\sqrt{4 \times 103}$

$C_c = 126.491$

$\zeta = \frac{C}{Cc} = \frac{40}{126.5} = 0.316$

Natural frequency of damped oscillation

$W_d = w_n\sqrt{1 - \zeta^2}$

$W_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1000}{4}} =$ 15.81Hertz or cps

$W_d = 15.81\sqrt{1 - (0.316)^2}$

= 15Hz (or) rpm

$f_d = \frac{1}{Td} = \frac{wd}{2\pi}$

$=\frac{15}{2\pi}$

=2.38 Hz (or) cycles per sec

Logarithmic decrement:

$\ln(\frac{xo}{x1})=\frac{2\,\pi\zeta}{\sqrt{1-\zeta^2}}$

$=\frac{2\pi\times(0.316)}{\sqrt{1-0.316^2}}$

= 2.092

No. of cycles after which original amplitude is reduced to 20%

$\ln[\frac{100}{20}]=2.092\times n$

If original is assumed as 100, it is reduced to 20

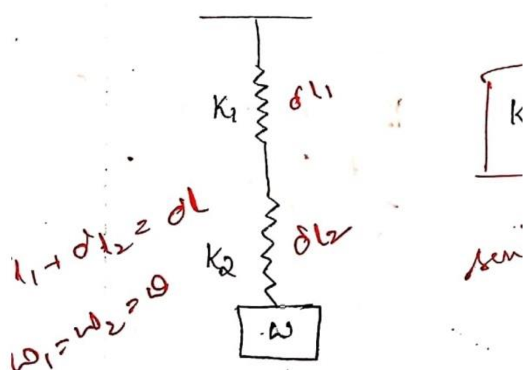
n= 0.334 cycle

Two springs of stiffness k_1 & k_2 are connected in series & carries a load w. Find equivalent spring stiffness of the system. If the springs are connected in parallel what is the equivalent spring stiffness.

SPRINGS CONNECTED IN SERIES

If the springs are connected in series, forces in each spring will be same and equal to externally applied force.

Series- forces applied are same, deflections different



$K = \frac{w}{\delta L}$

Since the force applied on springs are same, dl_1 is deflections of first spring and dl_2 is deflections of second spring.

Therefore, $K_1 = \frac{w}{\delta L1}$

$K_2 = \frac{w}{\delta L2}$

Total deflections $\delta L = \delta L_1 + \delta L_2$

$\delta L = \frac{w}{K1} + \frac{w}{K2}$

$w/k_e = w(\ 1/k_1 + 1/k_2)$

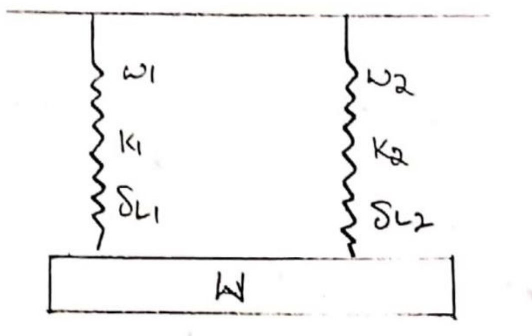
or, $w/k_e = w\ (\frac{k2 + k1}{k1\ k2})$ since, $k_e = 1/\ (1/k_1) + (1/k_2)$

$K_e = \frac{k1\ k2}{k1 + k2}$

SPRINGS CONNECTED IN PARALLEL

In case of springs in parallel the deflection is same in both the springs, then deflection of total system is equal to deflection of any spring.

Parallel- deflection same, forces shared are different



Let w_1 and w_2 are the loads shared by first and second spring respectively,

$K_1 = \frac{w1}{\delta L1} = \frac{w1}{\delta L}$

$W_1 = k_1 \delta L$

$k_2 = \frac{w2}{\delta L2} = \frac{w2}{\delta L}$

therefore, $w_2 = k_2\, \delta L$

$K_e = \frac{w}{\delta L}$

$W = k_e\, \delta L$

Total load $w = w_1 + w_2$

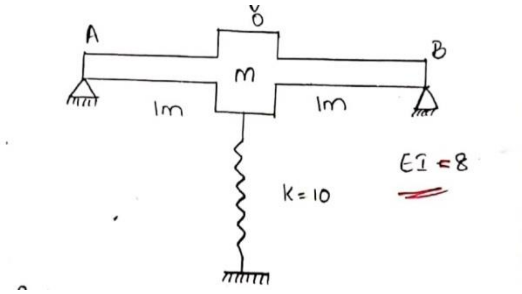
$w = (k_1 + k_2)\delta L$

$k_e\, \delta L = (k_1 + k_2)\delta L$

$k_e = k_1 + k_2$

Problem

Find the natural frequency of the simply supported beam shown in figure.



Let stiffness of beam AB be K_b and stiffness of spring is K_s

Therefore, total stiffness = $K_b + K_s$

Parallel- deflection same- $\boldsymbol{k_e = k_1 + k_2}$

$K_b = \frac{\rho}{\delta}$

$\delta = pl^3/ 48EI$

$K_b = 48EI/ l^3$

$K_b = (48 \times 8)/ 8$

$K_b = 48$units

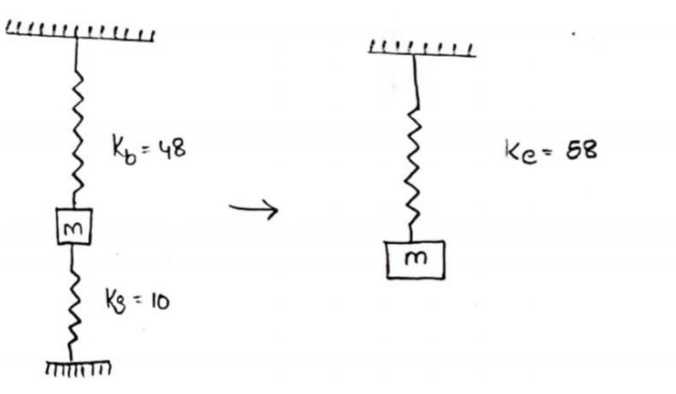
$K = K_b + K_s$

$= 48+10$

$= 58$ units

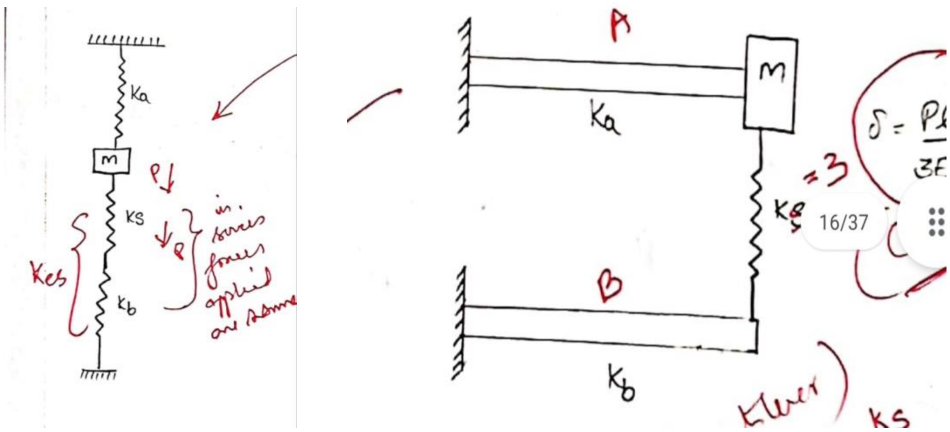
$W_n = \sqrt{k/m}$

$= \sqrt{58/m}$



Problem

Find the natural frequency of the cantilever beam with attached mass system as shown in the figure.



K_s = 3; L= 1m ; EI= 1

$K_a = \frac{P}{\delta}$ = 3EI/l^3

$K_b = \frac{3EI}{I}$ = 3

K_a = 3

K_b = 3

equivalent stiffness of parallel connected springs,

$K_e = K_a + K_{es}$ $\quad\quad$ $k_{es} = k_b\, k_a / (k_b + k_a)$ = (3x3)/(3+3) = 9/6 = 3/2

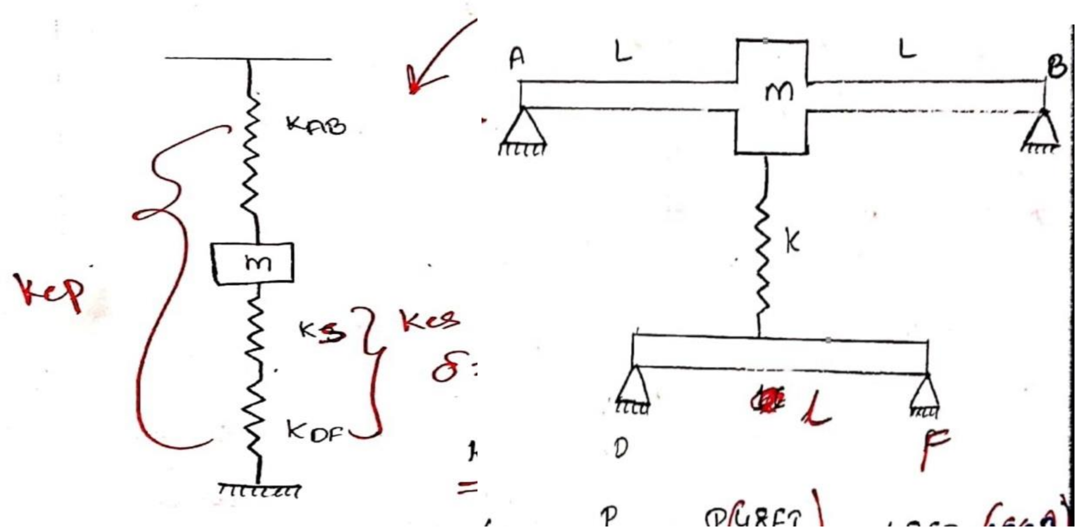
= 3 +(3/2)

= 9/2

Therefore, natural frequency= $w_n = \sqrt{\frac{k}{m}} = \sqrt{9/2m}$

Problem

The natural frequency of given mass system l=1, EI=1, K=48



$K_{AB} = \frac{P}{\delta}$ = P (48EI/Pl^{3}) = 48EI/ l^3

K_{AB}= 6

$K_{DF} = \frac{P}{\delta}$ = P(48EI/Pl^{3}) = 48(1)/ 1

K_{DF} = 48

K_{es} = (48x x48)/ (48+48) = 24

K_{ep}= 6+24 = 30

Natural frequency, $w_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{30}{m}}$

Example:

A single degree of freedom freedom system whose mass is 9100kgs with viscous damping is displaced from its position of rest by distance of 30m. the displacement on retrn swing is 20m on 0.5sec. determine spring constant, damping constant c.

x_o = 30mm x_1= 20mm

these are the two subsequent displacements

T= 0.5sec

$l_n = (x_o/x_1) = 2\pi\zeta / (\sqrt{1-(\zeta)2})$

$0.1760/ 0.164 (1- \zeta^2) = 6.2831^2 \zeta^2$

$1- \zeta^2 = 224 \zeta^2$

$1 / 225 = \zeta^2$

$\zeta = \sqrt{\frac{1}{225}}$

$\zeta = 0.066$ (damping factor)

$T_d = 2\pi/ w_d$

$0.5 = 2\pi/ w_d$

$W_d = 12.566$ (damped frequency)

$W_d = w_n \sqrt{1-(\zeta)2}$

$12.566 = w_n \sqrt{1-(0.066)(0.066)}$

$w_n = 12.593$ rad/sec

therefore, $w_n = \sqrt{\frac{k}{m}}$

$k= mw_n^2$

$k= 9100x\ 12.593^2$

$k= 1.44 \times 10^6$ kg/m

$k= 1.44 \times 10^7$ N/m

Damping constant 'c':

C_c=2 mw = 2 x 9100 x 12.593 = 229138

$\zeta= C/ C_c$

$C= \zeta\ C_c$

C= 0.066 x 229138

C= 15123.108

- Simple harmonic motion expressed in two forms harmonic sine oscillations and harmonic cos oscillations. The time t is measured from extreme positions(cos, sine)time t is measured from mean position(sin)

Case i) cosine oscillations

x= Acoswt (extreme x from centre)

x= v= -Awsinwt

x'= a = -Aw^{2}coswt

v_{max} = -Aw(at mean point)

a_{max} = -Aw^2(at extreme end)

case ii) sine oscillations

x= asinwt ----- x from centre

v= aw coswt

v= w$\sqrt{a^2 - x^2}$

a= - Aw^{2}sinwt

Problem

Find velocity and accerelation after 0.3sec from extreme position of body with SHM with an amplitude of 0.8m and period of complete oscillation is 1.68sec.

A= 0.8M

t= 0.38sec

T= 1.6sec

Cosine oscillation(from extreme position)

v = -2.9 m/s

x =a = -Aw2 cos(wt)

= -08 x 3.93 2 x cos(3.93 x 0.3 x 180 o/ pie

= - 4.71m/s

Problem

A body is moving in SHM and has amplitude of 1m and period of complete oscillation is 2sec. find the velocity and accerelation of body 2/5th of a second after paving the mid position.

A=1m

T= 2sec

l= 2/5sec = 0.4sec

sine oscillation, x = asinwt (from mean position)

T= 2pi/ w

w= pi/t = 2pi/2

= 3.14rad/sec= 180^{o}

x = v= Awcoswt

= 1x 3.14 x cos180 x 0.4

= 0.97m/s

a= -Aw^{2}sinwt

= -1 x 3.14^{2} sin(180 x 0.4)

= -9.37m/s

Problem

A body moving in SHM amplitude is 1m, period of oscillation 2 sec. what will be the velocity and accerelation at 0.4sec after passing the extreme position.

Problem

Find velocity and accerelation after 0.3sec from extreme position of body moving with SHM with an amplitude of 0.8m and period of complete oscillation is 1.68sec.

A= 0.8m

t= 0.3sec

T= 1.68sec

Cosine oscillation(from extreme position)

x= Acoswt

x= v= -Awsinwt

= -0.8 x 3.92 x sin 3.92 x 0.3 x $(180^o/\pi)$

v= -2.9m/s

T= 2 π/ w
ω= 2π/1.68 = 3.92rad/sec
= 3.92 x 0.3 x $(180^o/\pi)$

x' = - Aw^2sinwt

= -0.8 x 3.93^2 x cos{3.93 x 0.3 x $(180^o/\pi)$ }

= -4.71m/s

Problem

A body is moving in SHM and has amplitude of 1m and period of complete oscillation is 2sec. find the velocity and acceleration of body 2/5 th of a second after paving the mid position.

A= 1m

T=2sec

t= 2/5s = 0.4s

sine oscillation, x= asinwt(from mean position)

x = v = Awcoswt

T= 2π/w;
w= 2π/T = 2π/2 = 3.14rad/s= 180°

= 1 x 3.14x cos 180 x 0.4

= 0.97 m/s

a= - Aw^2sinwt

$= -1 \times 3.14^2 \sin(180 \times 0.4)$

Or, a= - 9.37m/s

Example

A piston of an engine moves with SHM the crank rotates @ 100RPM and the stroke is 180cm, find the velocity and accerelation of piston when it is at a distance of 60cm from centre.

N= 100rpm

$\omega = 2\pi n/60 = 2\pi \times 100/60 = 10.47$rad/ sec

$\omega = 2\pi n/T$; n=f

A= STROKE/2 = 180/2 = 90= 0.9m

x=60cm

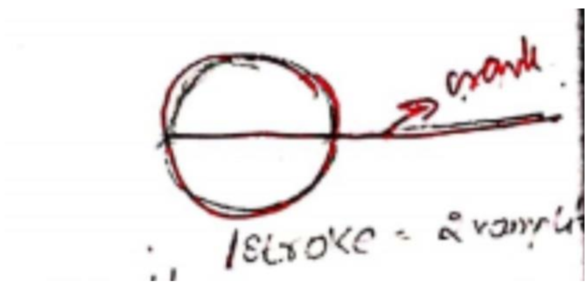
sine oscillation, x = Asinwt

0.6 = 0.9 sin wt

wt= 41.81

L 41.81

l= 0.069



Example

A body is moving with SHM and has velocity of 8m/s, 3m/s at a distance of 1.5m; 2.5m from centre. Find A, T.

$x_1 = 1.5$m $\quad$ $v_1 = 8$m/s

$x_2 = 2.5m$ $v_2 = 3m/s$

$v = w\sqrt{A^2 - x^2}$

$v_1 = w\sqrt{A^2 - x1^2}$ --------- (1)

$v_2 = w\sqrt{A^2 - x2^2}$ ------------(2)

$8 = w\sqrt{A^2 - 1.5^2}$

$3 = w\sqrt{A^2 - 3^2}$

Dividing eq (1) with (2),

$8/3 = w\sqrt{A^2 - 1.5^2} / w\sqrt{A^2 - 3^2}$

Square on both sides,

$2.67 = A^2 – 1.5^2 / A^2 – 3^2$

$2.67 A^2 – 16.6667 = A^2 – 1.5^2$

$1.67 A^2 = 14.41$

$A^2 = 8.632$

A= 2.93m

From eq 1,

$8 = w\sqrt{2.93^2 - 1.5^2}$

w= 3.71rad/s

T= 2π/w= 1.69s

Example

A body performing SHM has velocity of 12m/s when the displacement is 50mm and 3m/s. when displacement is 100mm displacement are measured from mid point. Calculate amplitude and frequency of motion and also find accerelation when displacement is 75mm.

$x_1 = 50mm$ $v_1 = 12m/s$

$x_2 = 100mm$ $v_2 = 3m/s$

$v_1 = w\sqrt{A^2 - x1^2}$

$12 = w\sqrt{A^2 - 50^2}$

$v_2 = w\sqrt{A^2 - x2^2}$

$3 = w\sqrt{A^2 - 100^2}$

$v_1 / v_2 = 12/3$

$4 = \sqrt{A^2 - 0.05^2} / \sqrt{A^2 - 0.10^2}$

$16 = A^2 - 0.05^2 / A^2 - 0.10^2$

$16A^2 - 0.04 = A^2 - 0.5^2$

A= 10.24cm

w= 134.16rad/s

T= 2π/w

f= 1/T = w/ 2π= 134.16 / 2π

f= 21.35 Hz

$a = x\ w^2$

$= 0.075 \times (134.16)^2$

$= 1350.07 m/s^2$

$V_{max} = Aw$

$= 0.102 \times 134.16$

$= 13.68 m/s$

$a_{max} = \pm aw^2$

$= 0.102 \times 134.16^2$

$= \pm 1835.8\ m/s^2$

Example

Particle is moving with SHM and performs 8 complete oscillation per minute i.e the body is 5cm from centre of oscillation. Find amplitude velocity and max accerelation. Given that the velocity of body at a distance of 7cm from centre is 0.6times max velocity.

1min = 8osc

? = 1 osc

= 1/8 x 60

= 7.5sec

T= 7.5sec

$x_1 = 0.07m$ $v_1 = v_{max} \times 0.6$

$x_2 = 0.05mm$ $v_2 = ?$

$v = w\sqrt{A^2 - x^2}$

$v_1 = w\sqrt{A^2 - x1^2} = 0.837\sqrt{A^2 - 0.07^2}$

$v_2 = w\sqrt{A^2 - x2^2} = 0.837\sqrt{A^2 - 0.05^2}$

$0.6Aw = 0.837\sqrt{A^2 - 0.07^2}$

$0.6A^2 = 4.9 \times 10^{-3}$

$V_{max} = Aw$

A = 0.0875m

$v_2 = 0.837\sqrt{0.0875^2 - 0.05^2}$

$v_2 = 0.06m/s$

$a_{max} = \pm aw^2$

$= 0.0875 \times 0.837^2$

$= 0.0614 m/s^2$

OSCILLATION OF A VERTICAL ELASTIC SPRING

This system is in SHM if amplitude should be less than static elongation or extension then only it will be in harmonic motion.

(underdamped) – if x less than e

Let 'L' be the length of spring if the weight is attached to spring it will extend upto some distance 'e' – static extension

When w is attached to spring it undergoes on extension of x

a- Accerelation

T- time period

e- displacement due to self wt.

x- displacement due to force

total extension due to weight = e+x

force required to produce unit deflection by weight 'w' = w/e

for (e+x) deflection by weight 'w' = W/e(e+x)

The force W/e(e+x) is balanced by the tension in the spring which is acting upwards to keep the weight 'W' in equilibrium.

The net force on the weight 'W' in upward direction is,

$= W/e(e+x) – W$

$= mg/e(e+x) – mg$

$= mgx/e$

This is the net force acting upward.

acceleration (a) = net force (f)/mass (m)

= mgx/e×m

a= gx/e = g/e×x

a= $(\sqrt{g/e})^2$ x

($\sqrt{}$a= w^2x)

∴w=$\sqrt{}$g/e

T=2π/w

T= 2π$\sqrt{}$m/x= 2π $\sqrt{}$l/g

Stiffness of spring k = w/e

e= w/k= mg/k

T=2π$\sqrt{}$m/k ⟶ mass/stiffness

frequency(f)= 1/T

Find the stiffness of spring if 50N is attached to it and its weight makes 40 oscillations per second, calculate stiffness

weight = 50N

1 sec= 40 oscillations

0.25 sec=1 oscillation

T= ¼= 0.25 sec

0.25 sec is time for one oscillation

T= 2π $\sqrt{}$e/g

0.25= 2π$\sqrt{}$e/9.81

<u>e= 1.55 cm</u>

w= mg

K=w/e= 50/1.55= <u>32.3 N/cm</u>

Q: The frequency of free vibration of weight "w" with spring constant "K" is 12 cycles per sec. When extra weight of 20 N is attached to weight W the frequency is reduced. Find the weight and stiffness.

Ans: w= f_1= 12 cps

$w+20 = f_2 = 10$ cps

$T=2\pi\sqrt{m/k}$ ⟶ mass/stiffness

$w = mg$

$m=w/g$

$f=1/T= 1/2\pi\sqrt{K/m}$

$f=1/2\pi\sqrt{K\times g/w}$

$12=1/2\pi\sqrt{K\times g/w}$ ⟶ equation 1

$10=1/2\pi\sqrt{K\times g/w+20}$ ⟶ equation 2

equation 1/equation 2 ⟶ $1.2= \sqrt{w+20/w}$

$1.44= \sqrt{w+20/w}$

$1.44\ w= w+20$

$0.44\ w=20$

$w= 45.45$ N

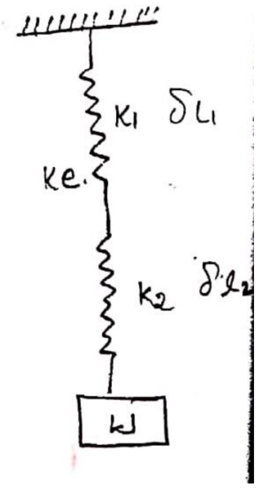
from, $12=1/2\pi\sqrt{K\times 9.81/45.45}$

$k= 26.34$ KN/m

Q: Two springs of stiffness K1 and K2 are connected in series, upper end of compound spring is connected ceiling and lower end carrier wight w. Find the equivalent weight spring stiffness of system.

Ans: a) series: -

If the springs are in series, they carry load "w"



$K_1= \sqrt{w/L_1}$

$K_2= \sqrt{w/L_2}$

$K= p/\delta$

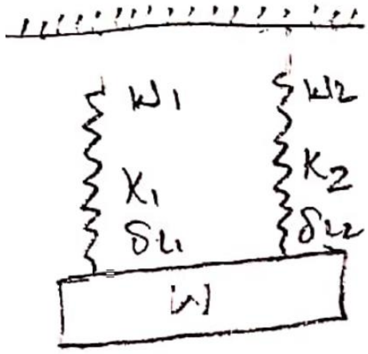
$e= \delta L1+ \delta L2$

$w/ K_1 + w/ K_2$

$1/ K_e = 1/ K_1+1/ K_2$

$K_e = K_1 K_2 / K_2 + K_{1 = e}$

b) Parallel:



$\delta L_1 = \delta L_2 = \delta L = e$

$K_1 = w_1 / \delta L_1$

$K_2 = w_1 / \delta L_2$

Total load= $w = w_{1+} w_2$

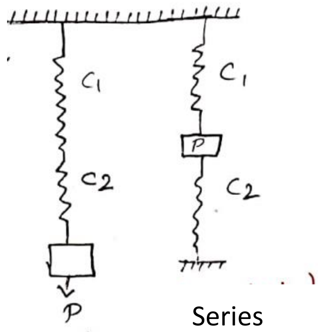
$w = (K_1 + K_2)\ \delta L$

$K_e \delta L = (K_1 + K_2)\ \delta L$

$K_e = (K_1 + K_2)$

Q: Find the periods of vibration for the following spring system

Ans: For series:



$e_1 = p/c_1$

$e_2 = p/c_2$

$e = e_{1+} e_2$

$e = p/c_{1+}\ p/c_2$

$e = p(c_{1+}c_{2)}/\ c_1 c_2$

$T = 2\pi\ \sqrt{}e/g$

$2\pi\ \sqrt{}p(c_{1+}c_{2)}/\ c_1 c_2/g$

For parallel

$p = p_1 + p_2$

$e_1 = p/c_1$

$e_2 = p/c_2$

$wKT_{=}\ e_{1=}\ e_2$

$p/c_{1\ =}\ p/c_2$

$p_{1=}\ p_2\ (c_{1/}\ c_2)$

$p = p_2\ (c_{1/}\ c_2) + p_2$

$p = p_2 (c_{1+} c_2 / c_2)$

$p / c_{1+} c_{2=} p_2 / c_2$

$e = p / (c_{1+} c_2)$

$T = 2\pi \sqrt{p / c_{1+} c_{2\times} g}$

Q: A 50 kg block is supported by 2 springs connected in series, if the spring constants are 4 kN/m and 6 kN/m. The block is pulled 40 mm down from the position of equilibrium. Find the priod of vibration, maximum velocity and acceleration.

Ans: b) When connected in parallel,

Given,

m= 50 kg

w= mg= 50×9.81= 490.5 N

K_1= 4 kN/m

K_2= 6 kN/m

Series= force same

parallel= force different

$e = w / K_{1,}\ e = w / K_2$

$e = e_{1+} e_2$

$e = w / K_1 + w / K_2$

e= 204.37 mm

$T = 2\pi \sqrt{e/g}$

$T = 2\pi \sqrt{0.204/9.81}$

= 0.9068 sec

$T = 2\pi / w$

$w = 2\pi / T$

= 6.93 rad/sec

$V_{max} = 2\ \omega = 0.277$ m/s

$= 0.04 \times 6.9$

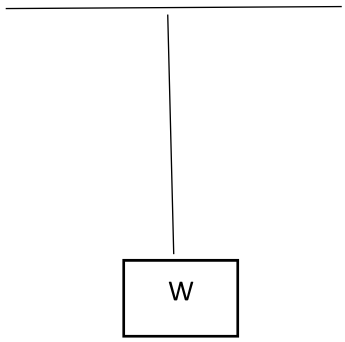
$a_{max} = \omega^2 = 1.92$ m/s

Notes:

free vibrations are;

- longitudinal
- Transverse
- Torsional vibration

Longitudinal vibration= (axial stress induced)



axial stress induced= $E = \sigma/e$

$E = \delta/L$

$\sigma = p/a$

$T = 2\pi \sqrt{e/g}$

$T = 2\pi / \omega$

$T = 2\pi \sqrt{m/k}$

$\omega = 1/\sqrt{\delta/g}$

$\omega = \sqrt{g/\delta}$

Transverse vibration: (bending stresses)

$\delta = wl^3/3EI$

$T = 2\pi \sqrt{\delta/g}$

$w = \sqrt{g/\delta}$

Torsional vibrations:

$T/J = N\theta/L$

$T = 2\pi \sqrt{I/q}$

I= momentum of inertia

I=mk2= radius of gyration

Torsional stiffness;

$q = T/\theta = NJ/L$

$q = \text{Torque}/\theta$

Q: A vertical shaft 100 mm in diameter and 1mm length is hang at one end and carries disc of weight 500N at another end. The radius of gyration is 450N. The modulus of rigidity of material is

0.8×10^5 N/mm^2. Find frequency of torsional and transverse vibrations if $6=2\times10^5$N

Ans: N or c = 0.8×10^5 N/mm^2

d= 100 mm

L=1m

w=500N

K=450mm

Transverse vibration:

$T= 2\pi \sqrt{\delta/g}$

$\delta= wl^3/3EI$

$= 500\times100^3/3\times2\times10^5 \times (\pi I/64)$

g=9.8 m/sec^2

$T= 2\pi \sqrt{\delta/g}$

$\delta= 1.69$

$T= 2\pi \sqrt{1.69/9.8\times10^3}$

f=1/T= 12.12 cps

Torsional vibrations:

$f= 1/2\pi \sqrt{g/\delta}$

$J= I_x + I_y$

T/θ= NJ/L= q== polar momentum. I

q= NJ/L

q/I= T/θI= g/ δ

I/q= g/δ

Circular:

$I_{x=}\ I_{y=}\ \pi D^4/64$

J or $I_J = I_x+I_y$

$_= \pi D^4/32$

$J= \pi 100^4/32= 9817477.042$

$q= 0.8\times10^5 \times9817477.042/ 1000$

$= 7.854\times10^8$ n

7.854×10^{11} kg

$f= 1/T= 1/ 2 \pi \sqrt{q/I}$

$I= ak^2$

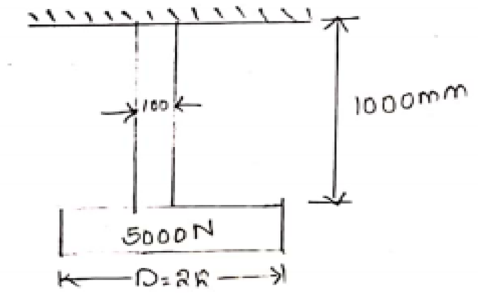
$I= mK^2$

$m= w/g$

$5000/9.81 \times450^2$

$= 103211009.2$ mm^2

Example :



$E=2\times10\ ^5$ N/mm^2

$K= 250$mm

$N= 8.16\times10\ ^5$ N/mm^2

find longitudinal and torsional vibration frequencies (f= 25.25 cps)

Ans:

f= 1/2π √ g/δ

δ= σ/E ×l

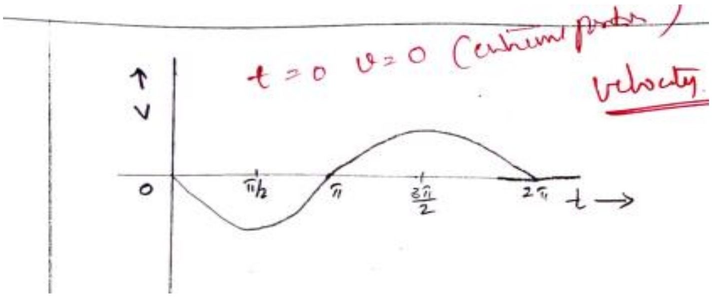
=3.18 ×10 $^{-7}$

σ= w/A= 5000/ π/4 ×d^{4}

= 6.366 ×10 $^{-5}$

E= δ/l= σ/E

Static analysis is special type of dynamic analysis, for dynamic analysis motion, static loading analysis plus dynamic loading analysis. Free vibration is one form of forced vibration.



from the plots it can be observed that,

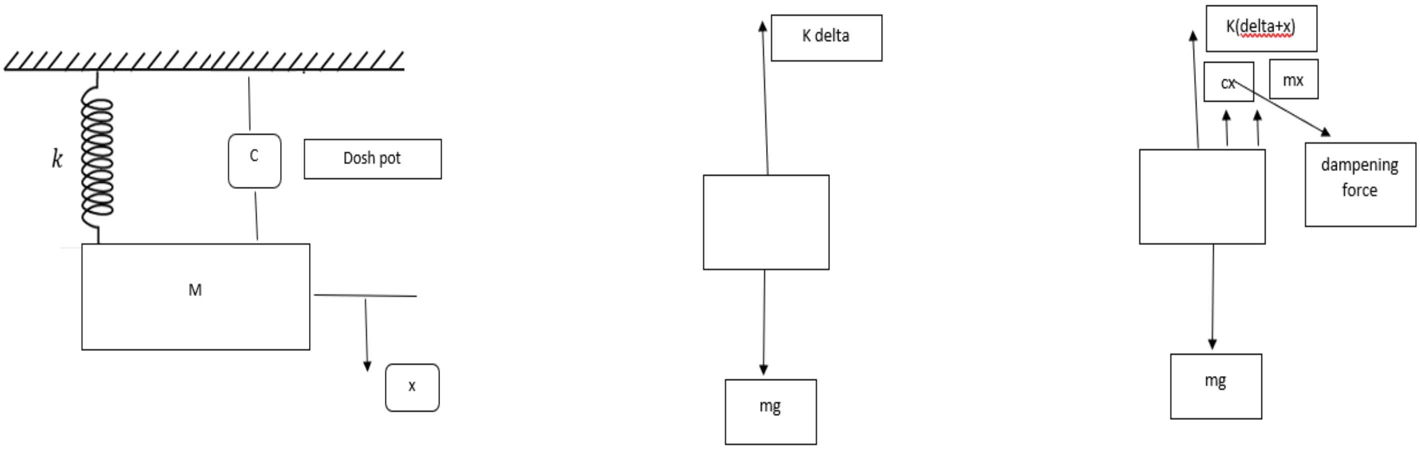
- If a body is disturbed it will never stop vibrating (Ideal case)
- When displacement is maximum (extreme portion), velocity is zero and acceleration is maximum in direction opp to displacement.
- When displacement is minimum at zero, velocity is maximum and acceleration is zero

Un-damped free vibrations EOM (pg- 92-93)

Equations of motion of SDOF system (damped free vibrations)

(3 element model): mass, spring, dampness

In a damped free vibration in addition to mass and spring there is an element called "dampness" is used. The element model is shown in figure.



C= dampening constant

K= Spring constant

m= Mass

Boundary conditions:

equation (a) can be written as

$x= X e^{-G\ \omega nt} \cos (w_d t +\Theta)$

$\therefore e^{ix} = \cos x + I \sin x$

$e^{-ix} = \cos x - I \sin x$

$x(t)=$

$e^{-c/2mt} (A \cos w_d t + B \sin w_d t$

$w_d = \omega_n \sqrt{1\text{-}G}$

where X and Θ are constants

X is amplitude and Θ is phase angle

<u>Introduce boundary conditions:</u>

x_0 (displacement) and v_o (velocity) at time period (t)= t

displacement (x) = x_0 solution for under damped case is;

$x_o = X\, e^{-G\, \omega nt} \cos (w_d t + \Theta)$ ____________(d)

after one time period the boundary conditions are;

t= t=tp/x=x_1 (boundary conditions)

Then the solution is

$x_{1=}\, Xe^{-G\, \omega n\, (t+tp)} \cos\, ^{(W_d (t+tp)} + \Theta$ _____________(e)

divide equation (d) and (e)

$x_o = X\, e^{-G\, \omega nt} \cos (w_d t + \Theta) /\ x_{1=}\, Xe^{-G\, \omega n\, (t+tp)} \cos\, ^{(W_d (t+tp)} + \Theta$

tp= time period = 1/fp ⟶ frequency= $2\pi/w_d$

$e^a/e^b = e^a$

$x_{o/}\ x_1 = e^{-G\, \omega ntp.} \cos (w_d t + \Theta) / \cos\, ^{(W_{dt}} + 2\pi + \Theta)$

$\therefore w_{dtp=}\ 2\pi$

$\text{Cos}\ \theta = \text{Cos}\ (2\pi + \theta)$

$\therefore x_{o/}\ x_{1=}\ e^{G\omega ntp}$

$\ln (x_{o/}\ x_1) = G\omega ntp = G\ \omega n \times 2\pi/\ wd$

$= G\ \omega n \times 2\pi/\ wn \sqrt{1-G^2}$

$\underline{\ln(x_{o/}\ x_1) = 2\pi G/\sqrt{1-G^2}}$

This equation is known as "logarithmic decrement" which is rate at which amplitude of free dampen vibration tend to be decreases.

After "n" number of oscillations of time period;

$n=t+nt_p$

where;

n= no. of cycles {reduction factor: pg 104}

The logarithmic decrement

$\underline{\ln (x_{o/}\ x_1) = 2\pi nG/\sqrt{1-G^2}}$

For over damped and vertically damped system, mass will return to its original position slowly and there is no vibration. Vibration is possible only under damped system because the roots of solution of underdamped case are complex and has periodic functions.

7.5.2015

Solution of differential Equation (Free Vibration - SDOF) (Undamped)

(having one num of natural frequency)

Differential Equation of SDOF undamped system is given by

$m\ddot{x} + kx + c\dot{x} = f(t)$

$$m\ddot{x} + Kx = 0$$

m = mass
K = stiffness of spring (or) spring constant
x = displacement (from equilibrium position)
$\ddot{x}$ = acceleration.

$$\ddot{x} + \frac{K}{m}x = 0$$

$$\omega_n = \sqrt{\frac{K}{m}} \quad ✓ \text{(Sa lyt)}$$

ω_n = natural Angular frequency (or) natural circular frequency

$$\ddot{x} + \omega_n^2 x = 0 \rightarrow (I)$$

This equation is satisfied by function $\sin\omega_n t$ & $\cos\omega_n t$. Then the solution of eq (I) can be written in the form of

$$x = A\sin\omega_n t + B\cos\omega_n t \leftarrow (1)$$

A & B are constants which can be determined from boundary condition (Initial Conditions)

The System shown in fig (A) can be disturbed in two ways

Case 'a':- → By pulling the mass by distance 'X'

Case 'b':- → By hitting the mass by means of fast moving object with a velocity of 'V'

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(a) **CASE 1:-** Pulling mass by distance 'x'

$t = 0$; $x = X$; $\dot{x} = 0$
time, displacement, velocity

(when you pull to 'x' distance and leave to oscillate, then at that point $t=0$, $v=0$)

Solution

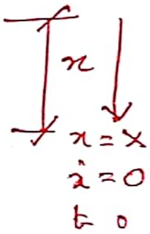
$x = A\sin\omega_n t + B\cos\omega_n t \rightarrow ①$

$\dot{x} = A\omega_n\cos\omega_n t - B\omega_n\sin\omega_n t \rightarrow ②$

sub $t, x, \dot{x}$ in ①

$X = A \times \sin 0 + B\cos 0$

$t = 0, x = X$ (B.C)



$X = B$

sub $t, x, \dot{x}$ in ②

$\dot{x} = A \times \omega_n \times \cos 0 - B \times \omega_n \sin 0$

$t = 0, \dot{x} = 0$ (BC)

$A = 0$
$X = B$ } →③

why $t = \frac{2\pi}{\omega}$?

sub ③ in ①

$\omega t = 0$

$\omega t = 2\pi$

$x = 0 \times \sin\omega_n t + X\cos\omega_n t$

$x = X\cos\omega_n t$

$x = a\cos\omega t$, $x = a\sin\omega t$; $\omega t = 2\pi$, $\omega t = 0$

(ωt cannot be zero because of oscillations)

(b) **CASE 2:-**

By hitting the mass by means of fast moving object with a velocity 'V'

Here, $t = 0$; $x = 0$; $\dot{x} = V$

(when body hits mass it will have some initial velocity)

Solution

$x = A\sin\omega_n t + B\cos\omega_n t$ — equ① → trigonometrical solutions.

$\dot{x} = A\omega_n\cos\omega_n t - B\omega_n\sin\omega_n t$

$0 = A\sin 0 + B\cos 0$ { $x = 0$, $t = 0$ }

$\therefore B = 0$

$\dot{x} = A \times \omega_n\cos 0 - B\omega_n\sin 0$ { $\dot{x} = V$, $t = 0$ }

$V = A\omega_n$

$$A = \frac{V}{\omega_n}$$

∴ from eqn ① we get

$$x = \frac{V}{\omega_n} \sin \omega_n t$$

* Case (i) represents the behaviour of undamped system in most practical cases

Solution for this case is

$$x = X \cos \omega_n t \quad — ①$$

$$\dot{x} = -X \omega_n \sin \omega_n t \quad — ②$$

$$= X \omega_n \cos\left(\omega_n t + \frac{\pi}{2}\right)$$

$$\ddot{x} = -X \omega_n^2 \cos \omega_n t \quad — ③$$

$$= X \omega_n^2 \cos(\omega_n t + \pi)$$

$\cos \omega t = 1$

$\omega t = 2\pi$ or 0

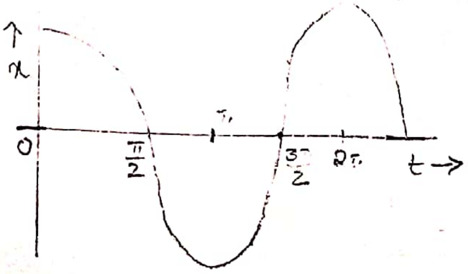
for x to be maximum

$2\pi = \omega t$

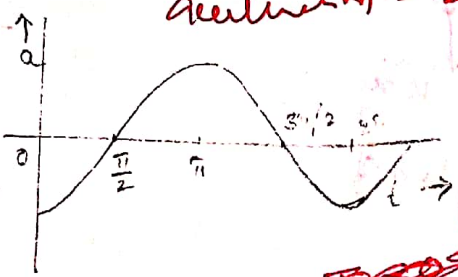
$t = \frac{2\pi}{\omega}$

* The expression indicates that velocity vector leads displacement vector by phase angle $\pi/2$. Similarly acceleration leads displacement by phase angle π. The maximum velocity is $\dot{x} = X\omega_n$, when $\cos(\omega_n t + \pi/2) = 1$, maximum acceleration is $\ddot{x} = X\omega_n^2$ → when $\cos(\omega_n t + \pi) = 1$

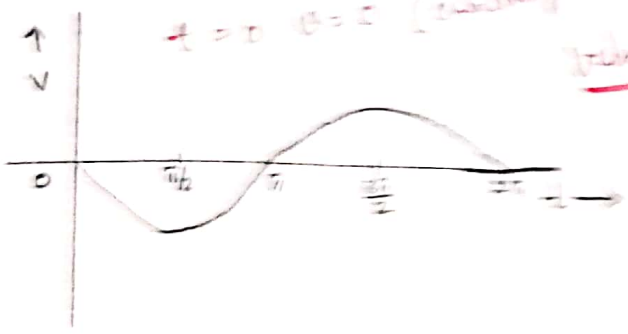
Following are the plots of displacement, velocity & acceleration w.r.t time



Displacement - time graph



acceleration - time graph



From the plots it can be observed that

→ If a body is disturbed it will never stop vibrating (ideal case)

→ when displacement is maximum, velocity is zero & acceleration is maximum in direction opp to displacement

→ when the displacement is zero, velocity is max and acceleration is zero.

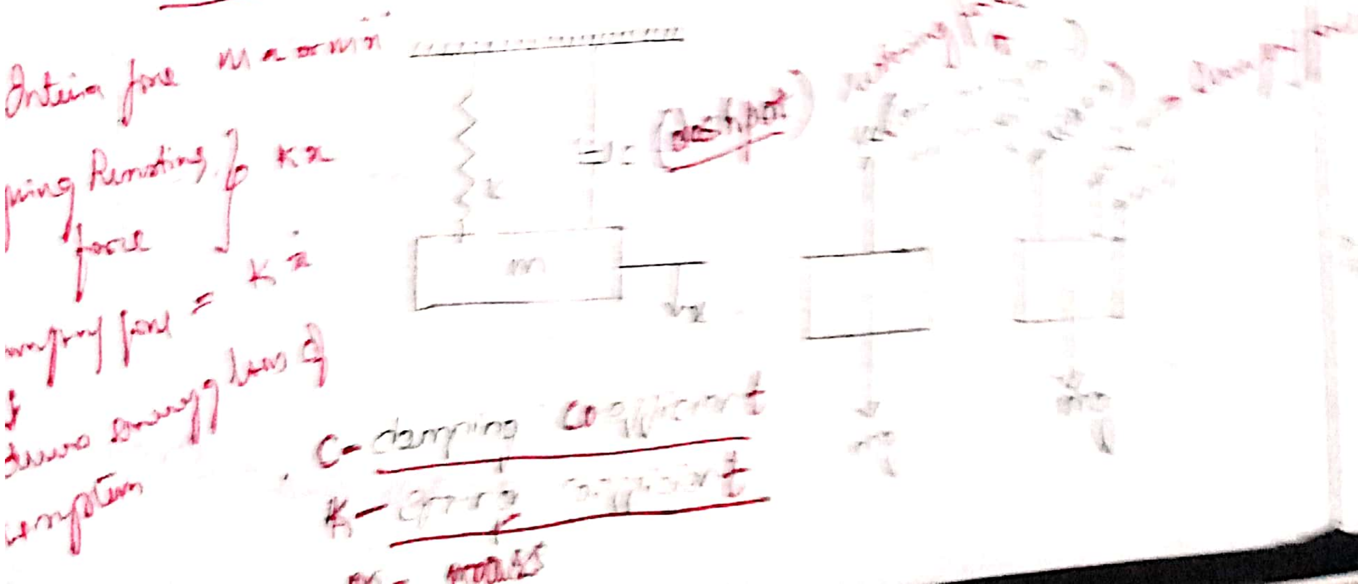
→ Undamped free vibration EOM

EQUATIONS OF MOTION OF SDOF SYSTEM

[Damped free vibrations] :- [3 element model]

pg 96

In damped free vibration in addition to "mass" & "spring" there is an element called damper is used. The element model is shown in figure



C - damping coefficient

K - spring coefficient

$F_g = f_i + f_D + f_K$

$K\Delta = mg$ 　　　　 $mg = K\Delta$

$mg = m\ddot{x} + K(\Delta + x) + c\dot{x}$

$$m\ddot{x} + c\dot{x} + Kx = 0 \rightarrow (1)$$

This is differential equation for SDOF system

- damped - free vibration *

The solution for this 2nd order Equation can be expressed as

(exponential solution) →

$x = Xe^{st}$ → constant amplitude

$\dot{x} = sXe^{st}$

$\ddot{x} = s^2Xe^{st}$

Sub these all above 3 in eqn (1)

$m \times s^2Xe^{st} + csXe^{st} + KXe^{st} = 0$

$$ms^2 + cs + K = 0$$

$$(1)s^2 + \left(\frac{c}{m}\right)s + \left(\frac{K}{m}\right) = 0$$

(a, b, c)

$ax^2 + bx + c = 0$

$\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ (or) $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$

The roots of Equation are

$$s_{1,2} = -\frac{c}{2m} \pm \frac{1}{2}\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{K}{m}\right)}$$

(superimpose the two roots solutions) The solution for eqn (1) is given by ⇒ $x = x_1(t) + x_2(t)$

$$x = X_1 e^{\left[-\frac{c}{2m} + \frac{1}{2}\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{K}{m}\right)}\right]t} + X_2 e^{\left[-\frac{c}{2m} - \frac{1}{2}\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{K}{m}\right)}\right]t}$$

$[\because e^{a+b} = e^a \cdot e^b]$

$x = Xe^{st}$

$$x = e^{-\left(\frac{c}{2m}\right)t}\left[X_1 e^{\frac{1}{2}\left[\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{K}{m}\right)}\right]t} + X_2 e^{-\frac{1}{2}\left[\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{K}{m}\right)}\right]t}\right] \rightarrow (2)$$

There are 3 possible cases of Solution based on the term in the determinant

this term is not considered why? pg 98.

Only this term is considered why? pg 98

*Case (i) :- Over damped Case

$$\left(\frac{c}{m}\right)^2 > 4\left(\frac{k}{m}\right)$$

Case (ii) :- Critically damped Case

$$\left(\frac{c}{m}\right)^2 = 4\left(\frac{k}{m}\right)$$

Case (iii) :- under damped Case

$$\left(\frac{c}{m}\right)^2 < 4\left(\frac{k}{m}\right)$$

C = damping constant
C_c = critical damping constant

Let 'C_c' be the critical damping co-efficient

Critical damped case :-

$$\left(\frac{C_c}{m}\right)^2 = 4\left(\frac{k}{m}\right)$$

$$C_c = 2\sqrt{km}$$

$$= 2\sqrt{\frac{k}{m}\times m^2}$$

$\omega_n^2 = \frac{k}{m}$

$\omega_n = \sqrt{\frac{k}{m}}$

$$C_c = 2\omega_n m$$

$$C_c = 2\omega_n m = 2\sqrt{km}$$ Critical Damping coefficient

Case (iii) :- Under damped

Usually practically all systems are under damped

$\sqrt{-x} = i\sqrt{x}$
$i = \sqrt{-1}$
imaginary part
$\sqrt{-x} = \sqrt{-1}\sqrt{x}$
$= i\sqrt{x}$

$$\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{k}{m}\right)} = i\sqrt{4\left(\frac{k}{m}\right) - \left(\frac{c}{m}\right)^2}$$

$$i\sqrt{4\omega_n^2 - \left(\frac{c}{C_c}\times\frac{C_c}{m}\right)^2}$$

Zeta $\zeta = \frac{c}{C_c}$

* Damping factor ζ is the ratio of damping co-efficient 'c' to the critical damping coefficient (C_c)

$$i\sqrt{4\omega_n^2 - \zeta^2 \times \left(\frac{2m\omega_n}{m}\right)^2}$$

$$\sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{k}{m}\right)} = i\left[2\omega_n\sqrt{1-\zeta^2}\right]$$

we are considering only inside part 'i' remai same.

$$x = e^{-\left(\frac{c}{2m}\right)t}\left[x_1 e^{\frac{1}{2}\left[i2\omega_n\sqrt{1-\zeta^2}\right]t} + x_2 e^{-\frac{1}{2}\left[i2\omega_n\sqrt{1-\zeta^2}\right]t}\right]$$

$$x = e^{-\left(\frac{c}{2m}\right)t}\left[x_1 e^{\left(i\omega_n\sqrt{1-\zeta^2}\right)t} + x_2 e^{-\left(i\omega_n\sqrt{1-\zeta^2}\right)t}\right] \quad (i)$$

Let us assume

$$\boxed{\omega_n\sqrt{1-\zeta^2} = \omega_d} \qquad \boxed{\zeta = \frac{c}{c_c}}$$

natural frequency of undamped free vibrations

where,

ω_d = damped natural angular frequency

ω_d is natural frequency of damped free vibration.

$\therefore$ For under damped case the solution is

$$x = e^{-\left(\frac{c}{2m}\right)t}\left[x_1 e^{i\omega_d t} + x_2 e^{-i\omega_d t}\right] \rightarrow (a)$$

Case (ii):- Critically damped

For critically damped case since two roots are equal the general solution can be expressed as follows:-

(one independent solution is)

$$x = x_1(t) + x_2(t)$$

$x_1(t)$ is one independent solution which is expressed as follows

For critically Damped Case, $\zeta = 1$

$$x_1(t) = x_1 e^{-\left(\frac{c}{2m}\right)t}$$

another independent solution $x_2(t)$ may be found using

$$x_2(t) = x_2 t e^{-\left(\frac{c}{2m}\right)t}$$

and these 2 equations satisfy eqn (1) $m\ddot{x} + c\dot{x} + kx = 0$ *

The General Solution for critically damped System is obtained by superimposing this 2 independent Solution

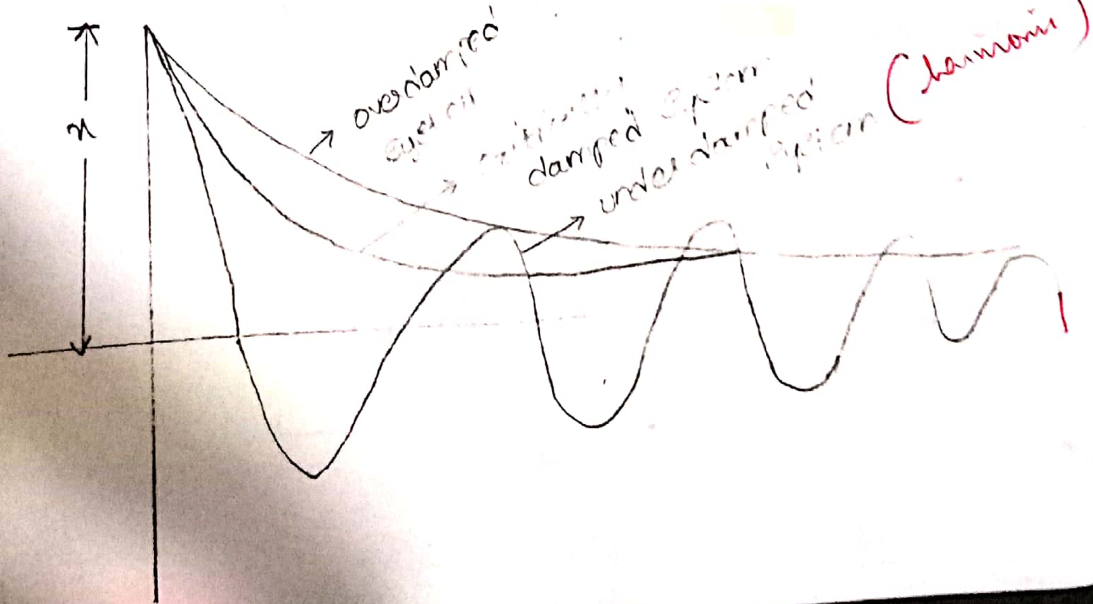
$$x = (x_1 + x_2 t) e^{-\left(\frac{c}{2m}\right)t} \quad \text{—(b)}$$

Case (iii) :- Over damped

$$x = e^{-\frac{c}{2m}t}\left[x_1 e^{\omega_n\left(\sqrt{\zeta^2 - 1}\right)t} + x_2 e^{-\omega_n\left(\sqrt{\zeta^2-1}\right)t}\right]$$

$$\because \frac{c}{2m} = \frac{c}{c_c} \times \frac{c_c}{2m} = \zeta \times \frac{2m\omega_n}{2m} = \zeta\omega_n$$

$$x = e^{-\zeta\omega_n t}\left[x_1 e^{\omega_n\left(\sqrt{\zeta^2-1}\right)t} + x_2 e^{-\omega_n\left(\sqrt{\zeta^2-1}\right)t}\right] \quad \text{(c)}$$



Boundary Conditions :-

Equation (a) can be written as

$e^{ix} = \cos x + i\sin x$
$e^{-ix} = \cos x - i\sin x$

$\zeta\omega_n = \frac{c}{2M}$

$$x = X e^{-\zeta\omega_n t}\cos(\omega_d t + \phi)$$

$x(t) = e^{-c/2m\, t}(A\cos\omega_d t + B\sin\omega_d t)$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

where X & ϕ are constants

X is amplitude &

ϕ is phase angle

Introduce boundary conditions :-

x_0 (displacement) & v_0 (velocity)

at time period $(t) = t$ & displacement $(x) = x_0$ the

Solution for under damped case is

$$x_0 = X e^{-\zeta\omega_n t}\cos(\omega_d t + \phi) \rightarrow (d)$$

after one time period the boundary conditions are

$t = t + t_p$ } B.C's
$x = x_1$

Then the solution is

$$x_1 = X e^{-\zeta\omega_n(t+t_p)}\cos(\omega_d(t+t_p) + \phi) \rightarrow (e)$$

divide Eqn(d) / Eqn(e)

$$\frac{x_0}{x_1} = \frac{X e^{-\zeta\omega_n t}\cos(\omega_d t + \phi)}{X e^{-\zeta\omega_n(t+t_p)}\cos(\omega_d(t+t_p) + \phi)}$$

t_p = time period $= \frac{1}{f_p \text{ frequency}} = \frac{2\pi}{\omega_d}$

$(\omega_d t_p = 2\pi)$

$\frac{e^a}{e^b} = e^{a-b}$

$$\frac{x_0}{x_1} = \frac{X e^{-\zeta\omega_n t}\cos(\omega_d t + \phi)}{X e^{-\zeta\omega_n t}\cdot e^{-\zeta\omega_n t_p}\cos(\omega_d(t+t_p) + \phi)}$$

$e^{-\zeta\omega_n t - (-\zeta\omega_n t \cdot e^{-\zeta\omega_n t_p})} = e^{\zeta\omega_n t_p}$

11-5-2019

$$\frac{x_0}{x_1} = e^{\zeta\omega_n t_p} \cdot \frac{\cos(\omega_d t + \phi)}{\cos(\omega_d t + 2\pi + \phi)}$$

$\omega_d t_p = 2\pi$

$\because \cos\theta = \cos(2\pi + \theta)$

$$\frac{x_0}{x_1} = e^{\zeta\omega_n t_p}$$

$a = e^b$
$b = \ln(a)$

ln to base e

log to base 10

$$\ln\left(\frac{x_0}{x_1}\right) = \zeta\omega_n t_p = \zeta\omega_n \times \frac{2\pi}{\omega_d}$$

$$= \zeta\omega_n \times \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$\boxed{\ln\left(\frac{x_0}{x_1}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}}$$

rate at which amplitude of free damped vibration decreases

This equation is known as logarithmic decrement

After 'n' number of oscillations time period is

$t = t + nt_p$ n→no. of cycles

Reduction factor pg 104

The logarithmic decrement

$$\boxed{\ln\left(\frac{x_0}{x_1}\right) = \frac{2\pi n\zeta}{\sqrt{1-\zeta^2}}}$$

pg 102

ln or loge

* For over damped & critically damped system mass will return to its original position slowly and there is no vibration.
Vibration is possible only in under damped system because the roots of the solution of underdamped case are complex & has periodic functions.

11-5-2015
absent

Example (*)

A damped system has following elements mass $m = 4\,kg$, $K = 1\,KN/m$, damping const $C = 40\,N\,sec/m$. Find damping factor, natural frequency of a damped oscillation, logarithmic decrement & no. of cycles after which the original amplitude is reduced to 20%.

$m = 4\,kg$

$K = 1\,KN/m$

$C = 40\,N\text{-}sec/m$

damping factor $\xi = \frac{c}{C_c}$

Critical Damping Coefficient.

$C_c = 2\sqrt{Km}$

$= 2\sqrt{4 \times 1 \times 10^3}$

$= 2\sqrt{4 \times 10^3}$

$C_c = 126.491$

$\xi = \frac{c}{C_c} = \frac{40}{126.5} = 0.316$

Natural frequency of damped oscillation.

$\omega_d = \omega_n\sqrt{1-\xi^2}$ ✓

✓ $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{1000}{4}} = 15.81$ Hertz or cps

$\omega_d = 15.81\sqrt{1-(0.316)^2}$

$= 15\,Hz$ (or) rpm

$$f_d = \frac{1}{T_d} = \frac{\omega_d}{2\pi}$$

$f = \frac{1}{T}$

$T = \frac{2\pi}{\omega}$

$= \frac{15}{2\pi}$

$= 2.38$ Hz (or) cycles per sec

logarithmic decrement :-

$$\ln\left[\frac{x_0}{x_1}\right] = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$= \frac{2\pi \times (0.316)}{\sqrt{1-0.316^2}}$$

$= 2.092$

* no. of cycles after which original amplitude is reduced to 20%.

$$\ln\left[\frac{100}{20}\right] = 2.092 \times n$$

∴ If original is assumed as 100, it is reduced to 20

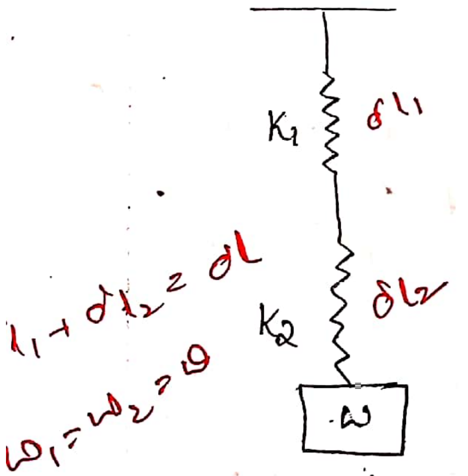
$\therefore n = 0.334 \simeq 1$ cycle ✓

* 2. Two springs of stiffness k_1 & k_2. are connected in series & carries a load ω, find equivalent spring stiffness of the system. If the springs are connected in parallel what is the equivalent spring stiffness

(a) SPRINGS CONNECTED IN SERIES :-

If the springs are connected in series, forces in each spring will be same & are equal to externally applied force

$K = \frac{W}{\delta L}$



$$K = \frac{W}{\delta L}$$

series — forces applied are same, deflections different.

Since the force applied on springs are same, δl_1 is deflections of 1st spring & δl_2 is deflections of 2nd spring.

$\therefore \quad K_1 = \frac{W}{\delta L_1} \qquad K_2 = \frac{W}{\delta L_2}$

$\delta L_1 = \frac{W}{K_1} \qquad \delta L_2 = \frac{W}{K_2}$

$\therefore$ Total deflections $\delta L = \delta L_1 + \delta L_2$ (Series)

$$\delta L = \frac{W}{K_1} + \frac{W}{K_2}$$

$$\frac{W}{K_e} = W\left[\frac{1}{K_1} + \frac{1}{K_2}\right]$$

$$\therefore \frac{W}{K_e} = W\left[\frac{K_2 + K_1}{K_1 K_2}\right]$$

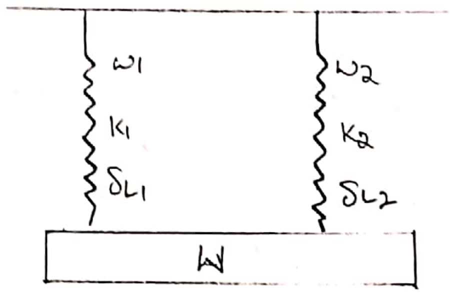
$$K_e = \frac{K_1 K_2}{K_2 + K_1}$$

$$\left[K_e = \frac{1}{\frac{1}{K_1} + \frac{1}{K_2}}\right]$$

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$$

SPRINGS CONNECTED IN PARALLEL:-

In case of spring in parallel the deflection is same in both the springs, then deflection of total system is equal to deflection of any spring.



Deflection same, forces shared are differ.

$$\delta l_1 = \delta l_2 = \delta l$$

let w_1 & w_2 are the loads shared by 1st & 2nd spring respectively

$$\therefore K_1 = \frac{w_1}{\delta L_1} = \frac{w_1}{\delta L}$$

$$w_1 = K_1 \delta L$$

$$K_2 = \frac{w_2}{\delta L_2} = \frac{w_2}{\delta L}$$

$$\therefore w_2 = K_2 \delta L$$

$$K_e = \frac{w}{\delta L}$$

$$w = K_e \delta L$$

Total load $w = w_1 + w_2$ (parallel)

$$w = (K_1 + K_2)\delta L$$

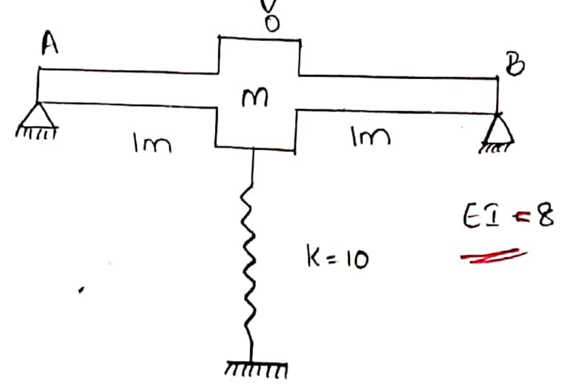
$$K_e \delta L = (K_1 + K_2)\delta L$$

$\therefore$ $K_e = K_1 + K_2$

Imp

* **Problem:-**

→ Find the natural frequency of the simply supported beam shown in figure



Let stiffness of beam AB be K_b & stiffness of spring K_S

$\therefore$ Total stiffness $= K_b + K_S$ ✓

parallel – (Deflection same)

$K_e = K_1 + K_2$

$\therefore$ $K_b = \frac{P}{\delta}$

$\delta = \frac{Pl^3}{48EI}$ (s.s.b) deflection

$\therefore$ $K_b = \frac{48EI}{l^3}$ [$l = 2m$]

$\therefore$ $K_b = \frac{48 \times 8}{8}$

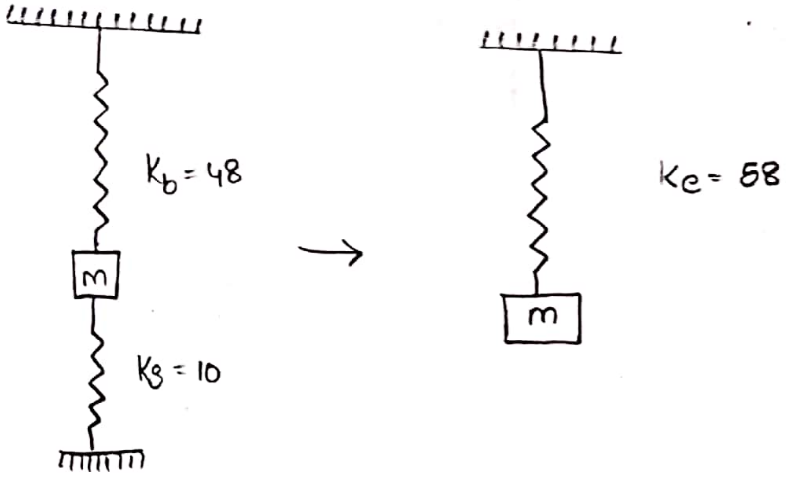
$K_b = 48$ units ✓

$\therefore$ $K = K_b + K_S$

$= 48 + 10$

$= 58$ units

$\therefore$ $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{58}{m}}$

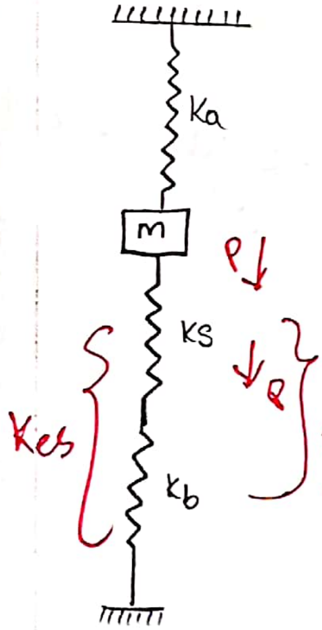


Example

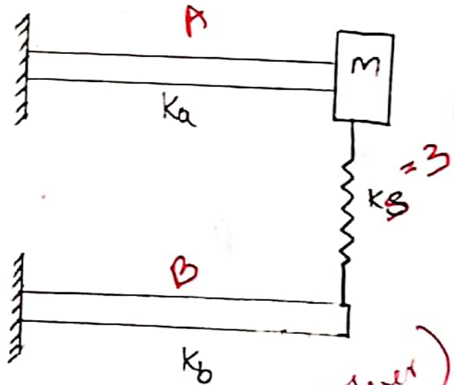
* Problem:-

Find the natural frequency of cantilever beam with attatched mass system as shown in figure

Data: $K_s = 3$, $L = 1m$, $EI = 1$



in series forces applied are same



$\delta = \frac{Pl^3}{3EI}$ (Cantilever)

$K_a = \frac{P}{\delta} = \frac{3EI}{l^3}$ (cantilever)

$K_a = \frac{3(EI)}{1} = 3$

$K_a = 3$ &

$K_b = 3$

$K_{es} = \frac{K_b K_s}{K_b + K_s} = \frac{3 \times 3}{3+3} = \frac{9}{6} = \frac{3}{2}$

Equivalent Stiffness of parallel Connected Springs

$K_e = K_a + K_{es}$ parallel

$K_{es} = \frac{3}{2}$

$= 3 + \frac{3}{2}$

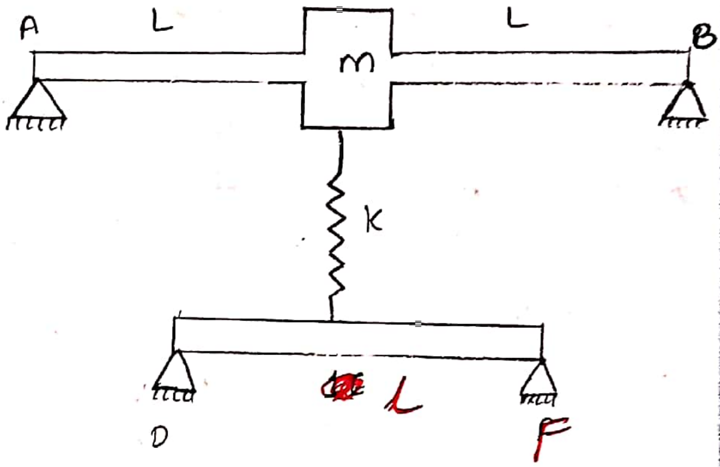
$= \frac{9}{2}$

$\therefore$ natural frequency $= \omega_n = \sqrt{\frac{K_{eff}}{m}} = \sqrt{\frac{9}{2m}}$

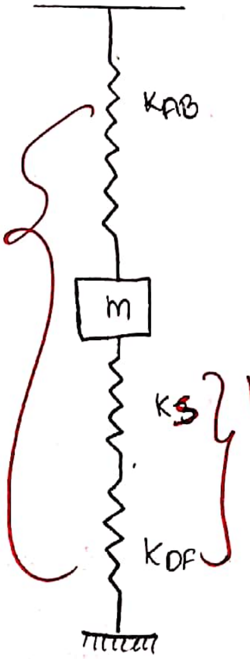
Problems *

The natural frequency of given mass system

$l = 1m$, $EI = 1$, $K = 48$



with



Kep

$K_{AB} = \frac{P}{\delta} = P\left(\frac{48EI}{Pl^3}\right) = \frac{48EI}{l^3}$ (SSB)

$\delta = \frac{Pl^3}{48EI}$

$l = 2$

$K_{AB} = 6$

$K_{DF} = \frac{P}{\delta} = \frac{P(48EI)}{Pl^3} = \frac{48(1)}{1}$

$K_{DF} = 48$

$\therefore K_{es} = \frac{48 \times 48}{48+48} = 24$ → K_S and K_{DF} (48, 48)

$K_{ep} = 6 + 24 = 30$

Natural frequency $\omega_n = \sqrt{\frac{K}{m}}$ $= \sqrt{\frac{30}{m}}$,

Example *

→ A single degree of freedom system whose mass is 9100 kgs with viscous damping is displaced from its position of rest. by distance of 30mm The max. displacement on return swing is 20mm on 0.5 sec. Determine spring ~~constant~~ constant damping constant C.

$x_0 = 30mm \quad x_1 = 20mm$

These are the two Subsequent displacements

$T = 0.5 sec$

$$\ln\left(\frac{x_0}{x_1}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

logarithm Decrement rate at which amplitude of free damped vibrations reduces.

*

$0.1760 \; (1-\zeta^2) = (6.2831)^2 \zeta^2$

0.164

$1-\zeta^2 = 224\zeta^2$

$\frac{1}{225} = \zeta^2$

$\zeta = \sqrt{1/225}$

$\zeta = 0.066$ damping factor

$$T_d = \frac{2\pi}{\omega_d}$$

$0.5 = \frac{2\pi}{\omega_d}$ damped frequency.

$\omega_d = 12.566$

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

$12.566 = \omega_n \sqrt{1-0.066^2}$

$\omega_n = 12.593 \; rad/sec$

$$\therefore \omega_n = \sqrt{\frac{K}{m}}$$

$\omega_n = \sqrt{\frac{k}{m}}$

$\omega_n^2 = \frac{k}{m}$

$K = m\omega_n^2$

$K = 9100 \times 12.593^2$

ments

$K = 1.44 \times 10^6$ Kg/m

$K = 1.44 \times 10^7$ N/m

1 Kg = 10 N

Damping Constant 'C':

$C_c = 2m\omega$ = $2 \times 9100 \times 12.593$ = 229138.

$\zeta = \frac{C}{C_c}$ Damping ratio. ω or ωn

$C = \zeta C_c$

$C = 0.066 \times 229138$

$C = 15123.108$

22.6.2015

Simple harmonic motion expressed in 2 forms
harmonic sine oscillations & harmonic cos oscillation
The time 't' is measured from extreme position (cos)
time is measured from mean position (sin)

Case a: Cosine oscillations

$x = A\cos\omega t$ (extreme) → x from centre

differentiate on both sides.

$\dot{x} = v = -A\omega\sin\omega t$

again differentiate

$\ddot{x} = a = -A\omega^2\cos\omega t$

A = Amplitude
v = velocity
a = acceleration

$v_{max} = -A\omega$ (at mean point) $\sin\omega t = 1$

$a_{max} = -A\omega^2$ (at extreme end) $\cos\omega t = 1$

$v_{max} = -a\omega$
$a_{max} = -a\omega^2$

$\frac{d}{d\theta}\cos\theta = -\sin\theta$

$\sin 0 = 0$
$\cos 0 = 1$

Case b: Sine oscillations.

$x = a\sin\omega t$ → x from extreme

$v = a\omega\cos\omega t$

$v = \omega\sqrt{a^2 - x^2}$

$a = -a\omega^2\sin\omega t$

$T = \frac{2\pi}{\omega}$

$f = \frac{1}{T}$ Hz / cps

(*) Find velocity & acceleration after 0.3sec from extreme position of body moving with SHM with an amplitude of 0.8m & period of complete oscillation is 1.6sec

v, a ?

$A = 0.8m$

$t = 0.3sec$

$T = 1.6sec.$

$\pi^c = 180^\circ$

$1^c = \frac{180^\circ}{\pi}$

Cosine oscillation. (from extreme position)

$x = A\cos\omega t$

$T = \frac{2\pi}{\omega}$

$\dot{x} = v = -A\omega\sin(\omega t)$

$\omega = \frac{2\pi}{1.6} \simeq 3.92 \, rad/sec.$

$= -0.8 \times 3.92 \times \sin 3.92 \times 0.3 \times \frac{180^\circ}{\pi}$

$\omega t = 3.92 \times 0.3 \times \frac{180^\circ}{\pi}$

$v = -2.9 m/s.$

$\ddot{x} = a = -A\omega^2\cos(\omega t)$

$= -0.8 \times 3.93^2 \times \cos\left(3.93 \times 0.3 \times \frac{180^\circ}{\pi}\right)$

$= -4.71 m/s$

(*) A body is moving in SHM & has amplitude of 1m & period of complete oscillation is 2sec find the velocity & acceleration of body 2/5th of a second after passing the mid position.

$A = 1m$ $v = ?$

$T = 2sec$ $a = ?$

$t = \frac{2}{5}s = 0.4sec.$

Sine oscillation (from mean position)

$x = a\sin\omega t$

$T = \frac{2\pi}{\omega}$

$\dot{x} = v = A\omega\cos\omega t$

$\omega = \frac{2\pi}{T}$

$= 1 \times 3.14 \times \cos 180 \times 0.4$

$= \frac{2\pi}{2}$

$= 0.97 m/s$

$= 3.14 \, rad/sec.$

$= 180^\circ$

$$a = -A\omega^2 \sin\omega t$$

$$= -1 \times 3.14^2 \sin(180 \times 0.4)$$

$$a = -9.37\ m/s$$

(*) A body moving in SHM amplitude is 1m. Period of oscillation 2sec. What will be the velocity and acceleration at 0.4sec after passing the extreme position.

$v = -2.98$

$a = -3.05$

(cosine oscillation).

lead and oscillations relation

Example

(*) A piston of an engine moves with SHM. The crank rotates @ 100RPM & the stroke is 180cm, find the velocity & acceleration of piston when it is at a distance of 60cm from centre

1 stroke = 2 ampli...

$N = 100 rpm$

$\omega = \frac{2\pi n}{T}$ → rpm

$T = 60 sec$

$n = f$

$$\omega = \frac{2\pi n}{60} = \frac{2\pi \times 100}{60} = 10.47\ rad/sec$$

$$A = \frac{stroke}{2} = \frac{180}{2} = 90$$

$= 0.9m.$

$x = 60cm.$

Sine oscillations

$x = A\sin\omega t$

$0.8 = 0.9\sin\omega t$

$\omega t = 41.81$

$t = \frac{41.81}{10.47} \times \frac{\pi}{180}$

$t = 0.069$

Example

A body is moving with SHM & has velocity of 8m/s, 3m/s at a distance of 1.5m ; 2.5m resp. from centre find A, T ?

$x_1 = 1.5m$ $v_1 = 8m/s$

$x_2 = 2.5$ $v_2 = 3m/s$

$$v = \omega\sqrt{a^2 - x^2}$$

$v_1 = \omega\sqrt{a^2 - x_1^2}$ — ①

$v_2 = \omega\sqrt{a^2 - x_2^2}$ — ②

$8 = \omega\sqrt{A^2 - 1.5^2} \to ①$ $3 = \omega\sqrt{A^2 - 3^2} \to ②$

$\frac{①}{②}$ $\frac{8 = \omega\sqrt{A^2 - 1.5^2}}{3 = \omega\sqrt{A^2 - 2.5^2}}$

$\frac{8}{3} = \sqrt{\frac{A^2 - 1.5^2}{A^2 - 2.5^2}}$

S.O.B.S. Square on Both sides

$2.67 = \frac{A^2 - 1.5^2}{A^2 - 2.5^2}$

$2.67A^2 - 16.6667 = A^2 - 1.5^2$

$1.67A^2 = 14.41$

$A^2 = 8.632$

$A = 2.93m$.

from eqn (1) . $8 = \omega\sqrt{2.93^2 - 1.5^2}$.

$\omega = 3.71$ rad/sec.

$T = \frac{2\pi}{\omega} = 1.69$ sec. ✓

Example 6.

A body performing SHM has velocity of 12 m/s when the displacement is 50 mm & 3 m/s when displacement is 100 mm displacement are measured from mid point. Calculate Amp & frequency of motion & also find acceleration when displacement is 75 mm

$x_1 = 50$ mm $\quad V_1 = 12$ m/s.

$x_2 = 100$ mm $\quad V_2 = 3$ m/s.

$V_1 = \omega\sqrt{A^2 - x_1^2} \rightarrow 12 = \omega\sqrt{A^2 - 50^2}$

$V_2 = \omega\sqrt{A^2 - x_2^2} \rightarrow 3 = \omega\sqrt{A^2 - 100^2}$.

$\frac{V_1}{V_2} \Rightarrow 4 = \sqrt{\frac{A^2 - 0.05^2}{A^2 - 0.10^2}}$.

$16 = \frac{A^2 - 0.5^2}{A^2 - 0.10^2}$.

$16A^2 - 0.04 = A^2 - 0.5^2$.

$A = 10.24$ cm .

$\omega = 134.16$ rad/sec.

$T = \frac{2\pi}{\omega}$

$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{134.16}{2\pi}$

$f = 21.35$ Hz (or) CPS

$a = x \cdot \omega^2$

$v = r\omega$
$a = r\omega^2$

$= 0.075 \times (134.16)^2$

$= 1350.07\ m/s^2$

$V_{max} = A\omega$

$= 0.102 \times 13416$

$= 13.68\ m/s$.

$a_{max} = \pm a\omega^2$

$= 0.102 \times 134.16^2$

$= \pm 1835.8\ m/s^2$

Example

(*) Particle is moving with SHM & performs 8 complete oscillation per minute i.e the body is 5cm from centre of oscillation find amplitude velocity & max acceleration : Given that the velocity of body at a distance of 7cm from centre is 0.6 times max velocity

1 min = 8 osc.

? = 1 osc

$= 1/8 \times 60 = 7.5 sec$

$T = 7.5 sec$.

$T = \frac{2\pi}{\omega}$, $\omega = \frac{2\pi}{T}$

$= 0.837$ rad/sec.

$x_1 = 0.07m$ $\quad$ $v_1 = V_{max} \times 0.6$

$x_2 = 0.05m$ $\quad$ $v_2 = ?$

$v = \omega\sqrt{A^2 - x^2}$

$v_1 = \omega\sqrt{A^2 - x_1^2}$ $\quad$ $v_2 = \omega\sqrt{A^2 - x_2^2}$

$v_1 = 0.837\sqrt{A^2 - 0.07^2}$ $\quad$ $v_2 = 0.837\sqrt{A^2 - 0.05^2}$

$a\omega 0.6 \sqrt{\ }$

$V_{max} = A\omega$

$0.6 A\omega = 0.837 \sqrt{A^2 - 0.07^2}$

$0.64 A^2 = 4.9 \times 10^{-3}$

$A = 0.0875 m$

$V_2 = 0.837 \sqrt{(0.0875)^2 - 0.05^2}$

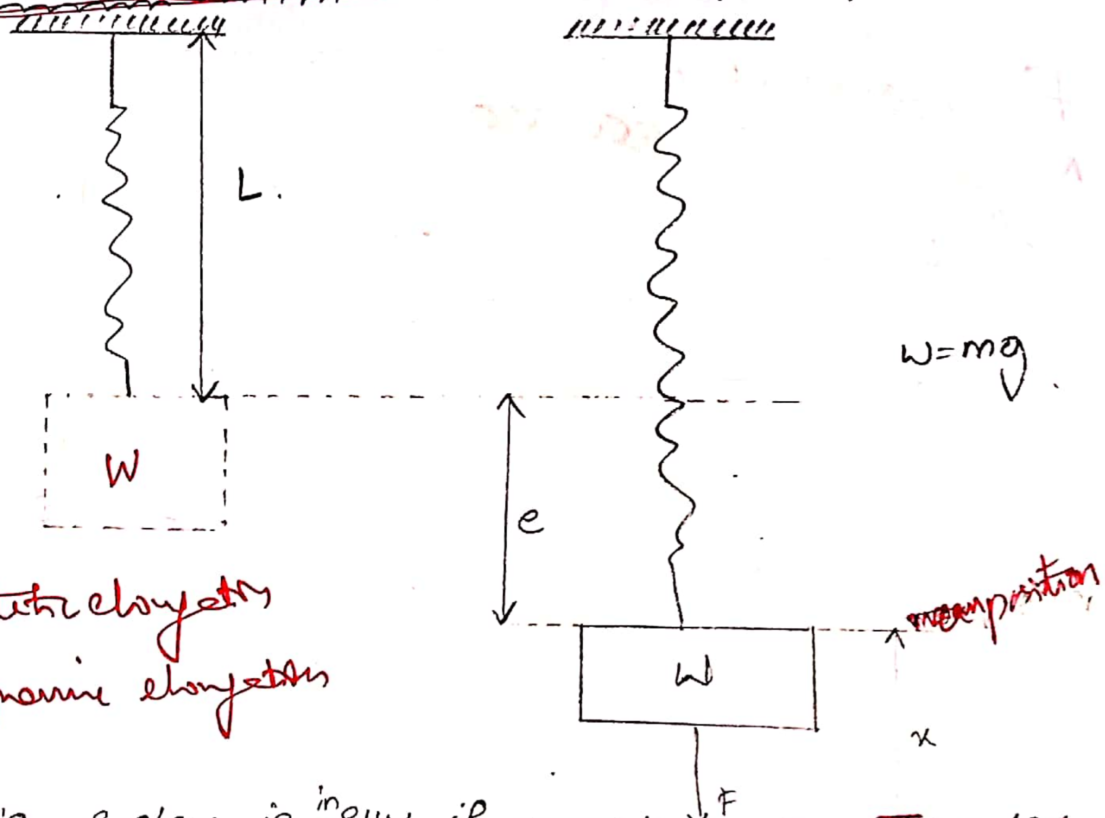
$V_2 = 0.06 m/s$

$a_{max} = \pm a\omega^2$

$= 0.0875 \times 0.837^2$

$= 0.0614 m/s^2$

* Oscillation of a vertical elastic spring :-



e = static elongation

a = dynamic elongation

* This system is in SHM if amplitude should be less than static elongation (or) extension then only it will be S.harmonic motion. (underdamped)

(if a less than e)

Let "L" be the length of Spring if the weight is attached to Spring it will extend up to some distance 'e' - static extension

When wt. ω is attached to spring it undergoes an extension of x

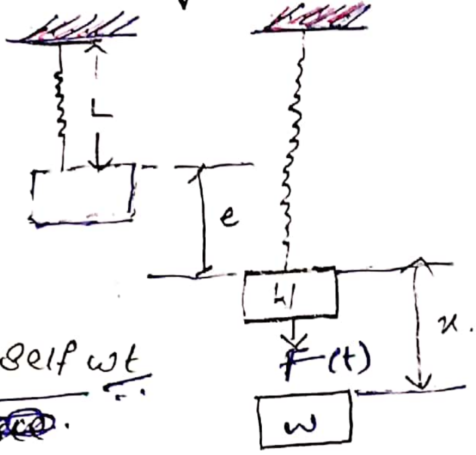
a - acceleration

T - Time period.

$\omega = mg$

→ e - ~~depth~~ dis due to self wt

→ x = displacement due to force



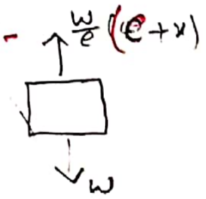
Total extension due to weight $= e + x$

force req. to produce unit deflection by weight 'ω' $= \frac{\omega}{e}$

Force required for $(e+x)$ deflection by wt 'ω' $= \frac{W}{e} \times (e+x)$

The force $\frac{\omega}{e}(e+x)$ is balanced by the tension in the spring which is acting upwards to keep the weight 'ω' in equilibrium.

The net force on the weight 'ω' in upward direction is



$$= \frac{W}{e}(e+x) - W$$

$(\omega = mg)$

$$= \frac{mg}{e}(e+x) - mg$$

$$= \frac{mgx}{e}$$

This is the net force acting upward.

acceleration $a = \frac{\text{net force}}{\text{mass}} \left(= \frac{f}{m}\right)$ $a = \frac{f}{M}$

$= \frac{mgx}{\ell \times m}$

$a = gx/\ell = \frac{g}{\ell} \times x$

$a = \left(\sqrt{\frac{g}{\ell}}\right)^2 x$

$\boxed{a = \omega^2 x}$

$a = \omega^2 x$ for max. acceleration

$\therefore \omega = \sqrt{g/\ell}$

pendulum $\omega = \sqrt{\frac{g}{\ell}}$

$T = \frac{2\pi}{\omega}$

$\boxed{T = 2\pi\sqrt{\frac{e}{g}}}$ (for structures) (for structures)

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{e}{g}}$

Stiffness of Spring 'K' $= \frac{W}{e}$

$e = \frac{W}{K} = \frac{mg}{K}$

$T = 2\pi\sqrt{\frac{mg}{Kg}}$

$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$ → mass / stiffness

Frequency $= f = \frac{1}{T}$

Example: Find the Stiffness of a spring ~~where~~ if 50N is attached to it & its weight makes 4 oscillations per second. Cal. Stiffness

weight = 50N.

$\boxed{T = 2\pi\sqrt{\frac{e}{g}}}$

$0.25 = 2\pi\sqrt{\frac{e}{9.81}}$

$e = 1.55\,cm$

1 sec → 4 oscillation.

0.25 sec = 1 oscillation

$T = \frac{1}{4}$ sec $= 0.25$ sec

Time for one oscillation

$W = mg \simeq$ force in Newtons

$$K = \frac{W}{e} = \frac{50}{1.55} = 32.3\ N/cm$$

Example

The frequency of free vibration of weight 'W' with Spring constant 'K' (stiffness) is 12 cycles per sec when extra weight of 20N is attached to weight 'W' the frequency is reduced to 10 cycles per second find the weight & stiffness

$W \rightarrow f_1 = 12\ cps$

$W + 20 \rightarrow f_2 = 10\ cps$

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$W = mg$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

$$m = \frac{W}{g}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K \times g}{W}} \Rightarrow 12 = \frac{1}{2\pi}\sqrt{\frac{K \times g}{W}} \rightarrow (1)$$

$f = (f_1, f_2)$

$$10 = \frac{1}{2\pi}\sqrt{\frac{K \cdot g}{W + 20}} \rightarrow (2)$$

$$\frac{(1)}{(2)} \quad 1.2 = \sqrt{\frac{W+20}{W}}$$

$$1.44 = \frac{W+20}{W}$$

$$1.44W = W + 20$$

$$0.44W = 20$$

$$W = 45.45\ N$$

from (1) $$12 = \frac{1}{2\pi}\sqrt{\frac{K \times 9.81}{45.45}}$$

$$K = 26.34\ KN/m$$

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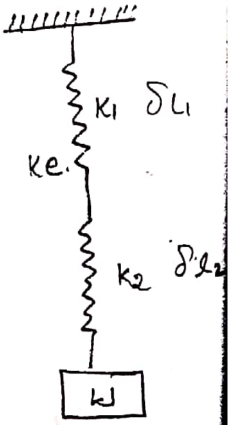
Example

Two springs of stiffness K_1, K_2 are connected in series upper end of compound spring is connected to ceiling & lower end carries weight 'W'. Find the Equivalent Spring Stiffness of system



a) Series:-

If the springs are in series they carry same load 'W'

Series:- forces/weights same, deflections different

$K_1 = \frac{W}{\delta L_1}$ $\quad K = \frac{P}{\delta}$

$K_2 = \frac{W}{\delta L_2}$.

$e = \delta L_1 + \delta L_2$

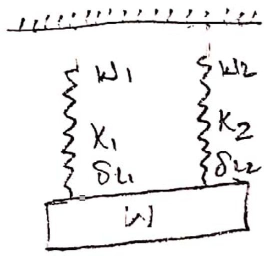
$= \frac{W}{K_1} + \frac{W}{K_2}$.

$\frac{W}{K} = \frac{W}{K_1} + \frac{W}{K_2}$.

$\frac{1}{ke} = \frac{1}{k_1} + \frac{1}{k_2}$.

$$ke = \frac{K_1 K_2}{K_2 + K_1}$$ ✓

b) Parallel:-



Parallel:- Deflections same, load carrying different

$\delta L_1 = \delta L_2 = \delta L = e$.

$K_1 = \frac{W_1}{\delta L_1}$; $K_2 = \frac{W_2}{\delta L_2}$

Total load $W = W_1 + W_2$.

$W = (K_1 + K_2)\,\delta L$

$K_e\,\delta L = (K_1 + K_2)\,\delta L$

$Ke = K_1 + K_2$

Example

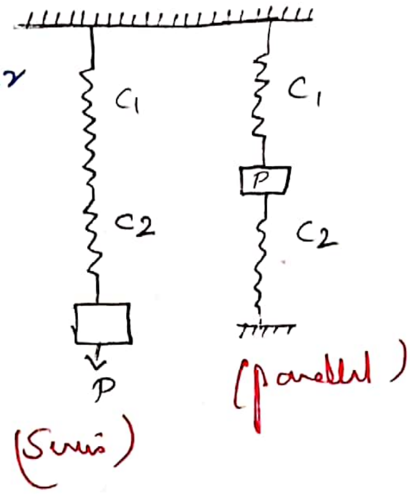
(4.) Find the periods of vibration for the following spring systems

For Series:-

$C_1 \simeq K_1$

$C_2 \simeq K_2$

$e_1, e_2 \simeq x_1, x_2$



$$e_1 = \frac{P}{C_1}$$

$$e_2 = \frac{P}{C_2}$$

$$e = e_1 + e_2$$

$$e = \frac{P}{C_1} + \frac{P}{C_2}$$

$$e = \frac{P(C_1 + C_2)}{C_1 C_2}.$$

$$T = 2\pi \sqrt{\frac{e}{g}}$$

$$= 2\pi \sqrt{\frac{P(C_1 + C_2)}{C_1 C_2 \times g}}.$$

Deflections different forces same

For parallel:-

$$P = P_1 + P_2.$$

$$e_1 = \frac{P_1}{C_1} \quad ; \quad e_2 = \frac{P_2}{C_2}$$

forces are different Deflections are same

WKT $e_1 = e_2$

$$\frac{P_1}{C_1} = \frac{P_2}{C_2}.$$

$$P_1 = P_2 \left(\frac{C_1}{C_2}\right)$$

$$P = P_2 \left(\frac{C_1}{C_2}\right) + P_2.$$

$$P = P_2 \frac{(C_1 + C_2)}{C_2}.$$

$$\frac{P}{(C_1 + C_2)} = \frac{P_2}{C_2}.$$

$$e = \frac{P}{(C_1 + C_2)}$$

$$T = 2\pi \sqrt{\frac{P}{(c_1+c_2)\times g}}$$

Example

A 50kgs block is supported by 2 springs connected in series if the spring constants are 4kN/m & 6 kN/m the block is pulled 40mm down from the position of equilibrium find period of vibrations, maximum velocity & acceleration.

(b) when connected in parallel (exercise)

$F = F_1 + F_2$

$1kg = 9.8N$

Given $m = 50kg$

$w = mg = 50\times 9.81 N$

$= 490.5N$

$k_1 = 4kN/m$ $\quad x = a = 40mm$

$k_2 = 6kN/m$ $\quad = 0.04m$

Series (Forces same) $\to e = e_1 + e_2 + \dots$

Parallel (Def. same) $\to F = F_1 + F_2 + \dots$

$e_1 = \frac{w}{K_1}$; $e_2 = \frac{w}{K_2}$.

$e = e_1 + e_2$

$= \frac{w}{K_1} + \frac{w}{K_2}$

$e = 204.37mm$.

$T = 2\pi\sqrt{\frac{e}{g}}$

$= 2\pi\sqrt{\frac{0.204}{9.81}}$

$= 0.906 sec.$

$T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T} = 6.93 rad/sec$

$V_{max} = x\omega = 0.277\ m/s$

$= 0.04 \times 6.9$

$a_{max} = x\omega^2 = 1.92\ m/s$

25.6.2015 absent

* Free vibrations

- Longitudinal
- Transverse
- Torsional vibration

Longitudinal Vibration :- [axial stresses induced]

$T = \frac{2\pi}{\omega}$

$\delta = x$

$T = 2\pi\sqrt{\delta/g}$

$T = 2\pi\sqrt{\frac{m}{K}}$

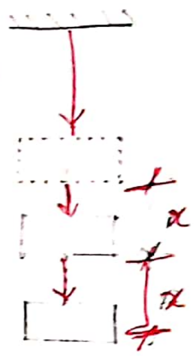
$\therefore \omega = \frac{1}{\sqrt{\delta/g}}$

$\omega = \sqrt{g/\delta}$

$E = \frac{\sigma}{\epsilon}$

$\epsilon = \frac{\delta}{l}$

$\sigma = \frac{P}{A}$



Transverse Vibration :-

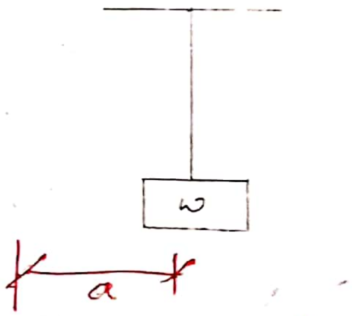
[bending stresses]

$\delta = \frac{wl^3}{3EI}$

$T = 2\pi\sqrt{\delta/g}$

$T = \frac{2\pi}{\omega} = \sqrt{\delta/g}$

$\omega = \sqrt{g/\delta}$



$\delta = \frac{Wl^3}{3EI}$

Torsional Vibration :- (... shear stress)

$\frac{T}{J} = \frac{N\theta}{L}$

$T = 2\pi\sqrt{\frac{I}{q}}$

I = mom. of Inertia

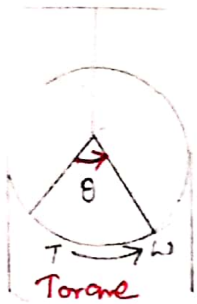
$= mk^2$

k → radius of gyration

Torsional Stiffness

$q = \frac{T}{\theta} = \frac{NJ}{l}$

$q = \frac{Torque}{\theta}$



Problem

A vertical shaft 100mm in dia. 1m in length is hung at one end & carries disc of weight 5000N at other end. The radius of gyration is 450mm. The modulus of rigidity of material is $0.8 \times 10^5 N/mm^2$. find frequency of torsional & transverse vibration if $E = 2\times10^5$ N/mm²

N or $C = 0.8 \times 10^5 N/mm^2$

$d = 100mm$

$L = 1m$

$W = 5000N$

$K = 450mm$

→ Transverse vibration :-

$$T = 2\pi\sqrt{\frac{\delta}{g}}$$

$$\delta = \frac{Wl^3}{3EI} = \frac{5000 \times 1000^3}{3 \times 2 \times 10^5 \times \left(\frac{\pi D^4}{64}\right)}$$

$g = 9.8 m/sec^2$

$$= 2\pi\sqrt{\frac{1.69}{9.8 \times 10^3}}$$

$\delta = 1.69$

$$I = \frac{\pi d^4}{64}$$

$T = 0.082 sec$

$$f = \frac{1}{T} = 12.12 cps$$

$9.8 m/sec^2$

→ Torsional vibrations

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

$$\frac{T}{J} = \frac{N\theta}{L}$$

Polar M.I

$$\frac{T}{\theta} = \frac{NJ}{L}$$

Polar mom...

$$q = \frac{NJ}{L}$$

Torsional Stiffness

$J = I_x + I_y$ Polar M.I

circular

$$I_x = I_y = \frac{\pi D^4}{64}$$

$$\frac{T}{\theta} = \frac{NJ}{L} = q$$

$$J \text{ or } I_J = I_x + I_y = \frac{\pi D^4}{32}$$

$$\frac{q}{I} = \frac{T}{\theta I} = \frac{g}{\delta}$$

$f = ma$

$1N = kg \times m$

$$J = \frac{\pi \times 100^4}{32} = 9817477.042$$

$$\frac{I}{q} = \frac{\delta}{g}$$

$$q = \frac{0.8 \times 10^5 \times 9817477.042}{1000} = 7.854 \times 10^8 \text{ N}$$

Torsional Stiffness

$= 7.854 \times 10^{11}$ Kg

$I = aK^2$

$m = W/g$

$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$

$= \frac{1}{2\pi}\sqrt{\frac{7.854\times10^8}{I}}$

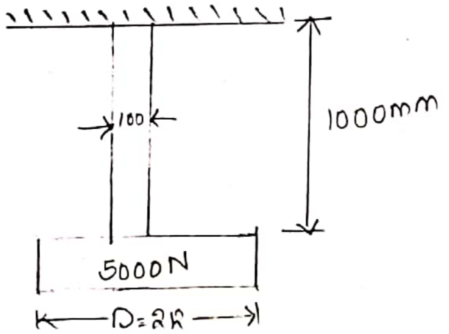
$=$

$I = mK^2$

$= \frac{5000}{9.81} \times 450^2$

$= 103211009.2 \ mm^2$

Example



$E = 2\times10^5 N/mm^2$

$K = 250 mm$

$N = 8.16\times10^5 N/mm^2$

find longitudinal & torsional vibration frequencies

↳ f 25.25 c/s

longitudinal

$f = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$

$\delta = \frac{\sigma}{E} \times l$ $\quad \sigma = \frac{W}{A} = \frac{5000}{\frac{\pi}{4}\times d^4} = 6.366\times10^{-5}$

$= 3.18\times10^{-7}$

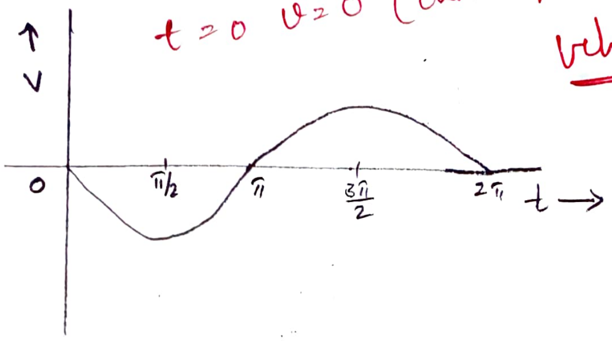
$\epsilon = \frac{\delta}{l} = \frac{\sigma}{E}$

Static analysis is special type of Dynamic analysis.

Dynamic analysis means static loading analysis + Dynamic loading analysis

Free vibration is one form of forced vibration t = 0 v = 0 (extreme position)

velocity time graph



From the plots it can be observed that

→ If a body is disturbed it will never stop vibrating (ideal case)

→ when displacement is maximum (extreme position), velocity is zero & acceleration is maximum in direction opp to displacement

→ when the displacement is zero (mean), velocity is max and acceleration is zero.

→ Undamped-free vibration EOM (pg 92-93)

EQUATIONS OF MOTION OF SDOF SYSTEM

pg 96 [Damped free vibrations] :- [3 element model] (mass, spring, damper)

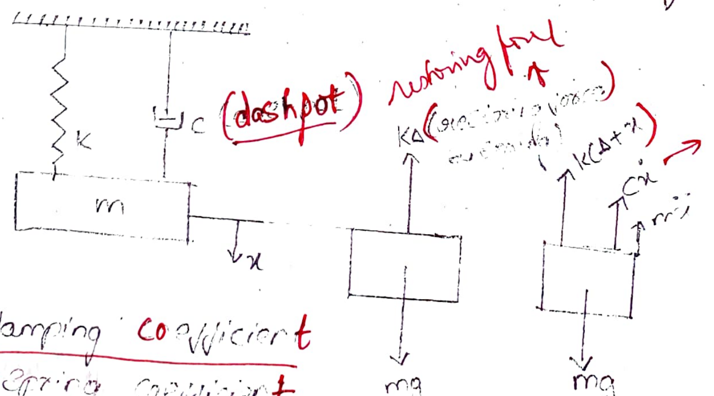
In damped free vibration in addition to "mass" & "spring" there is an element called "damper" is used. The element model is shown in figure.

Inertia force $m\ddot{x}$ or $m\ddot{x}$

Spring Resisting force } kx

Damping force = $k\dot{x}$

↓ induces energy loss of the system



C - damping coefficient

K - spring coefficient

m - mass

Boundary Conditions :-

Equation (a) can be written as

$x = X e^{-\zeta \omega_n t} \cos(\omega_d t + \phi)$ $\zeta \omega_n = \frac{c}{2M}$

where X & ϕ are constants

X is amplitude &

ϕ is phase angle

$e^{ix} = \cos x + i \sin x$
$e^{-ix} = \cos x - i \sin x$

$x(t) = e^{-c/2m t}(A \cos \omega_d t + B \sin \omega_d t)$

$\omega_d = \omega_n \sqrt{1-\zeta^2}$

Introduce boundary conditions :-

x_0 (displacement) & v_0 (velocity)

at time period $(t) = t$ & displacement $(x) = x_0$ the

solution for under damped case is

$x_0 = X e^{-\zeta \omega_n t} \cos(\omega_d t + \phi) \rightarrow$ (d)

after one time period the boundary conditions are

$t = t + t_p$
$x = X_1$ } B.C's

Then the solution is

$x_1 = X e^{-\zeta \omega_n (t + t_p)} \cos(\omega_d (t + t_p) + \phi)$ (e)

divide Eqn(d) / Eqn(e)

$$\frac{x_0}{x_1} = \frac{X e^{-\zeta \omega_n t} \cos(\omega_d t + \phi)}{X e^{-\zeta \omega_n (t + t_p)} \cos(\omega_d (t + t_p) + \phi)}$$

t_p = time period $= \frac{1}{f_p \rightarrow \text{frequency}} = \frac{2\pi}{\omega_d}$

$\omega_d t_p = 2\pi$

$\frac{e^a}{e^b} = e^{a-b}$

$e^{a+b} = e^a \cdot e^b$

$$\frac{x_0}{x_1} = \frac{X e^{-\zeta \omega_n t} \cos(\omega_d t + \phi)}{X e^{-\zeta \omega_n t} \cdot e^{-\zeta \omega_n t_p} \cos(\omega_d (t + t_p) + \phi)}$$

$e^{-\zeta \omega_n t - (-\zeta \omega_n t \cdot e^{\zeta \omega_n t_p})} = e^{\zeta \omega_n t_p}$

11-5-20..
absent

$$\frac{x_0}{x_1} = e^{\zeta \omega_n t_p} \cdot \frac{\cos(\omega_d t + \phi)}{\cos(\omega_d t + 2\pi + \phi)}$$

$\omega_d t_p = 2\pi$

$\cos\theta = \cos(2\pi + \theta)$

$$\frac{x_0}{x_1} = e^{\zeta \omega_n t_p}$$

$a = e^b$
$b = \ln(a)$

$$\ln\left(\frac{x_0}{x_1}\right) = \zeta \omega_n t_p = \zeta \omega_n \times \frac{2\pi}{\omega_d}$$

ln to base e

log to base n

$$= \zeta \omega_n \times \frac{2\pi}{\omega_n \sqrt{1-\zeta^2}}$$

$$\ln\left(\frac{x_0}{x_1}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

rate at which amplitude of free damped vibration decreases

This equation is known as logarithmic decrement

After 'n' number of oscillations time period is

$t = t + n t_p$ $n \rightarrow$ no. of cycles

{Reduction factor pg 104

The logarithmic decrement

$$\ln\left(\frac{x_0}{x_1}\right) = \frac{2\pi n\zeta}{\sqrt{1-\zeta^2}}$$

pg 102

ln or $\log_e$

* For over damped & critically damped system mass will return to its original position slowly and there is no vibration. Vibration is possible only in under damped system because the roots of the solution of underdamped case are complex & has periodic functions.

Harmonic excitation:- (forced vibration)

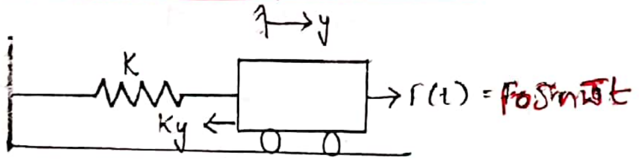
free vibration is special type of forced vibration

*Motion of a structure can be idealised as SDOF systems excited harmonically which means that structures are subjected to either *forces or displacements whose magnitudes are expressed sinusoidaly (sine or cos function)

In case of free vibration only* displacement are expressed sinusoidally because there is no force component in free vibration. Where as in forced vibration both are expressed sinusoidally.

Undamped forced vibration

Equations of motion for undamped harmonic excitation:- (forced vibration)



$\bar{\omega}$ = angular velocity for forced vibration

The differential equation can be written in the form of

force expressed sinusoidally

$$m\ddot{y} + ky = F_0 \sin\bar{\omega}t \rightarrow ①$$

$m\ddot{y} + c\dot{y} + ky = F(t)$, $c = 0$

F_0 - Peak amplitude

$\bar{\omega}$ - Angular velocity for forced vibration

The solution for this equation can be expressed in the form of

$$y(t) = y_c(t) + y_p(t) \rightarrow 1(a)$$

$y_c(t)$ - complimentary solution (real part) - belongs to free vibration

$y_p(t)$ - particular solution or (Imaginary part) - belongs to forced vibration

Complimentary solution - belongs to free vibration

$$y_c(t) = A\cos\omega t + B\sin\omega t - ②$$

ω - Angular velocity free vibration

$\bar{\omega}$ = angular velocity for forced vibration

$$\omega = \sqrt{\frac{k}{m}}$$

Because of force function, $y_p(t)$ exists

$$y_p(t) = Y \cdot \sin \bar{\omega} t \quad \text{—③}$$

→ $\bar{\omega}$ = forced vibration angular velocity

Y - peak amplitude value for particular solution.

Sub ③ in ①

$y = y_p$

$$y_p(t) = Y \sin \bar{\omega} t$$
$$\dot{y}_p(t) = \bar{\omega} \cdot Y \cos \bar{\omega} t$$
$$\ddot{y}_p(t) = -\bar{\omega}^2 Y \sin \bar{\omega} t$$

} eqn ①

$$m[-\bar{\omega}^2 Y \sin \bar{\omega} t] + k \cdot Y \sin \bar{\omega} t = F_0 \sin \bar{\omega} t$$

$$-m\bar{\omega}^2 Y + kY = F_0$$

$$Y = \left(\frac{F_0}{k - m\bar{\omega}^2}\right)$$

$$= \frac{F_0/k}{1 - \left(\frac{m}{k}\right) \cdot \bar{\omega}^2}$$

$\omega = \sqrt{\frac{k}{m}}$

$\frac{k}{m} = \omega^2 \Rightarrow \frac{m}{k} = \frac{1}{\omega^2}$

$$= \frac{F_0/k}{1 - \left(\frac{1}{\omega^2}\right) \bar{\omega}^2}$$

r = frequency ratio $= \left(\frac{\bar{\omega}}{\omega}\right)$

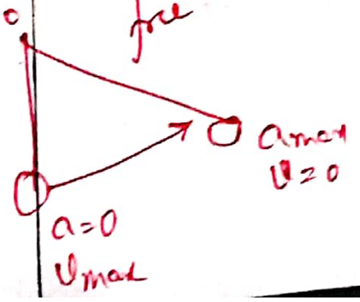
$$Y = \frac{F_0/k}{1 - \frac{\bar{\omega}^2}{\omega^2}} = \frac{F_0/k}{1 - r^2}$$

$$Y = \frac{F_0/k}{1 - r^2} \quad \rightarrow ④$$

where $r = \left(\frac{\bar{\omega}}{\omega}\right)$ } frequency ratio

forced

free



case

free

Sub eqn (2) & (3) in 1(a) — undamped force vibration — $y = y_c + y_p$

$$y(t) = A\cos\omega t + B\sin\omega t + \frac{f_0/k}{1-r^2}\sin\bar{\omega}t \rightarrow (5)$$

(the first term) corresponds to undamped free vibration → This term corresponds to undamped free vibration

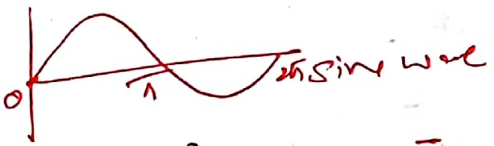
for sinewave at origin: $t=0$, $y_0 = 0$, $v_0 = 0$ } Initial conditions (BC)

{Time measured from mean → sine wave
{ " " from extreme → cosine wave

Sub in eqn (5)

$0 = A + 0 + 0$

$\boxed{A = 0}$



$$\dot{y}(t) = -\omega A\sin\omega t + \omega B\cos\omega t + \frac{f_0/k}{1-r^2}\bar{\omega}\cos\bar{\omega}t \rightarrow 5(a)$$

$\dot{y} \rightarrow v_0$

$$0 = 0 + \omega B + \frac{f_0/k}{1-r^2}\bar{\omega}$$

$$\frac{-f_0/k}{1-r^2} \times \left(\frac{\bar{\omega}}{\omega}\right) = B \quad \left(\frac{\bar{\omega}}{\omega} \rightarrow r\right)$$

$$\boxed{B = -\frac{(r) f_0/k}{1-r^2}}$$

$$\left\{ \begin{array}{l} A = 0 \\ B = -\frac{r (f_0/k)}{1-r^2} \end{array} \right.$$

Sub (A) & (B) in eqn (5)

$$y(t) = -\frac{(r) f_0/k}{1-r^2}\sin\omega t + \frac{f_0/k}{1-r^2}\sin\bar{\omega}t$$

$$y(t) = \frac{f_0/k}{1-r^2}\left[\sin\bar{\omega}t - r\sin\omega t\right] \rightarrow (6)$$

$= \left(\frac{f_0/k}{1-r^2}\times\sin\bar{\omega}t\right)$ {forcing frequency term} {free freq. term} $\frac{f_0/k}{1-r^2} r.\sin\omega t$

6.15

From Equation(6) we can observe that the response is given by superposition of 2 harmonic terms of different frequencies. The resulting motion is not harmonic because in practical tion

(5)

cases damping forces exists because of which

* free frequency term vanishes (or) disappears.

$$y(t) = \frac{f_0/k}{1-r^2}\left(\sin\bar{\omega}t - r\sin\omega t\right)$$

(forced) (free)

Solution for Undamped forced vibration (Undamped Harmonic excitation)

$$y(t) = \frac{f_0/k}{1-r^2}\sin\bar{\omega}t$$

* Free vibration term is also called transient response. which disappears with time

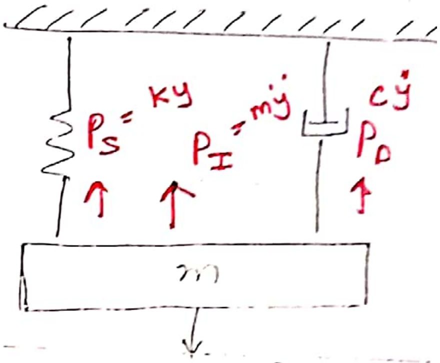
* The forced vibration is also called Steady state response

If 'r' becomes 1 means forcing frequency is equal to natural frequency then resonance occurs due to which amplitude of motion is infinite.

$\bar{\omega}\cos\omega t$

EOM for Damped Harmonic Excitation → forced vibration

⊛ Response of SDOF System to Harmonic Excitation:-



$P(t) = P_0 \sin \omega t$

P_0 = Peak amplitude

According to Newton's law of motion

$$P_I + P_D + P_S = P(t) \quad \text{--- (1)}$$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = P(t) \quad \text{--- (2)}$$

(or)

$$m\ddot{x} + c\dot{x} + kx = P_0 \sin \omega t \quad \text{--- 2(i)} \qquad \textbf{D.E}$$

(P(t) is impressed sinusoidally)

The solution for 2nd order differential equation

$$x = x_c + x_p \quad \text{--- (3)}$$

(C.S) (P.I)

where $x_c \rightarrow$ Complementary solution obtained by taking the RHS of equation (2) $= 0$ → free vibration $P(t) = 0$

$x_p \rightarrow$ particular solution for the given RHS → forced vibration

In earlier sections we have already obtained x_c

To get x_p:-

displacement

$$x_p = X \sin(\omega t - \phi) \quad \text{--- (3)}$$

(→ phase angle)

−ve	lag
+ve	leads

where $\phi \rightarrow$ phase lag or phase angle

dropping subscript 'p'

$$\dot{x} = \omega X \cos(\omega t - \phi)$$

$$\ddot{x} = -\omega^2 X \sin(\omega t - \phi)$$

x_p or $x = X \sin(\omega t - \phi)$

✓ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

✓ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

Substituting (3) in (2a) we get

$$-m\omega^2 X \sin(\omega t-\phi) + c\omega X\cos(\omega t-\phi) + KX\sin(\omega t-\phi) = P_0 \sin\omega t$$

$$[X(K-m\omega^2)][\sin(\omega t-\phi)] + c\omega X\cos(\omega t-\phi) = P_0\sin\omega t$$

sin A−B   cos(A−B)

$$X(K-m\omega^2)\sin(\omega t-\phi) + c\omega X\cos(\omega t-\phi) = P_0\sin\omega t + 0\cdot\cos\omega t$$

Zero

X, φ are unknowns

X, ϕ are unknowns.

$$X(K-m\omega^2)\{\sin\omega t\cos\phi - \cos\omega t\cdot\sin\phi\}$$
$$+ c\omega X\{\cos\omega t\cdot\cos\phi + \sin\omega t\cdot\sin\phi\}$$
$$= P_0\sin\omega t + 0\cdot\cos\omega t \quad \text{--- (3(a))}$$

Equating the coeffts of $\sin\omega t$ & $\cos\omega t$ on both sides, we get

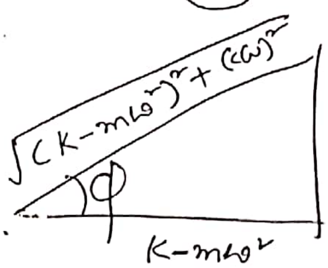
$$X[(K-m\omega^2)\cos\phi + c\omega\cdot\sin\phi] = P_0 \quad \text{--- (4)}$$

$$X[-(K-m\omega^2)\sin\phi + c\omega\cdot\cos\phi] = 0 \quad \text{--- (5)}$$

$$-(K-m\omega^2)\sin\phi + c\omega\cdot\cos\phi = 0$$

$$(K-m\omega^2)\sin\phi = c\omega\cos\phi$$

$$\tan\phi = \frac{c\omega}{K-m\omega^2} \quad \text{--- (6)}$$



Substituting for $\cos\phi$ & $\sin\phi$ in eqn (4)

$$X\left[\frac{(K-m\omega^2)(K-m\omega^2)}{\sqrt{(K-m\omega^2)^2+(C\omega)^2}}+\frac{C\omega\, C\omega}{\sqrt{(K-m\omega^2)^2+(C\omega)^2}}\right]=P_0$$

$$X\left[\frac{(K-m\omega^2)^2+(C\omega)^2}{\sqrt{(K-m\omega^2)^2+(C\omega)^2}}\right]=P_0$$

$$\therefore X=\frac{P_0}{\sqrt{(K-m\omega^2)^2+(C\omega)^2}} \quad \text{——— (7)}$$

Amplitude of steady state vibration.

The above eqn can be put in a non-dimensional form.

normalization

$$X=\frac{P_0/K}{\sqrt{\left(1-\frac{m\omega^2}{K}\right)^2+\left(\frac{C\omega}{K}\right)^2}} \quad \text{——— (8)}$$

from eqn (5)

$$\tan\phi=\frac{C\omega/K}{\left(1-\frac{m\omega^2}{K}\right)} \quad \text{——— (9)}$$

Let $\frac{P_0}{K}=X_0$ ——— (10) → Peak amplitude

$K=\frac{P_0}{X_0}$

Where X_0 is the static displacement.

$K=\frac{P}{X}$

$\frac{P}{K}=X$

$\omega_n = \sqrt{\frac{K}{m}}$

$K = m\omega_n^2$

$$\frac{m\omega^2}{K} = \frac{m\omega^2}{m\omega_n^2} = \left(\frac{\omega}{\omega_n}\right)^2 \quad \text{--- (11)}$$

$$\frac{C\omega}{K} = \frac{2\eta m \omega_n \cdot \omega}{m\omega_n^2} = 2\eta\left(\frac{\omega}{\omega_n}\right) \quad \text{--- (12)}$$

$C = 2\eta m \omega_n$

from eqn (9)

$$\tan\phi = \frac{2\eta \frac{\omega}{\omega_n}}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]} \quad \text{--- (13)}$$

$\eta = \frac{2\omega_n m}{\rho g (14)}$

Which is in a non-dimensional form.

Hence the 'phase angle' ϕ is known

$C = \frac{2 m \xi \omega_n}{\rho g (14)}$

eqn (8)

$$X = \frac{P_0/K}{\sqrt{\left(1-\frac{m\omega^2}{K}\right)^2 + \left(\frac{C\omega}{K}\right)^2}} \quad \text{--- (8)}$$

$\eta = \xi$ Damping factor or Damping Ratio

X_0 — static displacement

$$X = \frac{X_0}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\eta\frac{\omega}{\omega_n}\right)^2}}$$

D - Dynamic magnification factor

$$\therefore \frac{X}{X_0} = D = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\eta\frac{\omega}{\omega_n}\right)^2}} \quad \text{--- (14)}$$

(D is ratio of steady state amplitude to static deflection)

Ily by putting $\frac{X}{X_0} = D$ — 15

Called 'the dynamic ~~magnification~~ amplification factor' (or magnification) and

damping ratio $= \frac{C}{C_c}$

the frequency ratio $\frac{\omega}{\omega_n} = \beta$ — 16

Eqns (13) & (14) Can be represented in a Convenient non-dimensional form from eqn ③

$$\tan\phi = \frac{2\eta\beta}{(1-\beta^2)} \quad \text{--- (17)}$$

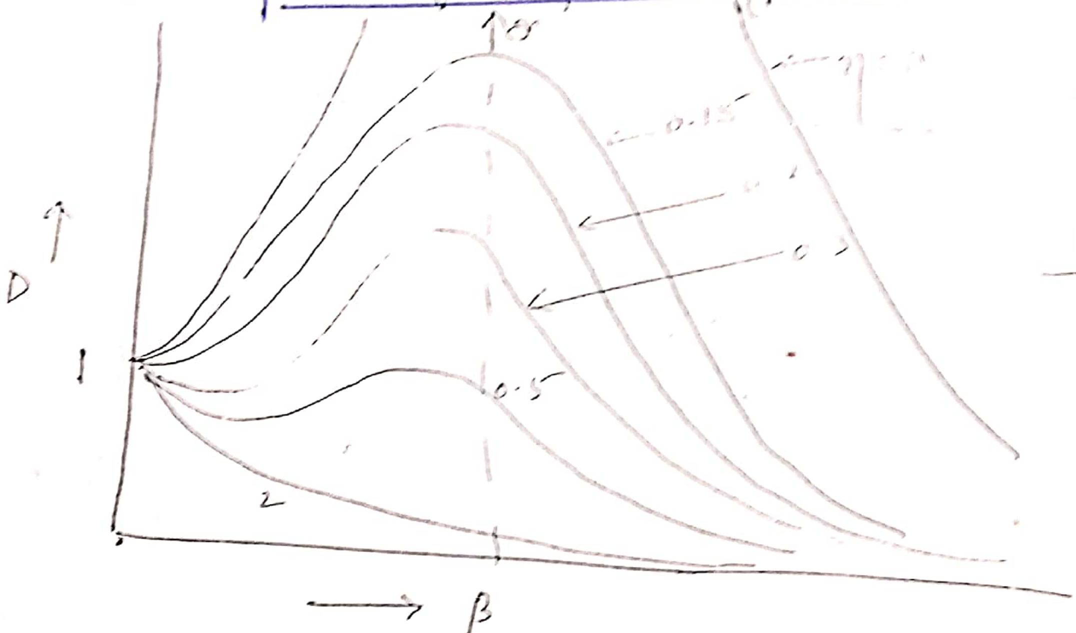
$\beta = 0$ means no damping

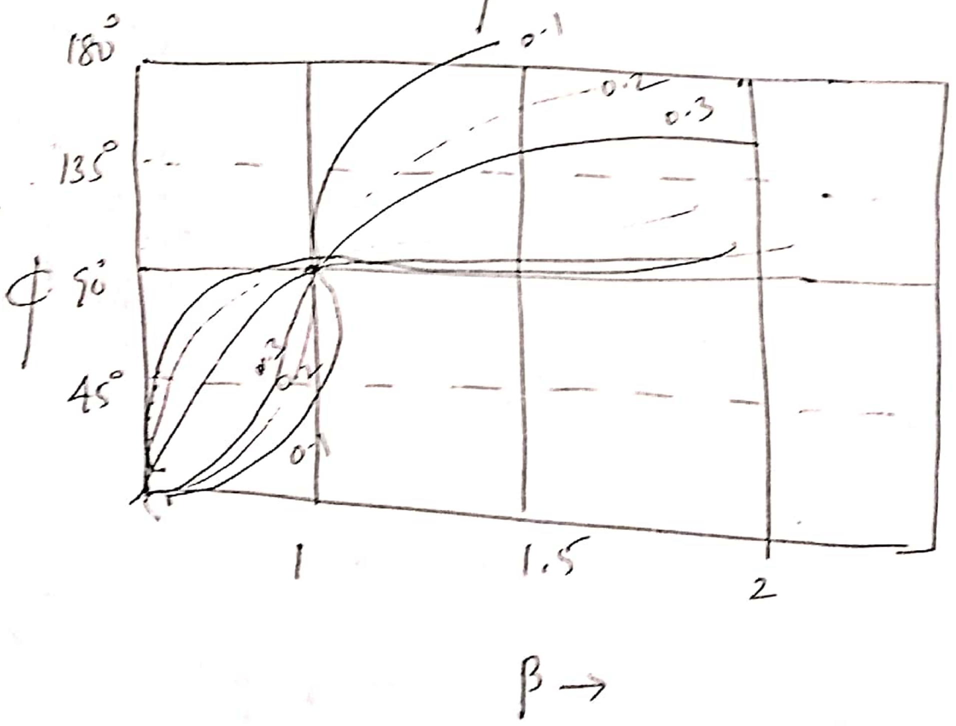
$$D = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\eta\beta)^2}} \quad \text{--- (18)}$$

If $\beta = 0$, $D = 1$

$X_0 \rightarrow$ static
$X \rightarrow$ dynamic

$$\frac{P_0}{K} = X_0$$





CLASS STRUCTURAL DYNAMICS:— 2nd Mid

Resonance case:-

$$D = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\eta\beta)^2}}$$

(• resonance) Applied frequency = natural frequency

at resonance $\omega = \omega_n$ & $\frac{\omega}{\omega_n} = \beta = 1$ — (16)

$$D_{resonance} = \frac{1}{2\eta^2} \quad — (17) \quad \frac{1}{2\eta}$$

also

$$D = (1+\beta^4 - 2\beta^2 + 4\eta^2\beta^2)^{-1/2}$$

Max. Dynamic amplification factor:-

for max D — (18)

$$\frac{dD}{d\beta} = 0 \text{ i.e. } -\frac{1}{2}(1+\beta^4 - 2\beta^2 + 4\eta^2\beta^2)^{-3/2}(4\beta^3 - 4\beta + 8\eta^2\beta) = 0$$

$$\therefore (4\beta^3 - 4\beta + 8\eta^2\beta) = 0$$

$$\beta^2 - 1 + 2\eta^2 = 0 \Rightarrow \beta^2 = 1 - 2\eta^2$$

$$\therefore \beta = \sqrt{1-2\eta^2} \quad — (19)$$

$$\omega_D = \omega_n\sqrt{1-\eta^2}$$

Substituting the value of β in eqn for D

$$D_{max} = \frac{1}{\sqrt{(2\eta^2)^2 + 4\eta^2(1-2\eta^2)}}$$

$$D_{max} = \frac{1}{2\eta\sqrt{(1-\eta^2)}}$$

Max. Dynamic amplification factor.

Hence max. amplification occurs @ frequency very near to resonance frequency. But for all practical purposes both will be the same.

Substituting for $\cos\phi$ & $\sin\phi$ in eqn (4)

$$X\left[\frac{(K-m\omega^2)(K-m\omega^2)}{\sqrt{(K-m\omega^2)^2+(C\omega)^2}}+\frac{C\omega \cdot C\omega}{\sqrt{(K-m\omega^2)^2+(C\omega)^2}}\right]=P_0$$

$$X\left[\frac{(K-m\omega^2)^2+(C\omega)^2}{\sqrt{(K-m\omega^2)^2+(C\omega)^2}}\right]=P_0$$

Amplitude of steady state vibration

$$\therefore X=\frac{P_0}{\sqrt{(K-m\omega^2)^2+(C\omega)^2}} \quad \text{--- (7)}$$

The above eqns can be put in a non-dimensional form.

normalization

$$X=\frac{P_0/K}{\sqrt{\left(1-\frac{m\omega^2}{K}\right)^2+\left(\frac{C\omega}{K}\right)^2}} \quad \text{--- (8)}$$

from eqn (6)

$$\tan\phi=\frac{C\omega/K}{\left(1-\frac{m\omega^2}{K}\right)} \quad \text{--- (9)}$$

Let $\frac{P_0}{K}=X_0$ --- (10) (Peak amplitude)

$K=\frac{P_0}{X_0}$

Where X_0 is the static displacement.

$K=\frac{P}{X}$

$\frac{P}{K}=X$

$\omega_n = \sqrt{\frac{K}{m}}$

$K = m\omega_n^2$

$$\frac{m\omega^2}{K} = \frac{m\omega^2}{m\omega_n^2} = \left(\frac{\omega}{\omega_n}\right)^2 \quad \text{--- (11)}$$

$$\frac{c\omega}{K} = \frac{2\eta m\omega_n \cdot \omega}{m\omega_n^2} = 2\eta\left(\frac{\omega}{\omega_n}\right) \quad \text{--- (12)},$$

$C = 2\eta m\omega_n$

$C_c = \frac{2\omega_n m}{} \quad$ pg (14)

$C = \frac{2m\xi\omega_n}{} \quad$ pg (15)

$\eta = \xi$ Damping factor or Damping Ratio

from eqn (9)

$$\tan\phi = \frac{2\eta\frac{\omega}{\omega_n}}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]} \quad \text{--- (13)}$$

which is in a non-dimensional form.

Hence the 'phase angle' ϕ is known

eqn (8)

$$X = \frac{P_0/K}{\sqrt{\left(1-\frac{m\omega^2}{K}\right)^2+\left(\frac{c\omega}{K}\right)^2}} \quad \text{--- (8)}$$

X_0 – static displacement

$$X = \frac{X_0}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\eta\frac{\omega}{\omega_n}\right)^2}}$$

D – Dynamic magnification factor

$$\therefore \frac{X}{X_0} = D = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\eta\frac{\omega}{\omega_n}\right)^2}} \quad \text{--- (14)}$$

(D is ratio of steady state amplitude to static deflection)

Illy by putting $\frac{X}{X_0} = D$ --- 15

Called 'the dynamic ~~magnification~~ amplification (or magnification) factor' and

damping ratio $= \frac{c}{c_c}$

the frequency ratio $\frac{\omega}{\omega_n} = \beta$ --- 16

Eqns (13) & (14) Can be represented in a Convenient non-dimensional form

from eqn (13)

$$\tan\phi = \frac{2\eta\beta}{(1-\beta^2)} \quad \text{--- (17)}$$

$\beta = 0$ means no damping

$$D = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\eta\beta)^2}} \quad \text{--- (18)}$$

If $\beta = 0$
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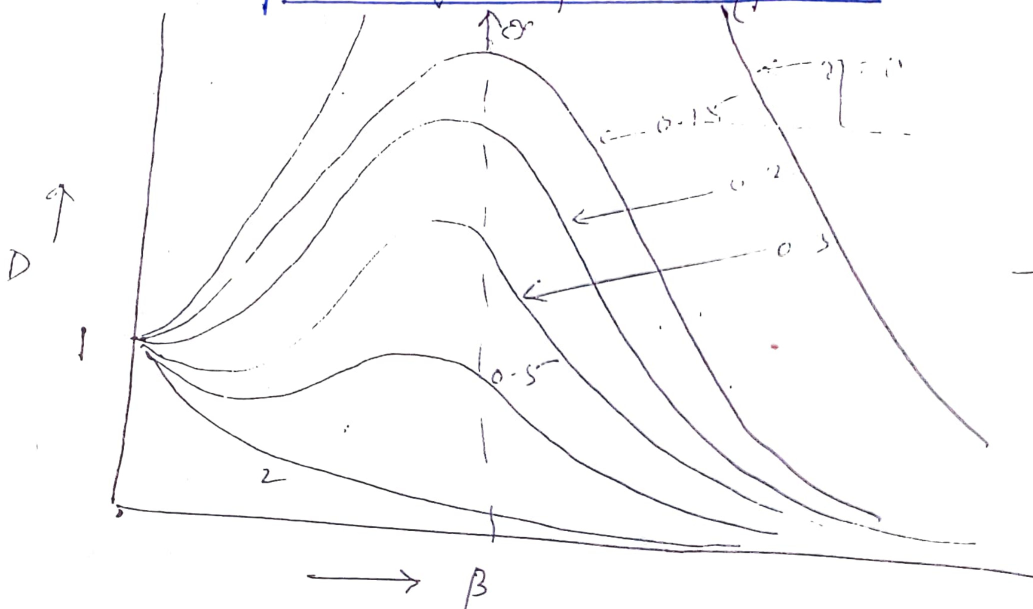
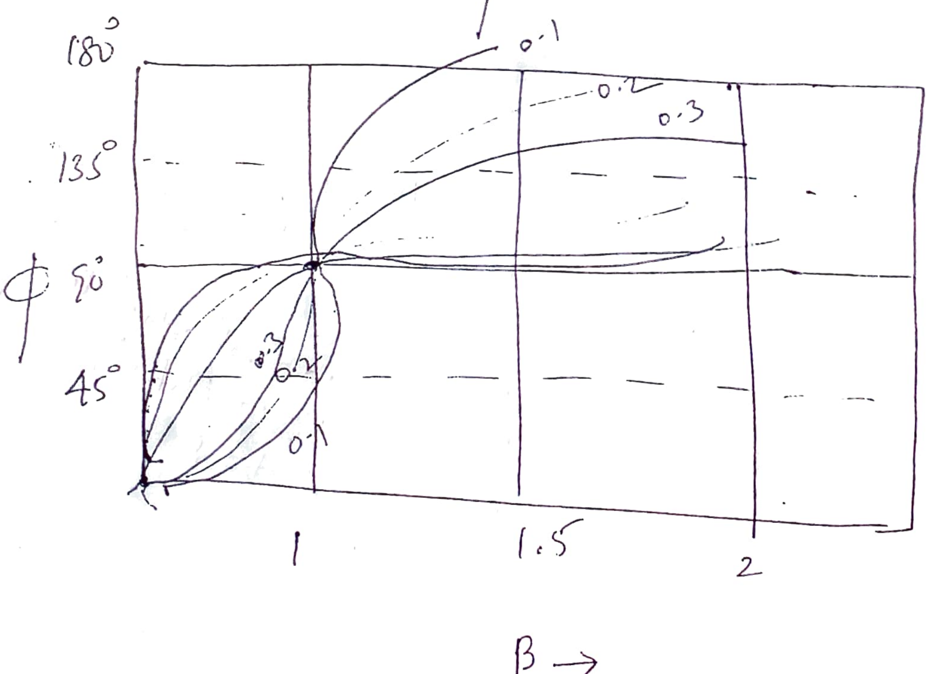




* Response to general dynamic loading

(or)

Equations of motions of SDOF system subjected to general dynamic loading

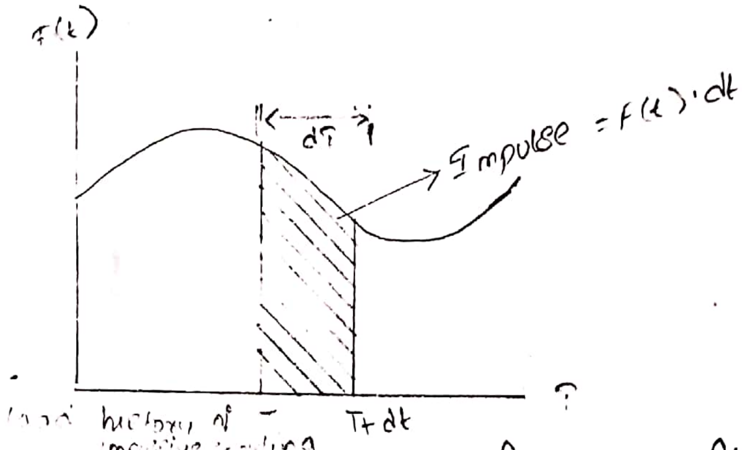
(or)

Response to SDOF system with impulsive loading

(or)

Duhamel's Integral

In real structures loads acting are not harmonic & not periodic. They will be subjected to different load functions (general dynamic loading) to which response can be obtained in terms of an integral. An Impulsive load is a load which is applied during a short duration of time. The impulse is calculated defined as a product of force & time of its duration



Shaded part is impulse of force F(T) at time during interval dT. This impulse acts on a body of mass 'm' producing change in velocity

$$F(t) = m\left(\frac{dv}{dT}\right)$$

$$dv = \frac{F(T)\,dT}{m} \rightarrow \text{Impulsive} \rightarrow (1)$$

where, dv = incremental velocity

This incremental velocity may be considered as initial velocity.

W.K.T Equation of motion for undamped SDOF System

$$m\ddot{y} + ky = 0 \rightarrow (2)$$

The Solution to this differential equation is given by

$$y = y_0 \cos \omega t + \frac{v_0}{\omega} \sin \omega t \rightarrow (3)$$

y_0 & v_0 = Initial displacement & velocity at any time 't'.

Sub Eqn ① in eqn ③ as initial velocity v_0 (i.e., dv = v_0)
& considering initial displacement $y_0 = 0$

$$dy(t) = \frac{F(\tau)\,d\tau}{m\omega} \sin\omega(t+\tau) \rightarrow ④$$

The loading history is represented as sum of short impulses at successive incremental times $d\tau$ which produces its own response at time 't' in the form of Eqn ④.

∴ we conclude that the total displacement at time 't' due to continuous action of force $F(\tau)$ is given by summation or integral of differential displacement $dy(t)$ from time $t=0$ to t

$$\int dy(t) = \int \frac{F(\tau)\,d\tau}{m\omega} \sin\omega(t+\tau). \rightarrow ⑤$$

$$y(t) = \frac{1}{m\omega} \int_0^{t} F(\tau) \sin\omega(t+\tau)\, d\tau \rightarrow ⑥$$

In this Equation the term $\int_0^t F(\tau) \sin\omega(t+\tau)\, d\tau$ is known as Duhamel's Integral

Eqn(5) represent total displacement which include both transient & steady state response.

To include initial condition effect (y_0, v_0 at time $t=0$) then the total displacement of undamped SDOF System subjected to arbitrary loading (impulse (or) general loading) is given by

$$y(t) = y_0 \cos\omega t + \frac{v_0}{\omega} \sin\omega t + \frac{1}{m\omega}\int_0^t F(\tau)\sin\omega(t-\tau)$$

30-6-15 Case (i):- If force is constant force (Point load).

(or) Impact ...

Case (ii):- ...

Case (iii):- ...

Consider the case of constant force of magnitude 'F_0' applied suddenly to undamped system at time $t=0$

For initial conditions the Equation '6' becomes

$$y(t) = \frac{1}{m\omega}\int_0^t F_0 \sin\omega(t-\tau)\, d\tau$$

$$y(t) = \frac{F_0}{m\omega^2}\left|\cos\omega(t-\tau)\right|_0^t$$

$$y(t) = \frac{F_0}{k}(1-\cos\omega t)$$

$$= y_{st}(1-\cos\omega t) \rightarrow (7)$$

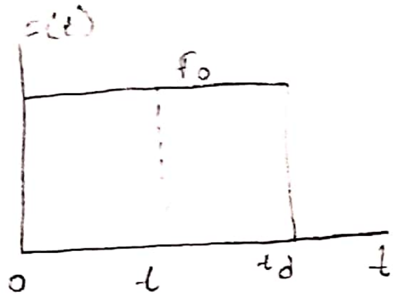
y_{st} - static deflection

From the response of suddenly applied constant load it is observed that the solution is very similar to the solution of free vibration of undamped system. It was found that the max displacement of linear elastic system for a constant force applied suddenly is twice the displacement caused by the sam

Force applied statically.

consider the case where constant force F_0 suddenly applied but only during a limited time t_d then the displacement & velocities @ time t_d is given by

$$y_d = \frac{F_0}{K}(1-\cos\omega t_d)$$



(velocity @ t_d) $v_d = \frac{F_0}{K}\omega \sin\omega t_d$

The response after t_d will be free vibration, so displacement after t_d is given by

$$y = y_0 \cos\omega t + \frac{v_0}{\omega}\sin\omega t \rightarrow (8)$$

Applying initial conditions for displacement & velocity @ time t_d the response is obtained as follows.

replacing $t \rightarrow t-t_d$, $y_0 \rightarrow y_d$, $v_0 \rightarrow v_d$ [conditions @ t_d] we get

$$y(t) = \frac{F_0}{K}(1-\cos\omega t_d)\cos\omega(t-t_d) + \frac{F_0}{K}\sin\omega t_d \cdot \sin\omega(t-t_d)$$

$$y(t) = \frac{F_0}{K}\left\{\cos\omega(t-t_d) - \cos\omega t\right\} \rightarrow (9)$$

DYNAMIC LOAD FACTOR:-

It is defined as displacement at any time 't' [Dynamic displacement / Static displacement] then Equations 7 & 9 can be written in the form of

$$\left.\begin{aligned} DLF &= 1-\cos\omega t \quad t \le t_d \\ DLF &= \cos\omega(t-t_d) - \cos\omega t \quad t \geq t_d \end{aligned}\right\} \quad -(10)$$

Equation 10 can be written as

$$DLF = 1-\cos\frac{2\pi}{T}\cdot t$$

$$DLF = \cos 2\pi\left(\frac{t}{T} - \frac{t_d}{T}\right) - \cos 2\pi\left(\frac{t}{T}\right)$$

CASE-3:-

TRIANGULAR LOAD:-

Consider a system represented by undamped oscillator initially at rest & subjected to force $F(T)$ which has initially value of F_0 & which decreases linearly to zero at time t_d

Response to this type of triangular loading can be calculated using Equation 6 in two integrals

For the 1st range $T \le t_d$.

the force is given by the initial conditions are

$$F(T) = F_0\left(1-\frac{T}{t_d}\right)$$

$$y_0 = 0 \quad ; \quad v_0 = 0$$

Substitute these values in Equation (6) on integration we get

$$y = \frac{F_0}{K}(1-\cos\omega t) + \frac{F_0}{Kt_d}\left(\frac{\sin\omega t}{\omega} - t\right) \rightarrow (11)$$

Equation (11) gives response to Δ^{le} load & Eqn (11) can be expressed in terms of dynamic loading factor as

$$DLF = \frac{y}{y_{st}} = 1-\cos\frac{2\pi t}{T} + \frac{\sin\frac{2\pi t}{T}}{2\pi t_d/T} - \frac{t}{t_d}$$

For second range $T \geq t_d$. from Eqn (11) the displacement & velocity at time t_d is given by

$$y_d = \frac{F_0}{K}\left(\frac{\sin\omega t_d}{\omega t_d} - \cos\omega t\right)$$

$$v_d = \frac{F_0}{K}\left(\omega\sin\omega t_d + \frac{\cos\omega t_d}{t_d} - \frac{1}{t_d}\right)$$

These are the initial conditions at time t_d for the range. Substituting these value in Eqn (8) & replacing t by $t-t_d$ & y_0 & v_0 by y_d & v_d ; $F(T)=0$

The response is obtained as

$$y = \frac{F_0}{K\omega t_d}\left(\sin\omega t + \cos(t-t_d)\right) - \frac{F_0}{K}\cos\omega t \rightarrow (13)$$

Equation 13 is expressed as

$$DLF = \frac{y}{y_{st}} = \frac{1}{\omega t_d}\left(\sin \omega t - \sin \omega (t - t_d)\right) - \cos \omega t \rightarrow (C$$

RESPONSE TO PERIODIC LOADING:

In order to find out the response of a structure to period loading it is necessary to express period load in terms of fourier series. The response of each term of series is expressed in terms of harmonic loading & by the principle of super position the total response is obtained by summation of response to the individual load terms.

Any periodic fun is expressed in terms of fourier series as

$$p(t) = a_0 + \sum_{r=1}^{\infty} a_r \cos \frac{2\pi r}{T_p} + \sum_{r=1}^{\infty} b_r \sin \frac{2\pi t}{r}$$

where T_p =

& co-eff

$$(2)\quad \begin{cases} a_0 = \frac{1}{T_p}\int_0^{T_p} p(t)\,dt \\ a_r = \frac{2}{T_p}\int_0^{T_p} p(t)\cdot \cos \frac{2\pi r}{T_p}\cdot t\cdot dt \\ b_r = \frac{2}{T_p}\int_0^{T_p} p(t)\sin\left(\frac{2\pi r}{T_p}\right) t\, dt \end{cases}$$

The arbitrary periodic loading is expressed in the form of fourier series by equation (1) which consists of 2 terms (1) & (2)

Term (1) represents constant load & term (2) represents series of harmonic loads of frequency ω_r & amplitudes a_r & b_r.

The steady state response produced in undamped SDOF system subjected to harmonic loading (sine term)

$$y = \left(\frac{F_0}{K}\right)\left(\frac{1}{1-r^2}\right)(\sin \bar{\omega} t - r \sin \omega t)$$

steady state — transient state

$$= y_r(t) = \frac{b_r}{k} \frac{1}{r^2} \sin r \bar{\omega} t \rightarrow (3a)$$

$$\therefore r_r = \frac{\omega_r}{\omega} = -\frac{rT}{T_p} = \frac{r\bar{\omega}}{\omega}$$

where $\omega_1 = \frac{2\pi}{T_p}$

Similarly the steady state response produced by undamped SDOF system subjected to harmonic loading in terms of cosine term is given by

$$y_r(t) = \frac{a_r}{K} \frac{1}{1-r_r^2} \cos r \bar{\omega}_1 t \rightarrow 3(b)$$

The steady state response due to constant load is given by static deflection

$$y_0 = \frac{a_0}{k} \rightarrow (4)$$

The total periodic response of undamped SDOF system is given by (3a) (3b) & (4) as follows

$$y(t) = \frac{1}{k}\left[a_0 + \sum_{r=1}^{\infty} \frac{1}{1-r_r^2}\left(a_r \cos r\bar{\omega} t + b_r \sin r\bar{\omega} t\right)\right]$$

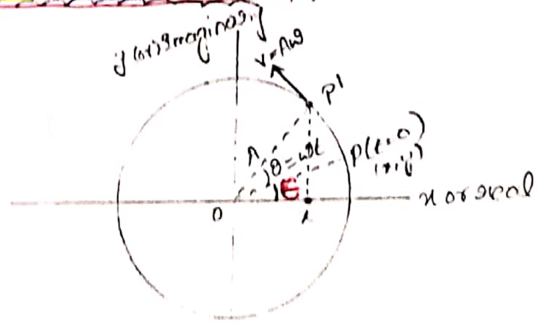
where a_0 a_r b_r can be computed from Eqn (2)

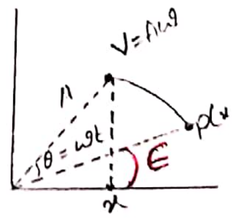
The total periodic response of damped

$$y(t) = \frac{1}{k}\left[a_0 + \sum_{r=1}^{\infty} \frac{1}{(1-r_r^2) + (2\xi r_r)^2}\left\{\left(a_r 2\xi r_r - b_r(1-r_r^2)\right)\sin r\bar{\omega}_1 t + \left(a_r(1-r_r^2) - b_r 2\xi r_r\right)\cos r\bar{\omega}_1 t\right\}\right]$$

6.7.15
absent

Vectorial representation of SHM:-





SHM which is sinusoidal in nature is expressed as $A \sin 2\pi f t + B \cos 2\pi f t$

where A, B - constants

f - frequency per sec or Hz

W.K.T
$\omega = 2\pi/T$
$\omega = 2\pi f$
$f = 1/T$

$\omega = \frac{2\pi}{T}$
$\omega = 2\pi f$
$f = 1/T$

Scanned with CamScanner

This sinusoidal equation can also be written in this form $C \cdot \cos(\omega t + \epsilon)$

where $C = \sqrt{A^2 + B^2}$ = amplitude

$\epsilon = \tan^{-1}\left(\frac{A}{B}\right)$ = phase constant

Quantity $(\omega t + \epsilon)$ = phase

The variation of this type is called harmonic (or) more precisely simple harmonic

Note:- $m\ddot{x} + c\dot{x} + Kx = F(t)/\omega$

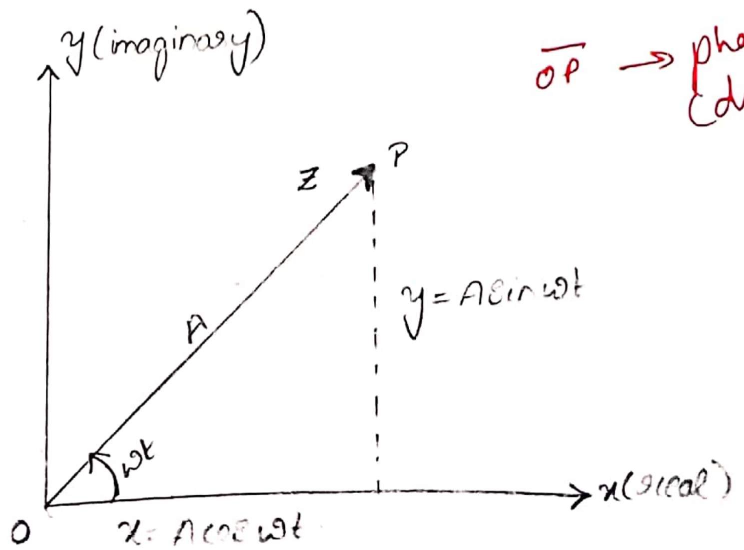
- $m\ddot{x}$ → force generated by force
- $c\dot{x}$ → force by damper
- Kx → force due to spring constant

$F_F + F_D + F_S = F/\omega$

(or) F_I. Inertial force

Figure shows graphical representation of SHM in which the vector of length A (OP) rotates from point P (initial position where $t=0$) to P'.

The vector $P(x, y)$ can be expressed as complex numbers as follows [representation of SHM as complex number]



$\overline{OP}$ → phasor (displacement phasor)

Vector $\vec{OP}$ is represented as complex number Z

where $z = a + ib$ (real) (imaginary)

$\bar{Z} = A(\cos\omega t + i\sin\omega t)$
(vector) ↓ real ↓ imaginary

In SHM we are interested only in real part of 'Z'

using Eulers theorem

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{i\omega t} = \cos\omega t + i\sin\omega t$$

$$A.e^{i\omega t} = A(\cos\omega t + i\sin\omega t)$$

above equation i.e, for ... , but we have many no. of waves so add phase constant

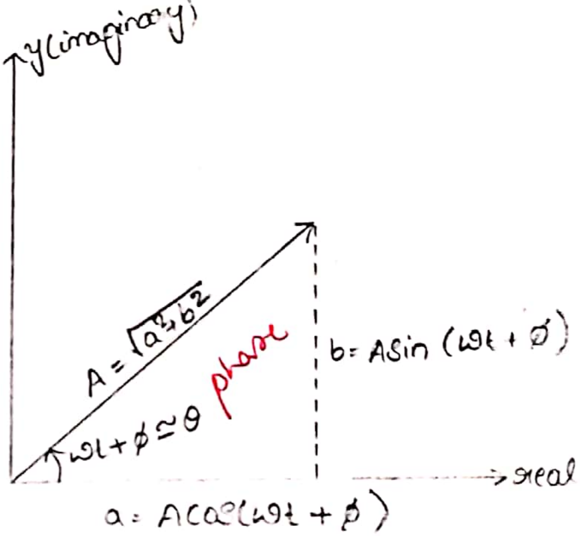
$$A.e^{(i\omega t + \epsilon)} = A(\cos(\omega t + \epsilon) + i\sin(\omega t + \epsilon))$$

Now SHM is expressed vectorially as

$A.e^{(i\omega t + \epsilon)}$

A- Amplitude

$i\omega t + \epsilon$ → angle with which it moves



$a = A\cos(\omega t + \phi)$

$\theta = \omega t + \phi = \tan^{-1}\frac{b}{a}$

$A = \sqrt{a^2 + b^2}$

$Ae^{i\omega}$ - phasor (phase - amplitude)

$x = A\cos(\omega t + \epsilon)$

$v = \dot{x} = -\omega A\sin(\omega t + \epsilon)$

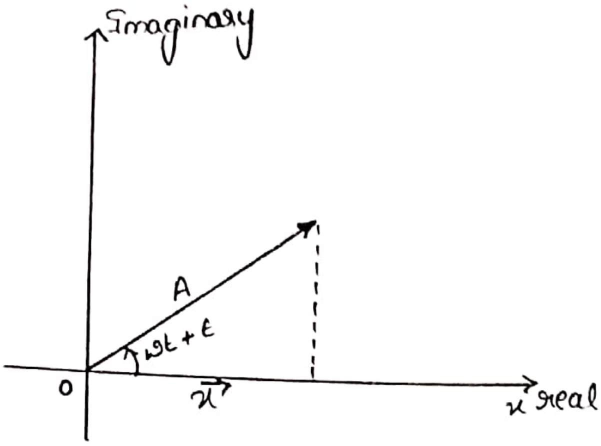
$a = \ddot{x} = -\omega^2 A\cos(\omega t + \epsilon)$

(vector - magnitude)

displacement vector $\vec{x} \Rightarrow z = Ae^{i(\omega t + \epsilon)}$

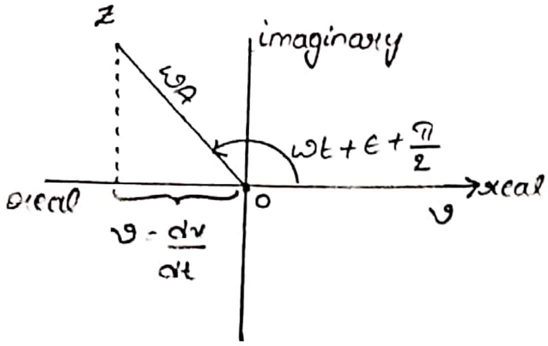
velocity vector $\vec{v} \Rightarrow \dot{z} = i\omega Ae^{i(\omega t + \epsilon)}$

acceleration vector $\vec{a} \Rightarrow \ddot{z} = i^2\omega^2 Ae^{i(\omega t + \epsilon)}$



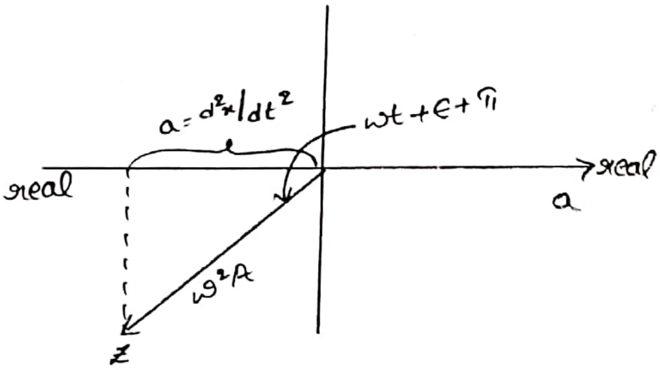
Displacement vector ($\vec{x}$)

$z = Ae^{i(\omega t + \epsilon)}$



$\vec{v} = \dot{z} = i\omega Ae^{i(\omega t + \epsilon)}$

velocity vector



$\vec{a} = \ddot{z} = i^2\omega^2 Ae^{i(\omega t + \epsilon)}$

acceleration vector

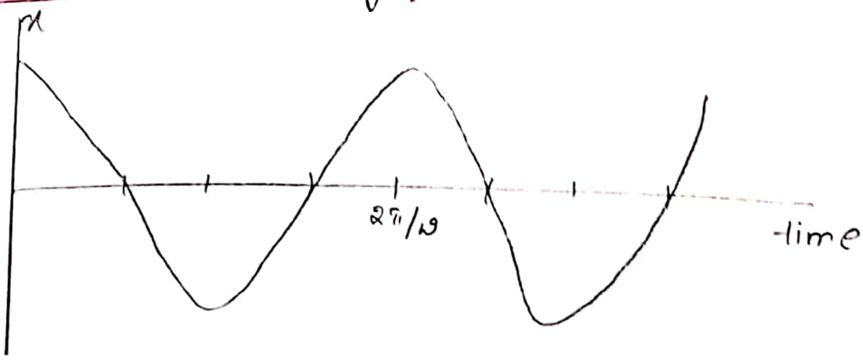
The constant $Ae^{i\omega}$ is known as Phasor

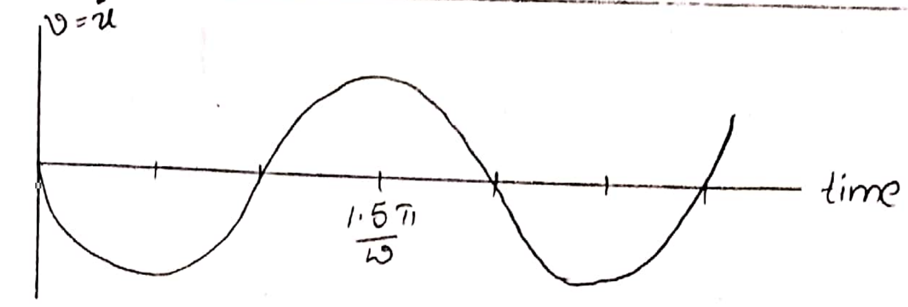
Vector – which has magnitude & direction

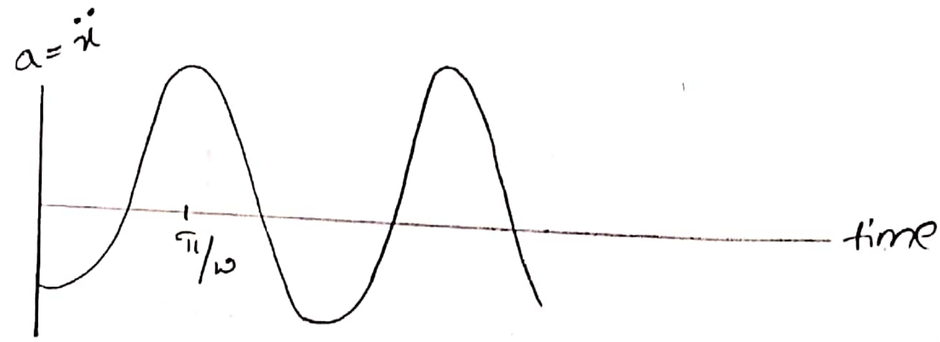
Phasor – Amplitude & phase angle

$= Ae^{i\omega t}$

Phase constant / phase angle :-







When we compare these 3 graphs each graph reaches its max +ve value at different times

acceleration reaches at $t = \pi/\omega$ } for example fig

velocity @ $t = \frac{1.5\pi}{\omega}$

displacement @ $t = 2\pi/\omega$

from this we notice that acceleration "leads" the velocity by a phase of $\pi/2$ (or) one quarter of cycle.

Similarly velocity leads displacement by phase of $\pi/2$ (or) displacement lags velocity by $\pi/2$ & velocity lags acceleration by $\pi/2$

when any 2 quantities varying sinusoidaly reaches max. value simultaneously

* we say they are "in phase".

When the phase difference is 180° (or) π we say they are in "out of phase" or antiphase".

Duhamels Integral:-

$$y(t) = \frac{1}{m\omega}\int_0^t F(t)\sin\omega(t-\tau)\,d\tau$$

Convolution Integral:-

from duhamels Integral we can write

$$y(t) = \int_0^t F(t)\, h(t-\tau)\,d\tau$$

where $h(t-\tau) = \frac{1}{m\omega}\sin\omega(t-\tau)$.

Dynamic magnification factor:-

The influence of dynamic character loading is expressed by ratio of dynamic response to the displacement that would be produced by static application of same load. This ratio is called D.L.F (or) response ratio.

$$D.L.F = \frac{\text{(displacement at any time t due to dynamic loading)}}{\text{Static displacement}}$$

$$= \frac{y_t}{y_{st}} = 1-\cos\omega t$$

D.L.F for undamped system harmonic loading

$$DLF = \frac{1}{1-r^2}\left(\sin\bar{\omega}t - \sin\omega t\right)$$

r - freq. ratio

$$r = \frac{\omega_d}{\omega_n} \text{ (or) } \frac{\bar{\omega}}{\omega}$$

The first term $\sin\bar{\omega}t/1-r^2$ has same frequency as the load & is called force vibration (or) steady state vibration.

Second term has same frequency as natural frequency of system is called free vibration (or) transient vibration.

$$\frac{y_t}{y_{st}} = D.L.F$$

$$y = D.L.F \times y_{st}$$

f - force req

f_0 - force at $t=0$

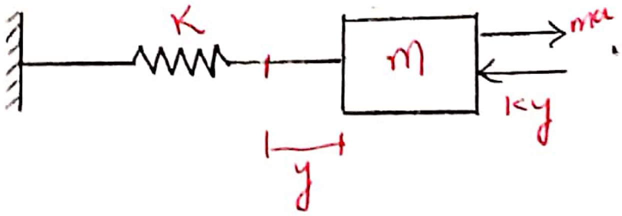
$$f = DLF \times P_0 \sin \omega t$$

D-Alemberts Principle:- It states that a system may be set in a state of dynamic equilibrium by adding to external forces a fictious force (virtual force) which is commonly known as inertia force

$$ma = ky$$

$$m\ddot{x} = -ky$$

$$m\ddot{x} + ky = 0$$

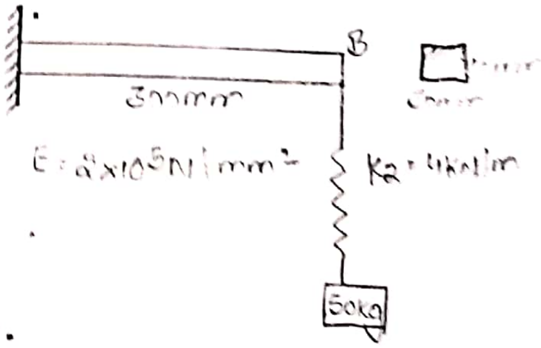


* D'alemberts Principle can be applicable for simple system

* For complex system along with DAlemberts Principle. principles of virtual work is used for analysis of dynamic system.

Determine natural frequency of system shown in figure which consists of 50kg is attached to horizontal cantilever beam through spring whose constant $K = 4\,kN/m$. The cantilever beam is having c/s of 30mm x 10mm & length of beam = 300mm

$E = 2 \times 10^5\, N/mm^2$



deflection of cantilever subjected to load at force end.

$$\delta_B = \frac{Wl^3}{3EI}$$

Stiffness $K = \frac{P}{\delta}$

$$I = \frac{Bd^3}{12} = \frac{30 \times 10^3}{12}$$

$$= 2500$$

$$K_1 = \frac{W}{\delta}$$

$K_1 = \frac{3EI}{l^3}$ K of beam

$= \frac{3 \times 2 \times 10^5 \times 2500}{300}$

$K_1 = 5 KN/m$

$f = \frac{1}{T}$

$T = 2\pi \sqrt{\frac{M}{K_e}}$ M → 5kg

$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$ (Series)

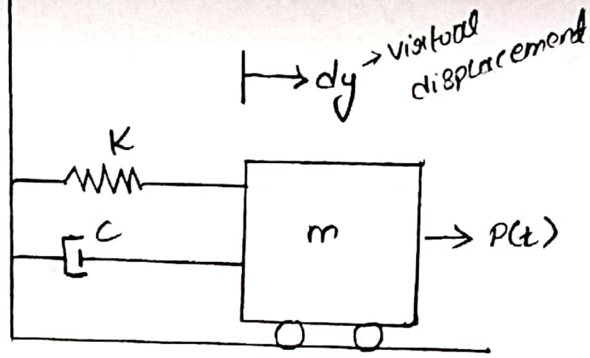
$K_e = \frac{K_1 K_2}{K_1 + K_2} = \frac{20}{9} = 2.22 KN/m$

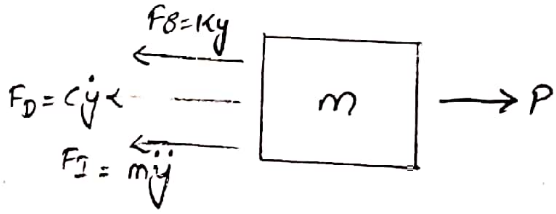
7.15 PRINCIPLE OF VIRTUAL WORK

The response to the dynamic loads on a structure is modelled using spring mass system. If the structural system consists of multiple interconnected rigid bodies are having distributed mass & elasticity the system can be modelled as SDOF system using the principle of virtual work. Since dynamic equilibrium is difficult to apply to large & complex structural systems & alternative approach is adopted to formulate equations of motion using principle of virtual work.

- The principle of virtual work states that for a deformable system in equilibrium under a set of forces the sum of total external virtual work & internal virtual work is zero, where virtual work is work done by the virtual displacement

$\delta W = 0$





$\delta W = 0$

$\delta W = \delta W_I + \delta W_s + \delta W_d + \delta W_p = 0$.

$$-F_I\,dy - F_s\,dy - F_d\,dy + P\cdot dy = 0$$

$$P = F_I + F_s + F_d$$

$$P(t) = F_I + F_s + F_d$$

$$m\ddot{y} + c\dot{y} + ky = P$$

This is the differential Equation for the motion of damped oscillating system.

Amplitude of Motion :-

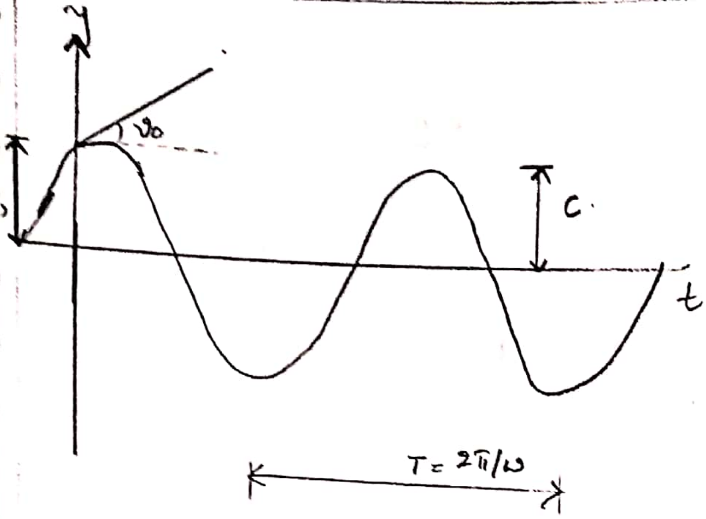
$$y = y_0 \cos\omega t + \frac{v_0}{\omega}\sin\omega t \rightarrow (1)$$

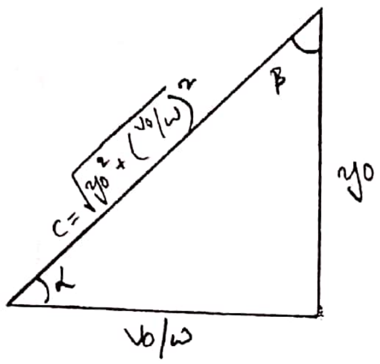
$$\left.\begin{aligned} y &= c\sin(\omega t+\alpha) \\ &(\text{or}) \\ y &= c\cos(\omega t-\beta) \end{aligned}\right\} \rightarrow (2)$$

$$c = \sqrt{y_0^2 + \left(\frac{v_0}{\omega}\right)^2}$$

Divide & multiply Equation (1) with constant

$$y = c\left(\frac{y_0}{c}\cos\omega t + \frac{(v_0/\omega)}{c}\sin\omega t\right)$$





from triangle

$\sin\alpha = y_0/c \qquad \cos\alpha = \dfrac{v_0/\omega}{c}$

Sub in above Equation

$$y = c(\sin\alpha \cos\omega t + \cos\alpha \cdot \sin\omega t)$$

$$y = c\sin(\alpha + \omega t)$$

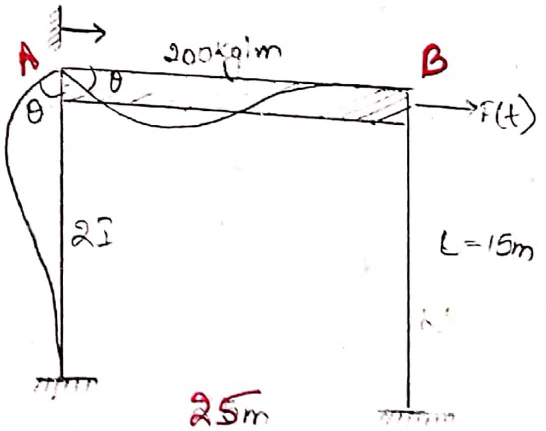
$\sin(A+B) = \sin A\cos B + \cos A \sin B$

Example:

Consider frame given in figure. A horizontal dynamic force subjected at upper level. find natural frequency of frame (Assume that masses of columns & walls are negligable & joints are rigid at top of columns.

$W = 200 \times 25$
$= 5000$ kg

$I = \dfrac{bd^3}{12}$
$= \dfrac{0.25 \times 0.4^3}{12}$
$= 1.33 \times 10^{-3}$



b = 250mm
d = 400mm

Stiffne

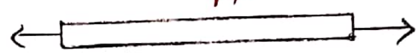
E

40

N 3) * Stiffness of various members :-

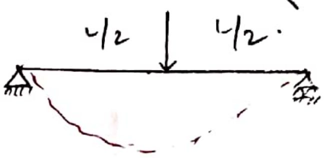
$K = \frac{AE}{L}$

A, E

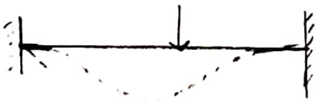


$K = \frac{AE}{L}$

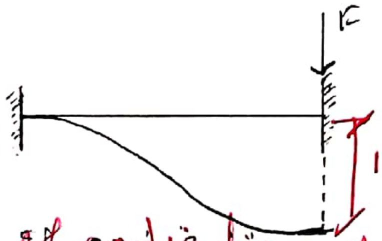
$K = \frac{3EI}{l^3}$



$K = \frac{48EI}{l^3}$

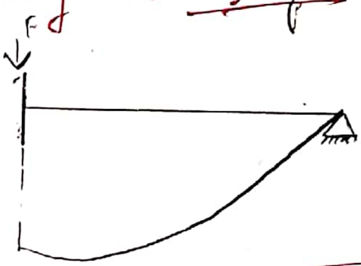


$K = \frac{192EI}{l^3}$



$K = \frac{12EI}{l^3}$

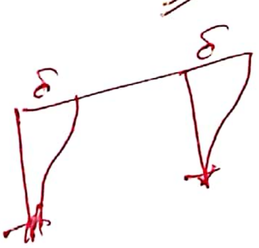
If end is hinge & force act at rigid joint



$K = 3EI/l^3$

$E = 2\times10^5 MPa = 2\times10^5\times10^{-6} N/mm^2$

column $K = 12EI/l^3 = \frac{12\times2\times10^5\times10^{-6}\times1.33\times10^{-3}}{25^3}$



$= 2.047488\times10^{-7}$

$T = 2\pi\sqrt{\frac{m}{k}}$

$w = mg$

$m = \frac{w}{g}$

$= 2\pi\sqrt{\frac{w}{k\cdot g}}$

$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{gk}{w}}$

$= \frac{1}{2\pi}\sqrt{\frac{2.047488\times10^{-7}\times9.81}{5000}} = 3.189\times10^{-6}$

Problems

A vibratory system consists of wt. 10 kg & spring with stiffness 1 kN/m is viscously damped. System so that the ratio two consecutive amplitudes is 1.00 to 0.85. Find the 1) natural frequency of the undamped system (2) the logarithmic decrement

3) damping ratio
4) damping coefficient
5) damped natural frequency

1) natural frequency of the undamped system

$\rightarrow \omega = \sqrt{\frac{K}{m}}$

$$m = \frac{w}{g} = \frac{10}{9.81} = 1.019$$

$$\omega = \sqrt{\frac{1}{1.019}} \quad \begin{matrix} \rightarrow K \\ \rightarrow m \end{matrix}$$

$= 0.99$ rad/sec

2) logarithmic decrement

$$\delta = \ln \frac{y_1}{y_2} = \ln \frac{1.00}{0.85}$$

$$= 0.1625$$

3) damping ratio

$(2\pi\xi)^2 = \delta^2(1-\xi^2)$

$$\xi = \frac{c}{c_{cr}}$$

$$\frac{2\pi\xi}{\sqrt{1-\xi^2}} = \ln \frac{x_0}{x_1} \quad \text{(or)} \quad \delta = 2\pi\xi$$

$$\xi = \frac{\delta}{2\pi}$$

$$= \frac{0.162}{2\pi} = 0.0257$$

4) damping coefficient

$c = \xi \times C_{cr}$

$C_{cr} = 2\sqrt{KM} = 2\, m \cdot \omega_n$

$= 2 \times \sqrt{1.019 \times 1}$

$= 2.0189$

$C = 0.050$

5) damped natural frequency. $\omega_D = \omega_n\sqrt{1-\xi^2}$

$\omega_D = 0.99\sqrt{1-(0.0257)^2}$

$= 0.989$

$\omega_n = \sqrt{\frac{K}{m}}$

1kg = 2.2 pound

Problem

2. A platform of weight 'w' 2000 kg is supported by 4 equal columns fixed to a foundation experimentally it has been determined that the static force of F = 500 kg is applied horizontally to the platform which produces the displacement $\Delta = 2.54mm$. It is estimated that the damping of the structure is 5% of critical damping. Determine the following

1) undamped natural frequency
2) absolute damping coefficient
3) logarithmic decrement
4) Number of cycles & the time required for the amplitude of motion to be reduced from initial value of 2.54 mm to 0.254 mm

1) $k = \frac{F}{\delta}$

$= \frac{5000}{2.54 \times 10^{-3}}$ mm to m

$= 1.96 \times 10^{6}$

$W = 2000 kg$

$m = \frac{2000}{9.81}$

$\omega = \sqrt{\frac{K}{m}}$

$= \sqrt{\frac{1.96 \times 10^{6}}{(2000/9.81)}}$

$= 98.04$

2) damping coefficient

$\zeta = \frac{c}{C_{cr}}$

$C_{cr} = 2\sqrt{Km} = 2m\omega_n$

$= 2 \times \frac{2000}{9.81} \times 98.04$

$= 39.975 \times 10^{3}$

3) logarithmic decrement

$\ln\left(\frac{x_0}{x_1}\right) = 0.314$

$\frac{x_0}{x_1} = e^{0.314}$

$= 1.36$

$\frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \ln\left(\frac{x_0}{x_1}\right)$

$\zeta = 5\% = 0.05$

3.

The ratio of 1st amplitude

$$\ln = \frac{x_0}{x_k} = \ln\left(\frac{x_0}{x_1} \times \frac{x_1}{x_2} \cdots\cdots \times \frac{x_{k-1}}{x_k}\right)$$

Ans:

$$\ln \frac{x_0}{x_k} = \delta_1 \times \delta_2 \cdots\cdots \delta_k = n\delta$$

$$\ln \frac{x_0}{x_k} = n \cdot \delta$$

$$\therefore \ln \frac{2.64}{0.254} = n \times 1.37$$

$n = 1.68 \simeq 2$ cycles. 7.32 ≃ 8 cycles

$$\omega_D = \omega\sqrt{1-\xi^2} \Rightarrow 98.04\sqrt{1-0.05^2} = 97.91$$

$$T_D = \frac{2\pi}{97.91} = 0.0641$$

Total time required = 2 × 0.0641
= 0.1288 sec.

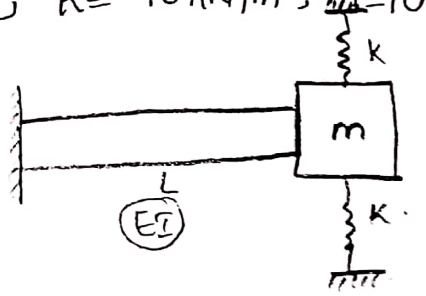
Example

3. Calculate natural frequencies and ~~times~~ natural period for following structures

$L = 3.6\,m$; $E = 22000\,MPa$; $K = 40\,KN/m$; $m = 10\,KN$

$I = 1.2\times10^{4}\,m^4$

$$\omega_n = \sqrt{\frac{k_e}{m}}$$ $k_e \to$ Equivalent stiffness



$$m = \frac{10000}{9.81} = 1019.36\ \frac{Ns^2}{m}$$

$$E \times I = 2.64\times10^{6}\ Nm^2$$

$$k_e = k_b + K + K.$$

$$= \frac{3EI}{L^3} + K + K.$$

$$= \frac{3\times2.64\times10^{6}}{3.6^{3}} + (40\times10^{3}) + (40\times10^{3})$$

$$= 249.753\times10^{3}\ N/m.$$

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$$\omega_n = \sqrt{\frac{K_e}{m}}$$

$$= \sqrt{\frac{249.753 \times 10^3}{1019.4}}$$

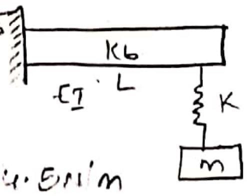
$$= 15.652 \text{ rad/sec}$$

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{15.652}$$

$$= 0.401$$

(ii) $K_b = \frac{3EI}{l^3}$

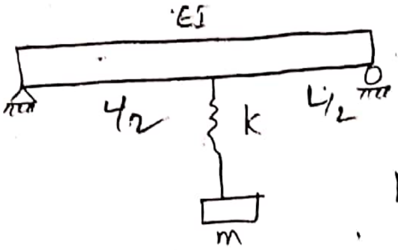


$$\frac{1}{K_e} = \frac{1}{K} + \frac{1}{K_b}$$

$$K_e = 32344.5 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{K_e}{m}} = 5.630 \text{ rad/sec}$$

(iii)



→ series.

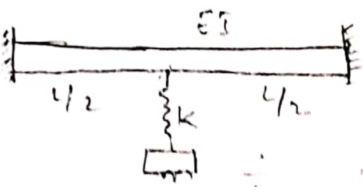
$$K_b = \frac{48EI}{l^3}$$

$$K = 40 \times 10^3 \text{ N/m}$$

$$\frac{1}{K_e} = \frac{1}{K_b} + \frac{1}{K}$$ … $\times 10^3$ N/m

$$\omega_n = \sqrt{\frac{K_e}{m}}$$ …

(iv)



$$K_b = \frac{192EI}{l^3}$$

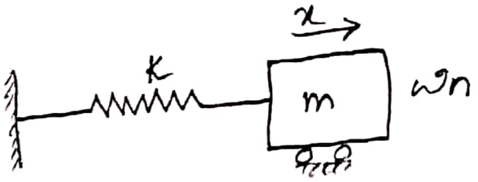
(v)

$$K_b = \frac{3LEI}{a^2(L-a)^2}.$$

16.7.2015

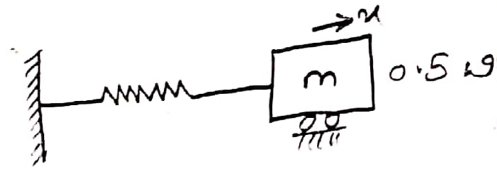
Example

1\. A SDOF system of mass 'm' & stiffness K is found to vibrate with natural frequency of 12 Hz. If stiffness is decreased by 1000 N/m the natural frequency is changed by 50%. Find mass & stiffness of original system



$\omega_n = 2\pi \times 12$

$= 75.39$ rad/sec



$\omega_n^2 = \frac{K}{m}$ — (1)

$0.5\,\omega_n^2 = \frac{K - 1000}{m}$ — (2)

$\frac{(1)}{(2)} = \frac{\omega_n^2}{(0.5\,\omega_n^2)} = \frac{K}{K-1000}$

$\frac{75.39^2}{(0.5(75.39))^2} = \frac{K}{K-1000}$

$4K - 4000 = K$

$4K - K = 2000$

$3K = 2000$

$K = 1333.33$ N/m

$m = 0.23$ Ns^2/m

Example

2\) A dynamic system has max velocity of 200 mm/sec and natural period of 1 sec. If the initial displacement 10 mm find the amplitude, initial velocity & max. accelerat

$T = 1sec.$ max velocity = 200mm/s ≃ 0.2m/s

$\omega_n = \frac{2\pi}{T} = \frac{2\pi}{1} = 6.284$ rad/sec.

initial displacement $x_0 = 10mm$

$\dot{x}_{max} = A \cdot \omega_n$

$0.2 = A \times 6.284$

$A = 0.032m.$

$A = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2}$ $\dot{x}_0$ = initial velocity

$0.032 = \sqrt{(10)^2 + \left(\frac{\dot{x}_0}{6.284}\right)^2}.$

$\dot{x}_0 = 0.189 m/s$

acceleration $\ddot{x}_{max} = A\omega_n^2$

$= 0.032 \times 6.284^2 = 1.25 m/s^2$

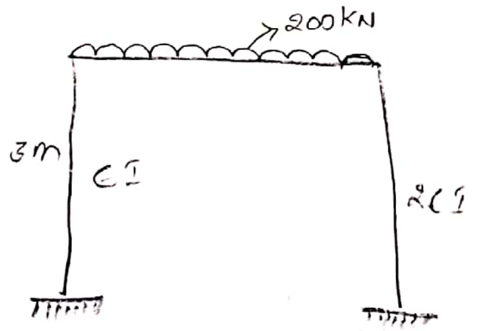
Problem

3) Compute the natural frequency in side sway for the frame shown in fig. If the initial displacement is 25mm & $\dot{x}_0$ = 25mm/s. what is amplitude & displacement at t = 1sec.

$x_0 = 25mm$

$\dot{x}_0 = 25mm/sec$

$t = 1sec$



EI = 30×10^{12} N mm²

4.

Equivalent lateral stiffness of column

$K_1 = \frac{12EI}{l^3}$ $K_2 = \frac{12(2EI)}{l^3}$

$K_e = K_1 + K_2$

$= 40000 \, N/mm$

$\omega_n = \sqrt{\frac{K_e}{m}} = \sqrt{\frac{40000}{200\times10^3/9810}}$ ($m = \frac{W}{g}$)

$= 44.29 \, rad/sec$

$A = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2}$

$= \sqrt{(25)^2 + \left(\frac{25}{44.29}\right)^2}$

$= 25 \, mm.$

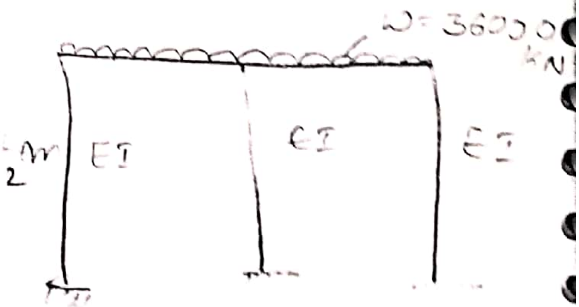
$x = x_0 \cos\omega_n t + \frac{\dot{x}_0}{\omega_n} \sin\omega_n t$

$= 25 \times \cos 44.29 \times 1 + \frac{25}{44.29} \sin 44.29 \times 1$

$x = 18.28 \, mm$

Problem 4. Calculate the natural frequency side sway for the frame shown in fig. initial displacement $x_0 = 30 \, mm$, initial velocity is $\dot{x}_0 = 30 \, mm/s$. calculate the ~~lat~~ displacement at time $t = 2$ sec

$EI = 30 \times 10^{12} \, N\cdot mm^2$



$K_e = K_1 + K_2 + K_3$.

$$K_e = \frac{12EI}{l^3} + \frac{12EI}{l^3} + \frac{3EI}{l^3}$$

$= 12650$ N/mm.

$$\omega_n = \sqrt{\frac{k_e}{m}} = \sqrt{\frac{12650}{(36000 \times 10^3/9810)}} \simeq 1.856$$

$m \to (w/g)$

$$x = x_0 \cos\omega_n t + \frac{\dot{x}_0}{\omega_n} \sin\omega_n t$$

$$= 30 \times \cos(1.856 \times 2) + \frac{30}{1.856} \sin 1.856 \times 1$$

$= 30.60$ mm.

Problem

5) The stiffness of damping characteristics are found using free vibration test. For dynamic system In this test mass of 18 kgs is set into free vibration by displacing $x = 25$ mm free vibration at the end of 20 complete cycles the elapsed time is 2sec and the amplitude is 5 mm find the damping & stiffness of the system

$m = 18$ kg

$x_1 = 25$ mm

$x_{20} = 5$ mm

6)

20 cycles - 3 sec

$T_D = \frac{3}{20}$ sec

1 cycle - $\frac{3}{20}$ sec

$\delta = \ln\left(\frac{x_1}{x_{20}}\right) = \ln\left(\frac{25}{5}\right) = 1.609$

$\delta = \frac{1.609}{20} = 0.08$

$\delta = 2\pi\zeta$

$\zeta = \frac{\delta}{2\pi} = \frac{~~1.609~~ 0.08}{2\pi} = ~~0.256~~ 0.0128$

$= 1.28\%$

$\omega_D = \frac{2\pi}{T_D} = \frac{2\pi}{0.15} = 41.88$ rad/sec

$\zeta = \frac{c}{c_r} = \frac{c}{2m\omega_n}$

$0.0128 = \frac{c}{2\times18\times41.88}$

$C = 19.3$ N·s/m.

$\omega_D = \omega_n\sqrt{1-\zeta^2}$

$\omega_n = 41.87$.

$\omega_n = \sqrt{\frac{K}{m}}$.

$K = 31.45$ N/mm.

Problem

6) In SDOF system consists of mass with weight 6KN, k = 5N/mm, damping force $F_D = 200$ N, velocity v = 250 mm/s. Find the damping ratio, logarithmic decrement & damped frequency, ratio of two consecutive amplitude.

$W = 6$ KN $\Rightarrow m = \frac{6000}{9.81} = 611.82$ Ns²/m

$K = 5$ N/mm $= 5000$ N/m

$c\dot{x} = F_D = 200N$

$\dot{x} = v = 250mm/s = 0.25m/s$

$\omega_n = \sqrt{\frac{K}{m}} = 2.86 \ rad/sec.$

$c = \frac{c\dot{x}}{\dot{x}} = \frac{F_D}{v} = \frac{200}{0.25m/s} = 800 Ns/m$

$C_{cr} = 2m\omega_n = 2 \times 611.82 \times 2.86 = 3499.6 Ns/m$

$\zeta = \frac{c}{C_{cr}} = \frac{800}{3499.6} = 0.23$

$\omega_D = \omega_n\sqrt{1-\zeta^2} = 2.86\sqrt{1-0.23^2} = 2.75 \ rad/sec$

$\delta = 2\pi\zeta = 2\pi \times 0.23 = 1.44$

$\delta = \ln\left(\frac{x_1}{x_2}\right)$

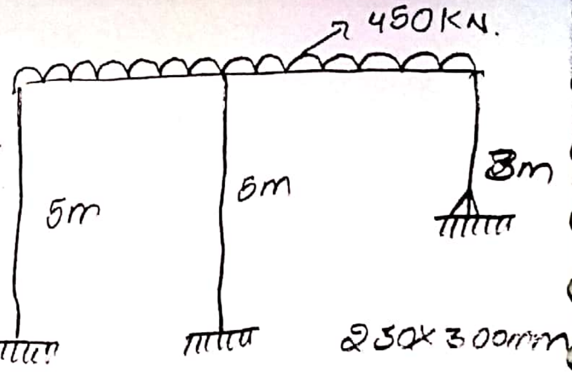
$\ln\left(\frac{x_1}{x_2}\right) = 1.44$

$\frac{x_1}{x_2} = e^{1.44} = 4.22.$

Problem

7. Compute the natural frequency in the side way for the frame shown if the initial displacement ~~shown~~ is 20mm, $\dot{x}_1 = 20mm/sec$. Calculate the amplitude & write the expression for displacement. Damping is 10% of critical; c/s of each column is given as 230 × 300mm

$Ke = K_1 + K_2 + K_3$



$ke = \frac{12EI}{l^3} + \frac{12EI}{l^3} + \frac{3EI}{l^3}$

$= 3500N/mm$

$\omega_n = \sqrt{\frac{k_e}{m}}$

$\sqrt{\frac{3500}{(450\times10^3/9810)}} = 8.75$ rad/sec.

$I = \frac{bd^3}{12} = \frac{230\times300^3}{12}$

$= 517\times10^6 mm^4$

$EI = 11.58\times10^{12}$

$\omega_D = \omega_n\sqrt{1-\xi^2}$

$\omega_D = 8.75\sqrt{1-0.1^2}$ $\quad \frac{c}{c_r} = 10\% \, c_r.$

$= 8.71$ rad/sec $\quad \frac{c}{c_r} = 10\% = 0.1.$

$A = \sqrt{x_0^2 + \left(\frac{\dot{x}_0 + \omega_n \xi x_0}{\omega_D}\right)^2}$

$A = \sqrt{20^2 + \left(\frac{20 + 8.75\times0.1\times20}{\omega_D = 8.71}\right)^2} = 20.01mm$

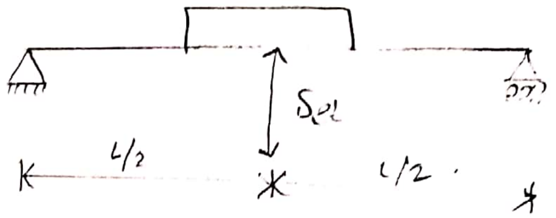
$x = e^{-\omega_n \xi t}\left[x_0 \cos\omega_D t + \left(\frac{\dot{x}_0 + \omega_n \xi x_0}{\omega_D}\right)\sin\omega_D t\right]$

$x = e^{-8.75\times0.1\times t}\left[20\cos 8.71t + \left(\frac{20 + 8.75\times0.1\times20}{8.71}\right)\sin\omega_D t\right]$

27.7.2016.

8.

1.) Problem



compute the magnification factor of forced vibration produced by a machine operating at speed of 600rpm installed at the middle of the beam static deflection is 0.25mm. wt of machine 5000N. Neglect the wt. of beam & consider viscous damping force of 500N @ velocity of $v = 25 mm/s$

$\delta_{st} = 0.25 mm$

$W = 5000N$.

Damping force $= 500N$.

$v = 25 mm/s$.

$\omega = 600 rpm$

$\omega = 2\pi n \rightarrow rpsec(n)$.

$= \frac{2\pi n}{60} = \frac{2\pi \times 600}{60} = 62.83 rad/sec$.

$c\dot{x} = 500N$

$\dot{x} = v = 25 mm/s$

damping coefficient $c = \frac{500}{25} = 20 N \cdot s/mm$

ation
o
n
ks

Stiffness $K = \frac{w}{\delta st} = \frac{5000}{0.25} = 20000 N/mm$

$\omega n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K \times g}{w}} = \sqrt{\frac{20000 \times 9.18 \times 10^{-3}}{5000}}$

$= 198$ rad/sec.

$C_{cr} = 2m \cdot \omega n$

$= 2 \cdot \frac{w}{g} \cdot \omega n.$

$= 2 \times \frac{5000}{9810} \times 198$

$= 201.83$ N.S/m

damping ratio.

$\zeta = \frac{c}{C_{cr}} = \frac{20}{201.83} = 0.099 \approx 0.1$ (10%)

frequency ratio $r = \frac{\bar{\omega}}{\omega} = \frac{62.23}{198} = 0.31$

$DMF = \mu = \frac{1}{\sqrt{(1-r^2)^2 + (2r\zeta)^2}}.$

$= \frac{1}{\sqrt{(1-0.32^2)^2 + (2 \times 0.32 \times 0.1)^2}}$

$= 1.11$

Pratham

2. A machine of weight 80KN is placed central on simply supported beam which produces a harmonic force of magnitude 140KN at a frequency of 60 rad/s. assume 15% of critical damping. determine the amplitude of machine motion of machine

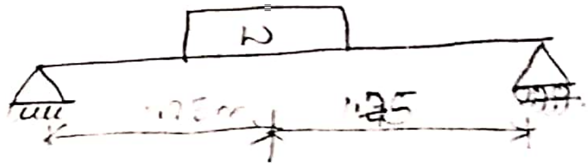
& force transmitted to support

$\zeta = 15\%$

$F_r = 140\,KN$

$\omega = 60$ rad/s.

$E = 2\times10^5\,N/mm^2$



$I = 30\times10^6\,mm^4$ $\quad l = 3.5m = 3500\,mm$

Transmissability: It is ratio of transmitted force to the excited force

$f(t) = F_0 \sin\omega t$ $\quad F_0 = 140\,KN.$

$$K = \frac{48EI}{l^3} = \frac{48\times(2\times10^5)\times(30\times10^6)}{3500^3} \quad \omega = 60\,rad/s$$

$$= 6717.2\,N/mm.$$

$$m = \frac{w}{g} = \frac{80000}{9810} = 8.15\,Ns^2/mm.$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{6717.2}{8.15}}$$

$$= 28.70$$

$$\text{frequency} = \frac{\omega}{\omega_n} = \frac{60}{28.70} = 2.1$$

$\zeta = 15\%$

$$Y = \frac{y_{st}}{\sqrt{(1-r^2)^2 + (2r\zeta)^2}}$$

$$y_{st} = \frac{F_0}{K}.$$

$$Y = \frac{140\times10^3/6717.2}{\sqrt{(1-2.1^2)^2 + (2\times2.1\times0.15)^2}}$$

$= 6.1\,mm$

Transmissibility $= T = \frac{F_t(t)}{F(t)}$

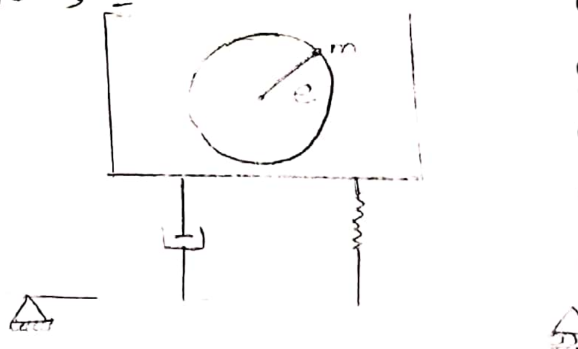
$$= \frac{\sqrt{1+(2r\zeta)^2}}{\sqrt{(1-r^2)^2+(2r\zeta)^2}}$$

$= 0.341$

Transmitted force

$F_t(t) = 140\times0.341 = 47.7\,KN$

Problem 3. A simply supported beam having a span of 4m supports a machine whose weight is 80KN. The motor of the machine runs at 250 rpm & the rotor is out of balance to the extent of w = 200N at radius of r = 250mm (eccentricity) what will be the amplitude of steady state response. Assume 10% critical damping. $E = 2.1\times10^5$, $I = 58\times10^6\,mm^4$.



$w = 250$ rpm

$\omega = \frac{2\pi}{60}\times250$

$= 26.17$ rad/sec

$F(t) = F_0 \sin\omega t$

$F_0 = m\times e\times\omega^2$

$= \left(\frac{200}{9810}\right)\times250\times26.17^2 = 3490.6N$

$K_b = \frac{48EI}{l^3} = 8347.5 \text{ N/mm}$

$\underset{(\text{machine})}{m} = \frac{80 \times 1000}{9810} = 8.155 \text{ N s}^2/\text{mm}$

$\omega_n = \sqrt{\frac{K_b}{m}} = \sqrt{\frac{8347.5}{8.155}} = 31.9 \simeq 32 \text{ rad/s}$

$r = \frac{\omega}{\omega_n} = \frac{26.17}{32} = 0.81$

$\zeta = 0.10$

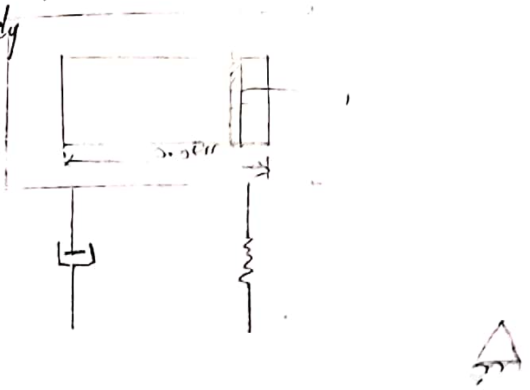
$x_{max} = \frac{F_0/K}{\sqrt{(1-r^2)^2 + (2r\zeta)^2}}$

$= \frac{3490.6 / 8347.5}{\sqrt{(1-0.81^2)^2 + (2 \times 0.81 \times 0.10)^2}}$

$= 1.1 \text{mm}$

Problem

4. A 100kg machine is mounted on spring with $K = 12 \times 10^5$ N/m with damping factor 20% with 2kg piston within machine reciprocates with a stroke of 0.08m & speed of piston 3500 rpm. Assuming motion of piston is harmonic. Determine steady state amplitude of vibrating force transmitting to fdn.



$N = 3600 rpm$

$$\omega = \frac{2\pi n}{60} = 366.52 \text{ rad/s}.$$

$$F_0 = m \times e \times \omega^2.$$

$$= 2 \times 0.04 \times 366.52^2$$

$$= 10746.8 \text{ N}.$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{12 \times 10^5}{100}} = 109.5 \text{ rad/sec}$$

$K = 12 \times 10^5$ N/m

$m = 100$ kg

$$r = \frac{\omega}{\omega_n} = \frac{366.52}{109.5} = 3.345.$$

$$\zeta = 0.20$$

$$x_{max} = \frac{F_0/K}{\sqrt{(1-r^2)^2 + (2r\zeta)^2}} = 0.871 \text{ mm}.$$

$$T = \frac{F_t(t)}{F(t)} = \frac{\sqrt{1 + (2r\zeta)^2}}{\sqrt{(1-r^2)^2 + (2r\zeta)^2}} = 0.163.$$

$$F_t = F_0 \times 0.167 = 1947.3 \text{ N}$$

Dynamic Analysis
- Model
- Differential equation
- Solutions / BC's
- Interpretation of results.

The arbitrary periodic loading is expressed in the form of fourier series by equation (1) which consists of 2 terms (1) & (2)

Term (1) represents constant load & term (2) represents series of harmonic loads of frequency ω_r & amplitudes a_r & b_r.

The steady state response produced in undamped SDOF system subjected to harmonic loading (sine term)

$$y = \left(\frac{F_0}{K}\right)\left(\frac{1}{1-r^2}\right)(\sin \bar{\omega} t - r \sin \omega t)$$

steady state transient state

$$= y_r(t) = \frac{b_r}{K} \frac{1}{r^2} \sin r \bar{\omega} t \rightarrow (3a)$$

$$\therefore r_r = \frac{\omega_r}{\omega} = -\frac{r T}{T_p} = \frac{r\bar{\omega}}{\omega}$$

where $\omega_1 = \frac{2\pi}{T_p}$

6.7.15 absent

Similarly the steady state response produced by undamped SDOF system subjected to harmonic loading in terms of cosine term is given by

$$y_r(t) = \frac{a_r}{K} \frac{1}{1-r_r^2} \cos r \bar{\omega}_r t \rightarrow 3(b).$$

The
load
The
Eqs
wh
Eq
Vecto
expre

$K_1 = \frac{3EI}{l^3}$ K of beam

$= \frac{3 \times 2 \times 10^5 \times 2500}{300}$

$K_1 = 5 KN/m$

$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$ (series)

$K_e = \frac{K_1 K_2}{K_1 + K_2} = \frac{20}{9} = 2.22 KN/m$

$f = \frac{1}{T}$

$T = 2\pi \sqrt{\frac{m}{K_e}}$ m → s.d.o.f.

2.15 ## PRINCIPLE OF VIRTUAL WORK

The response to the dynamic loads on a structure is modelled using spring mass system. If the structural system consists of multiple interconnected rigid bodies are having distributed mass & ~~electricity~~ elasticity the system can be modelled as SDOF system using the principle ~~&~~ of virtual work. Since dynamic equilibrium is difficult to apply to large & complex structural systems. & alternative approach is adopted to formulate equations of motion using principle of virtual work.

- The principle of virtual work states that for a deformable system in equilibrium under a set of forces the sum of total external virtual work & internal virtual work is zero. where virtual work is work done by the virtual displacement

$\delta W = 0$

Multi degree of freedom (MDOF) Systems:

The theory of MDOF system is applicable to all kinds of structural systems whether discrete structural system (or) continous structural system. The dynamic response will be in a more generalised & realistic of the structures are idealised as MDOF systems. Structures represented by SDOF model doesnot describe the dynamic behaviour of the structure adequatly. Normally multi storey buildings are modeled & analysed as MDOF system. To transform the building structure into MDOF with lumped masses at the floor level. the following are the assumptions.

lumped mass

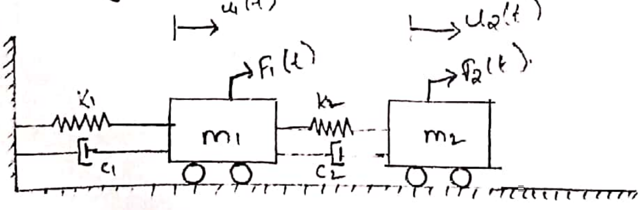
Assumpt.

1. The entire mass of the building is concentrated at floor levels.
2. The axial force do not contribute significantly for the deformation of structures
3. The floors with slabs & beams are infinitely rigid compared to the columns & remain horizontal without rotation

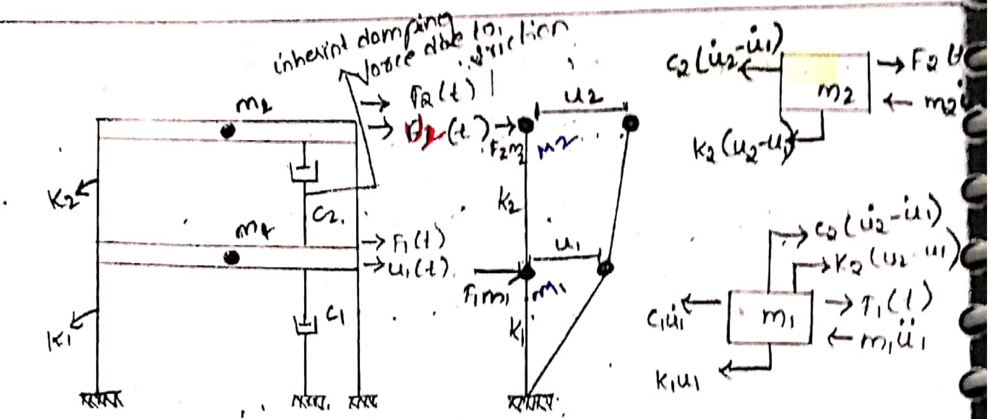
Equations of motion in MDOF systems:-

case (i):- 2-degree of freedom without damping

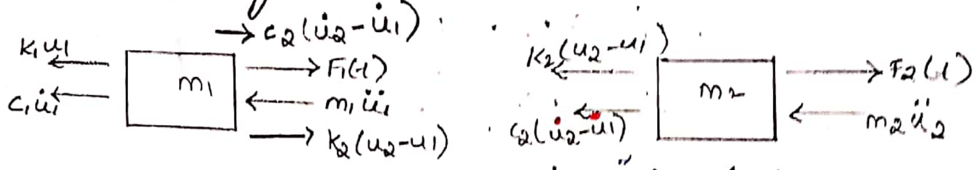
Step-1: Modelling:-



Dynamic analysis
(i) Model
(ii) D. Equations
(iii) Solutions of D.E
(iv) Interpretations



Free body diagram



For each mass, forces acting horizontally = 0

$$\Sigma F_x = 0$$

$$m_1 \rightarrow m_1\ddot{u}_1 + c_1\dot{u}_1 + k_1u_1 + c_2(\dot{u}_1-\dot{u}_2) + K_2(u_1-u_2) = F_1 \rightarrow ①$$

$$m_2 \rightarrow m_2\ddot{u}_2 + c_2(\dot{u}_2-\dot{u}_1) + K_2(u_2-u_1) = F_2(t) \quad -②$$

$$\ddot{u}_1 m_1 + \dot{u}_1(c_1+c_2) + \dot{u}_2(-c_2) + u_1(K_1+K_2) + u_2(-K_2) = F_1(t)$$

$$\ddot{u}_2 m_2 + \dot{u}_1(-c_2) + \dot{u}_2(c_2) + u_1(-K_2) + u_2(K_2) = F_2(t)$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} + \begin{bmatrix} K_1+K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

Matrix $\leftarrow$ $\boxed{[M]\ddot{u} + C\dot{u} + Ku = F}$ → ⓐ F = load vector

M = mass matrix (diagonal matrix)

$\ddot{U}$ = Vector of accelerations for each degree of freedom

C = damping matrix [Symmetrical matrix]

$\dot{u}$ = vector of velocities for each degree of freedom

K = Stiffness matrix [Symmetrical matrix]

u = Vector of displacements for each degree of freedom

Dynamic Analysis

(1) for free vibration $\underline{F=0}$

$$M\ddot{u}+C\dot{u}+K\ddot{u}=0$$

(2) for free vibration - undamped $\underline{C=0,\ F=0}$

$$M\ddot{u}+Ku=0$$

(3) for forced - undamped vibration $\underline{C=0}$

$$M\ddot{u}+Ku=F$$

(4) for static analysis $m=0\ \ c=0$

$$\underline{Ku=F}$$

(*) Free undamped vibrations for 2 DOF systems

Solution:- The Equation of Motion $\left.\begin{matrix}F=0\\ C=0\end{matrix}\right\}$

Matrix form $M\ddot{u}+Ku=0$ —— (1)

The Solution to the Equation (1) is in the form of

$$u=a\sin(\omega t+\phi)$$

where, a = vector of amplitudes corresponding to each degree of freedom

Substitute $\ddot{u}$ & u in equ(1)

$$\Rightarrow \ddot{u}=-\omega^2 a\sin(\omega t+\phi)=-\omega^2 u$$

$$\Rightarrow -\omega^2 ma\sin(\omega t+\phi)+Ka\sin(\omega t+\phi)=0$$

$$[K-\omega^2 m]a=0 \quad —— (2)$$

In dynamic problems, the amplitudes of each degree of freedom will never be zero.

Hence $\underline{a\neq 0}$

It is observed that this dynamic problem is a standard eigen values problem

Chk Notes @ end

Hence by Cramers rule

$$|k-\omega^2 m| = 0 \quad — (2)(a)$$

for 2 degrees of freedom

$$\left| \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}_{2\times2} - \omega^2 \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}_{2\times2} \right| = 0$$

$$\begin{vmatrix} k_1+k_2-\omega^2 m_1 & -k_2 \\ -k_2 & k_2-\omega^2 m_2 \end{vmatrix}_{2\times2} = 0$$

$\begin{vmatrix} a & c \\ b & d \end{vmatrix}$ $ad - bc = 0$

$$[(k_1+k_2)-\omega^2 m_1][k_2-\omega^2 m_2] - k_2^2 = 0 \rightarrow (3)$$

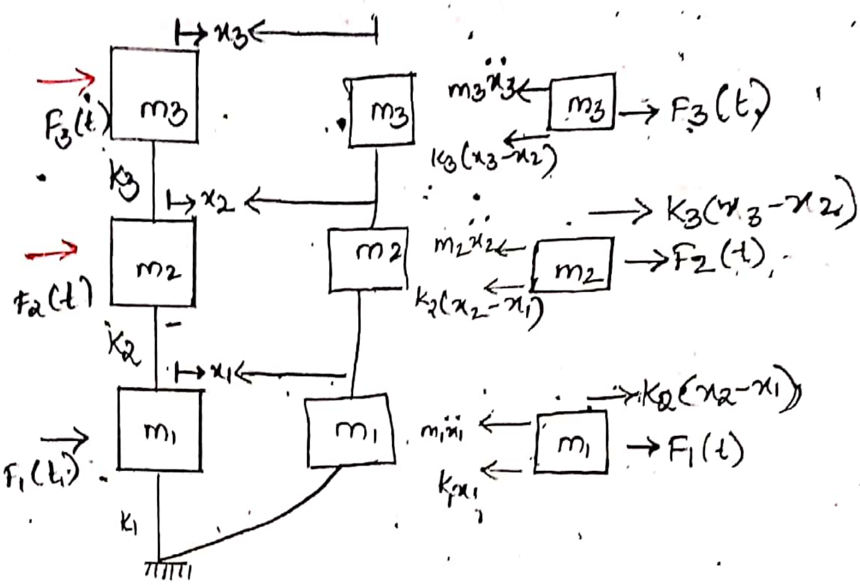
Equation 3 is called characteristic polynomial of system. The solution for 'ω' in Eqn (3) are eigen values of $|k-\omega^2 m|$

∴ when there are 2 solutions (or) roots for 'n' freedom system will have 'n' solution.

In the present case there are 2 values for ω^2 (ω_1^2, ω_2^2) which will satisfy the Equation (3). The lowest of these 2 solutions will be fundamental frequency. Substitute these roots in Eqn (2) to get amplitude vector 'a' which means that each frequency will have its own characteristic displacement shape of DOF called mode shape.

* Mode shapes are expressed as vector of relative amplitudes [relative to first value].

Develop Equations of motion for given MDOF system (No Damping or Undamped) $c=0$



$\Sigma f_x = 0$ (for m_1)

$$m_1\ddot{x}_1 + K_1x_1 + K_2(x_1 - x_2) = F_1(t) \quad \text{— (1)}$$

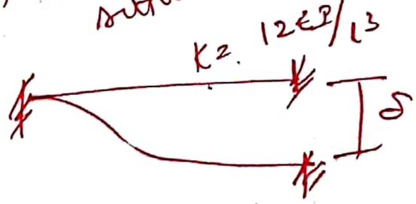
for m_2

$$m_2\ddot{x}_2 + K_2(x_2 - x_1) + K_3(x_2 - x_3) = F_2(t) \quad \text{— (2)}$$

for m_3

$$m_3\ddot{x}_3 + K_3(x_3 - x_2) = F_3(t) \quad \text{— (3)}$$

K = for each storey = $\frac{12EI}{l^3}$ (Both ends fixed and one end settles or sways) $\Sigma x = 0$



Above Equations can be written as

$$m_1\ddot{x}_1 + (K_1+K_2)x_1 - K_2x_2 = F_1 \rightarrow (1)$$

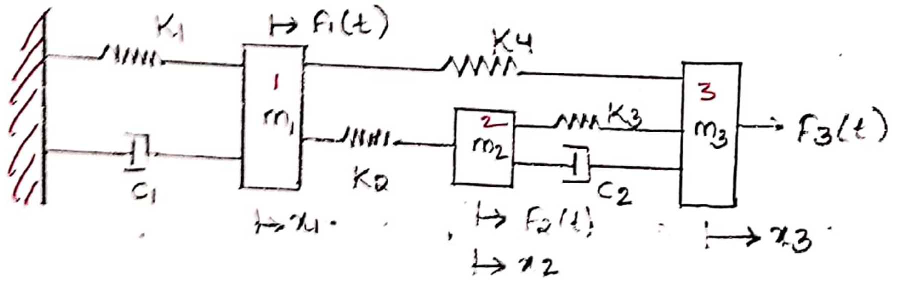
$$m_2\ddot{x}_2 - K_2x_1 + (K_2+K_3)x_2 - K_3x_3 = F_2 \rightarrow (2)$$

$$m_3\ddot{x}_3 - K_3x_2 + K_3x_3 = F_3 \rightarrow (3)$$

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} K_1+K_2 & -K_2 & 0 \\ -K_2 & K_2+K_3 & -K_3 \\ 0 & -K_3 & K_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

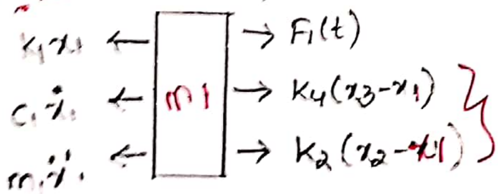
$$[M]\{\ddot{x}\} + [K]\{x\} = \{F\}$$

\# (9) Develop Equations of motion for given 3 degree of freedom system: (With Damping or Damped system)



Free body diagram

for m_1 :-

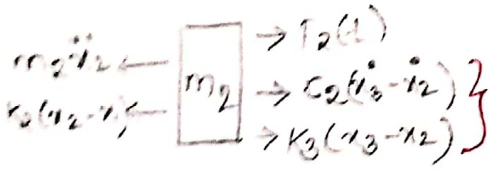


$\Sigma x = 0$

$$\Rightarrow m_1\ddot{x}_1 + C_1\dot{x}_1 + K_1x_1 + K_4(x_1 - x_3) + K_2(x_1 - x_2) = F_1(t)$$

$$\Rightarrow m_1\ddot{x}_1 + C_1\dot{x}_1 + (K_1+K_2+K_4)x_1 + x_3(-K_4) + (-K_2)x_2 = F_1(t)$$

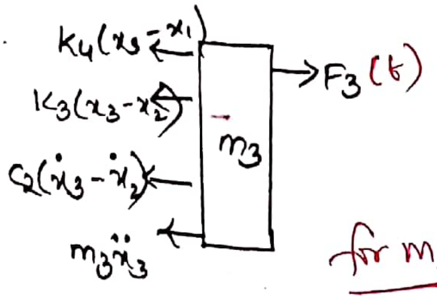
for m_2 :-



$$m_2\ddot{x}_2 + K_2(x_2 - x_1) + C_2(x_2 - x_3) + K_3(x_2 - x_3) = F_2(t)$$

$$m_2\ddot{x}_2 + (-K_2)x_1 + (-K_3)x_3 + (K_1+K_2)x_2 + C_2x_2 - C_2x_3 = F_2(t)$$

(2)



for m_3 :-

$$m_3\ddot{x}_3 + C_2\dot{x}_3 - C_2\dot{x}_2 + K_4(x_3 - x_1) + K_3(x_3 - x_2) = F_3(t)$$

$$m_3\ddot{x}_3 + C_2\dot{x}_3 - C_2\dot{x}_2 + (K_4 + K_3)x_3 - K_4x_1 - K_3x_2 = F_3(t) \quad \text{③}$$

writing Eqn ①, ②, ③ in matrix form.

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & -C_2 \\ 0 & -C_2 & C_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} K_1+K_2+K_4 & -K_2 & -K_4 \\ -K_2 & K_2+K_3 & -K_3 \\ -K_4 & -K_3 & (K_4+K_3) \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \quad \text{ⓐ}$$

Generate Equation of motion from direct matrix formation

(or)

Direct matrix formulation of MDOF System

From the Eqn ⓐ of 3 DOF System it can be observed that all diagonal terms are positive & contain terms that are directly attached to corresponding elements.

All non-diagonal terms are negative & Symmetric. They are symmetric because they are attached to two elements & the effects are same in this two elements. They are -ve (relative) because of relative displacements (or) velocities of two attached elements.

Steps to find direct matrix formation:-

① Determine the number of degrees of freedom for the problem [1 DOF is associated with mass] This step will determine the size of mass, damping & stiffness matrix

② **Formulation of mass matrix:**

Enter the mass values [follow the order of DOF] in to the diagonal of the mass matrix all other values of 'm' matrix are zero.

$$[m] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

③. For each mass sum up the damping attached to that mass can enter this value in the damping matrix at the diagonal location

④ Identify dampers (or) dashpots attached to 2 masses. label them as m & n. write down (-ve) damping at (m,n) & (n,m) locations in damping matrix. Repeat for all dampers any remaining terms in damping matrix are kept zero

$$C = \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & -c_2 \\ 0 & -c_2 & c_2 \end{bmatrix}$$

⑤ For each mass sum up all the stiffnesses from all springs attached to that mass & enter the value in stiffness matrix at the diagonal location corresponding to the mass in mass matrix

(6) Identify springs attached to 2 masses. Let the masses be m & n, write down −ve spring stiffness at (m,n) & (n,m) locations in the stiffness matrix. Repeat for all springs; any remaining terms in stiffness matrix are kept zero.

$$K = \begin{bmatrix} K_1+K_2+K_4 & -K_2 & -K_4 \\ -K_2 & K_2+K_3 & -K_3 \\ -K_4 & -K_3 & K_3+K_4 \end{bmatrix}$$

(1,2) (1,3) (2,1) (2,3) (3,1) (3,2) — m_1, m_2, m_3

b/n 2 masses

K_2 b/n m_1 & m_2.
(1,2), (2,1)
K_3 b/n m_2 & m_3
K_4 b/n m_1 & m_3

(7) Sum up all external forces applied in each mass, enter this value in force vector at row location corresponding to that of mass matrix.

$$\begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix}$$

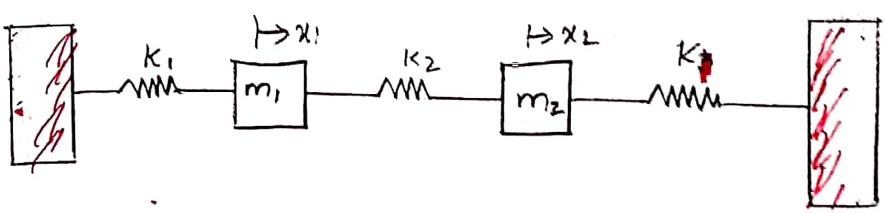
(8) The resulting matrix equation of motion.

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\}$$

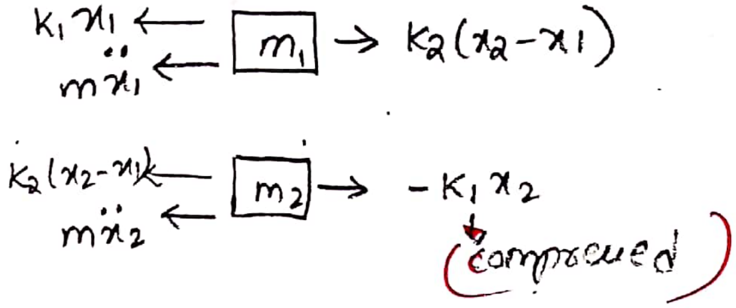
(*) Solutions of Eigen value problems for natural frequencies & mode shape

eigenvalue – f
eigenvector – mode shape

Let us consider an eigen value-vector problem as follows



$m_1 = m_2 = m$



$$m\ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 = 0$$

$$m\ddot{x}_2 + (k_2 + k_1)x_2 - k_2 x_1 = 0.$$

$$\ddot{x}_1 = -\left(\frac{k_1 + k_2}{m}\right)x_1 + \frac{k_2}{m}x_2,$$

$$\ddot{x}_2 = \frac{k_2}{m}x_1 + \left(-\left(\frac{k_1 + k_2}{m}\right)\right)x_2$$

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} -\left(\frac{k_1 + k_2}{m}\right) & \frac{k_2}{m} \\ \frac{k_2}{m} & -\left(\frac{k_1 + k_2}{m}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

displacement vector

let $\alpha = \frac{k_2}{m}$, $\beta = \frac{k_1 + k_2}{m}$.

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} -\beta & \alpha \\ \alpha & -\beta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$[\ddot{x}] = \begin{bmatrix} -\beta & \alpha \\ \alpha & -\beta \end{bmatrix} [x]. \quad \text{--- eqn (i)}$$

Solution for this Equation (1), [No damping: purely oscillatory solution]

$$X = V e^{i\omega t} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{i\omega t}$$

or a

$$\ddot{x} = -\omega^2 V e^{i\omega t}$$

$$\boxed{\ddot{x} = -\omega^2 x}$$

$a = -\omega^2 x$

using eqn (1)

$$\begin{bmatrix} -\beta & \alpha \\ \alpha & -\beta \end{bmatrix} x = -\omega^2 x$$

see back pg 165

$$\boxed{Ax = \lambda x}$$

eigenvalue

Here $\lambda = -\omega^2$ (eigenvalue) $A = \begin{bmatrix} -\beta & \alpha \\ \alpha & -\beta \end{bmatrix}$ → eigenvector

$$Ax - \lambda x = 0$$

$$(A - \lambda)x = 0$$

In dynamic problems, amplitudes of each DOF will be non zero

$$\therefore x \neq 0$$

for $A - \lambda = 0$, the det of $|A - \lambda|$ should be zero. To get the matrix form multiply 'λ' with I matrix

$$(A - \lambda) = 0$$

$$|A - \lambda| = 0$$

$$|A - \lambda I| = 0$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$|A + \omega^2 I| = 0$$

$$\begin{bmatrix} -\beta & \alpha \\ \alpha & -\beta \end{bmatrix} + \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix} = 0$$

$$\begin{vmatrix} -\beta + \omega^2 & \alpha \\ \alpha & +\omega^2 - \beta \end{vmatrix} = 0$$

$$(\omega^2 - \beta)^2 - \alpha^2 = 0$$

$$\omega^4 - 2\beta\omega^2 + \beta^2 - \alpha^2 = 0$$

$$\omega^4 - 2\beta\omega^2 + (\beta^2 - \alpha^2) = 0$$

$$\omega^2 = \frac{2\beta \pm \sqrt{4\beta^2 - 4(\beta^2 - \alpha^2)}}{2}$$

$$= \frac{2\beta \pm \sqrt{4\beta^2 - 4\beta^2 + 4\alpha^2}}{2}$$

$$= \frac{2\beta \pm 2\alpha}{2}$$

$$\omega^2 = \beta \pm \alpha \text{ (roots)}$$

The ω^2 has 2 roots, say $\beta \pm \alpha$ in roots

$$\omega_1^2 = \beta + \alpha = \frac{K_1 + 2K_2}{m}$$

$$\omega_2^2 = \beta - \alpha = \frac{K_1}{m}$$

$$\beta = \frac{\omega_1^2 + \omega_2^2}{2} = \frac{4}{2} = 2$$

$$\alpha = \frac{3 - 1}{2} = 1$$

let $K_1 = K_2 = m = 1$

Then $\omega_1^2 = 3$ (λ_1), $\omega_2^2 = 1$ (λ_2)

for the 1st eigen vector (or) to find 1st eigen vector, we consider 1st eigen value (ω_1^2)

$$\left[A + \omega_1^2 I\right][v_1] = 0 \quad \leftarrow (A - \lambda_1)v = 0$$

$$\left[\begin{bmatrix} -\beta & \alpha \\ \alpha & -\beta \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right](v_1) = 0$$

$$\left[\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] v_1 = 0$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}_{2\times2} \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix}_{2\times1} = 0$$

$$v_{11} + v_{12} = 0$$

$$v_{11} = -v_{12}$$

$v_{11} = 1$ (say)

$v_{12} = -1$

$$v_1 = \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

To get 2nd eigen vector

$$[A + \omega_2^2 I][v_2] = 0$$

$$\left[\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = 0$$

$$-v_{21} + v_{22} = 0$$

$$v_{21} = v_{22}$$

Second eigen vector

$$v_2 = \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

displacement vectors (eigenvectors) → based on which mode shapes are drawn

V. imp

Orthogonality of normal modes:-

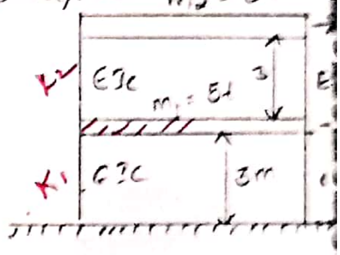
For each eigen value there will be eigen vector $[\omega_n^2]$ & vector (v). Eigen value represent natural frequency & Eigen vector represent mode shapes. (it is deflected shape).

It is to be noted that the modes corresponding

Example 1.

A Two Storey building is shown in figure has very stiff floor slabs compared to column cal. the natural freq. & mode shapes

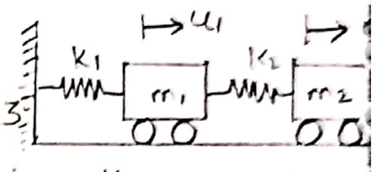
where $EI_c = 4.5 \times 10^3\ kN \cdot m^2$



effective lateral stiffness

$$k_e = k_1 + k_2$$

for whole floor ← $K_1 = \frac{12EI}{l^3}$ $\quad K_2 = \frac{12EI}{l^3}$



for whole floor ← $K_1 = \frac{12 \times 4.5 \times 10^3}{3^3}$ $\quad K_2 = \frac{12 \times 4.5 \times 10^3}{3^3}$

$K = k_1 + k_2$ (para

$K_1 = 2 \times 10^6\ N/m \quad K_2 = 2 \times 10^6\ N/m$

$$K_e = K_1 + K_2 = 4 \times 10^6\ N/m$$

The characteristic polynomial of system pg 51 back

$[K] - \omega^2 [m] = 0$

$$((K_1 + K_2) - \omega^2 m_1)(K_2 - \omega^2 m_2) - K_2^2 = 0$$

$m_1 = 5t \simeq 5 \times 10\ KN \simeq 50 \times 10^3\ N$

$m_2 = 3t \simeq 30 \times 10^3\ N$

$[(4 \times 10^6) - \omega^2 \times 50 \times 10^3][2 \times 10^6 - \omega^2\, 30 \times 10^3] - (2 \times 10^6$

$8 \times 10^{12} - \omega^2\, 1.2 \times 10^{11} - 1 \times 10^{11}\, \omega^2 + \omega^4 \times 15 \times 10^8 - 4 \times 10^{12} = 0$

$$ax^2 + bx + c$$

$$x = \omega^2$$

$$\omega^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \omega_n^2 \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$$

$\omega_1^2 = 425.3 \quad \omega_2^2 = 2508$ $\quad \omega_{n_1}^2 \quad \omega_{n_2}^2$

$$\omega_n^2 = \begin{bmatrix} 425.3 \\ 2508 \end{bmatrix} \qquad \omega_n = \begin{bmatrix} 20.6 \\ 50.1 \end{bmatrix} \text{ rad/sec}$$

$f = \frac{\omega_n}{2\pi} = \begin{bmatrix} 3.28 \\ 7.97 \end{bmatrix}$ Hz. freq. (fundamental)

To cal mode shape we use the equation

$[K - \omega^2 m]\, a = 0$ pg 57

$a \neq 0$ (Never in dynamics)

in 2DOF system let $E = [[K] - \omega^2 [m]]$ $\quad Ea = 0$

Stiffness matrix $= \begin{bmatrix} K_1 + K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} - \omega_n^2 \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$ $\quad (\omega_n^2 \to \omega_1^2)$

$$\begin{bmatrix} 4\times10^6 & -2\times10^6 \\ -2\times10^6 & 2\times10^6 \end{bmatrix} - 20.6^2 \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 5.87 & -4 \\ -4 & 2.72 \end{bmatrix} \times 10^6$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$E_1 a_1 = 0$

$$\begin{bmatrix} 5.87 & -4 \\ -4 & 2.72 \end{bmatrix} \times 10^6 \begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix} = 0$$

$5.87\, a_{11} - 4 a_{12} = 0$

$-4 a_{11} + 2.72 a_{12} = 0$

$a_{11} = 0.681\, a_{12}$

$$[(K_1 + K_2) - \omega_n^2 m_1]$$

$$[\phi_1] = \begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix} \begin{matrix} \to u_1 \\ \to u_2 \end{matrix} = \begin{bmatrix} 1 \\ 1.468 \end{bmatrix}$$

$$\begin{bmatrix} 0.681 \\ 1 \end{bmatrix}$$

mode shape 1

for $\omega_n = \omega_2^2$

for $\omega_n = \omega_2^2$

$$\begin{bmatrix} 8\times10^6 & -4\times10^6 \\ -4\times10^6 & 4\times10^6 \end{bmatrix} - 2508 \begin{bmatrix} 5000 & 0 \\ 0 & 5000 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} -4.54 & -4 \\ -4 & -3.52 \end{bmatrix} \times 10^6$$

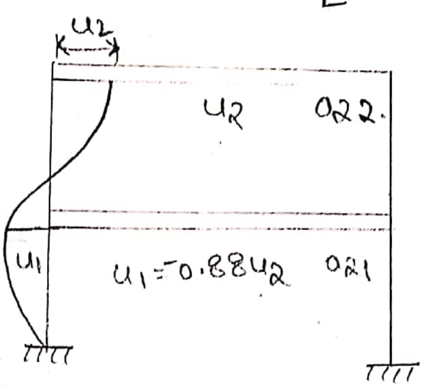
$$E_2 a_2 = \begin{bmatrix} -4.54 & -4 \\ -4 & -3.52 \end{bmatrix} \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix} = 0$$

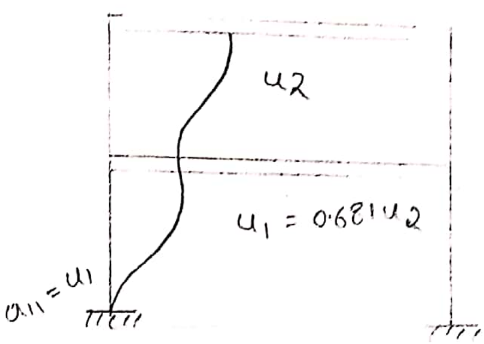
$$a_{21} = -0.88 a_{22}.$$

mode shape 2 $\phi_2 = \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix} = \begin{Bmatrix} 1 \\ -1.136 \end{Bmatrix}$ $\quad 0.88 \quad -1$

The complete solution for this problem can be given using following Eigen values & modeshape vector

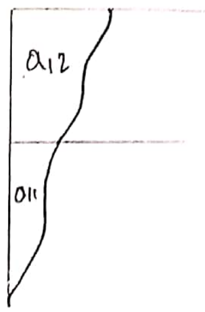
$$[\omega_n]^2 = \begin{bmatrix} 425.3 \\ 2508 \end{bmatrix} \qquad \phi = \begin{bmatrix} 1/1.468 & 1/1.136 \\ \frac{1.468}{1.468} & \frac{-1.136}{-1.136} \end{bmatrix}$$





mode shape for

$f = 7.91$ Hz



mode shape for

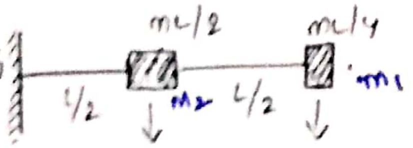
$f = 3.28$ Hz

Derive the Equation of motion for free vibration of cantilever beam with DOF as shown

Here m - mass per unit length

l - length of beam

EI - flexural rigidity of beam



$$[M][\ddot{x}] + [K][x] = 0 \qquad [C] = 0 \qquad [F] = 0$$

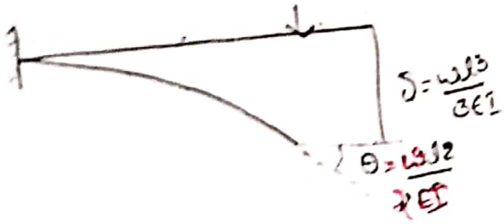
$$[m] = \begin{bmatrix} mL/4 & 0 \\ 0 & mL/2 \end{bmatrix} \qquad \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}$$

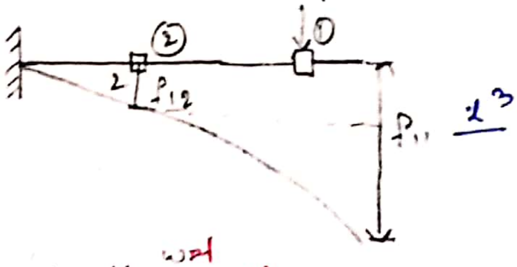
acceleration vector $[\ddot{x}] = \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix}$ $\qquad [x] = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

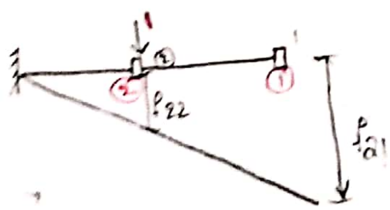
$f_{12} =$

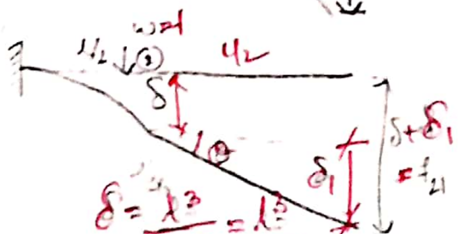
$f_{21} = \frac{5l^3}{48EI}$

$[x] = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ amplitude vector









$$\delta = \frac{l^3}{3EI} = \frac{l^3}{24EI}$$

$$f = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}_{2\times 2}$$

$$f_{11} = \frac{Wl^3}{3EI} \quad (W=1)$$

$$= \frac{l^3}{3EI}$$

$$\tan\theta = \frac{\delta_1}{l/2} \approx \theta$$

$$\delta_1 = \left(\frac{l}{2}\right)\theta$$

$$\delta_1 = \left(\frac{l}{2}\right)\left(\frac{Wl^2}{2EI} \to l/2\right)$$

$$\delta_1 = \frac{l^3}{16EI}$$

$$f_{21} = \delta + \delta_1 = \frac{l^3}{24EI} + \frac{l^3}{16EI}$$

$$f_{22} = \frac{\omega l^3}{3EI} \qquad \omega = 1,\ l = l/2$$

$$= \frac{l^3}{24EI}$$

$$f_{21} = \frac{l^3}{24EI} + \frac{l^3}{16EI} = \frac{5l^3}{48EI}$$

$$f_{12} = \frac{5l^3}{48EI}$$

$$[F] = \frac{l^3}{48EI}\begin{bmatrix} 16 & 5 \\ 5 & 2 \end{bmatrix}.$$

$$[K] = [F]^{-1}$$

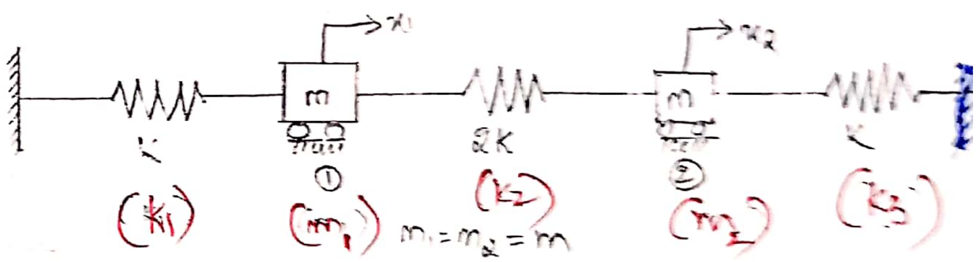
$$= \frac{48EI}{7l^3}\begin{bmatrix} 2 & -5 \\ -5 & 16 \end{bmatrix}.$$

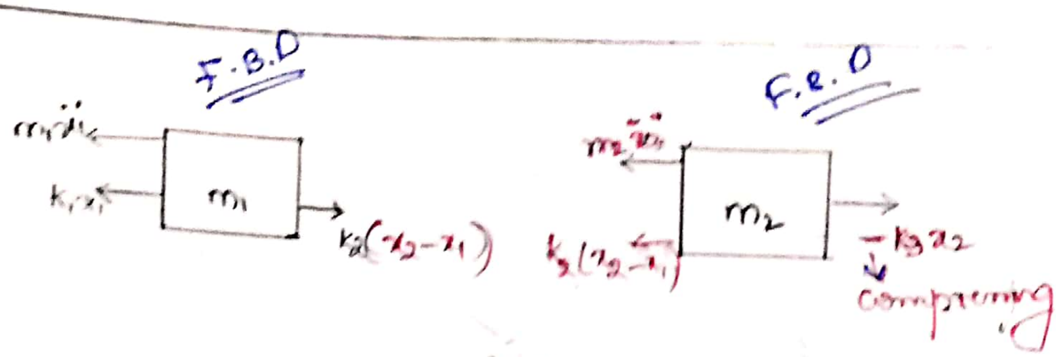
$$[m][\ddot{u}] + [K][x] = 0$$

$$\begin{bmatrix} ml/4 & 0 \\ 0 & ml/2 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \frac{48EI}{l^3}\begin{bmatrix} 2 & -5 \\ -5 & 16 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0$$

Example

2. Write equations of motion & also determine the natural frequencies & the mode shapes for the given system





Apply D-Alemberts Equation of Equilibrium or equation of Dynamic equilibrium

for m_1

$$m_1\ddot{x}_1 + K_1 x_1 + K_2(x_1 - x_2) = 0$$

$$m_1\ddot{x}_1 + (K_1 + K_2)x_1 - K_2 x_2 = 0$$

$$m_1\ddot{x}_1 + 3Kx_1 - 2Kx_2 = 0 \rightarrow (1)$$

for m_2

$$m_2\ddot{x}_2 + K_2(x_2 - x_1) + K_3 x_2 = 0$$

$$m_2\ddot{x}_2 + K_2 x_2 - K_2 x_1 + K_3 x_2 = 0$$

$$m_2\ddot{x}_2 - K_2 x_1 + (K_2 + K_3)x_2 = 0$$

$$m\ddot{x}_2 - 2Kx_1 + 3Kx_2 = 0 \rightarrow (2)$$

eqn (1) and eqn (2) are written in matrix form

Matrix form

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \overbrace{\begin{bmatrix} 3K & -2K \\ -2K & 3K \end{bmatrix}}^{[K]}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

→ free vibration

This case can be expressed as eigen value problem

$$\left[[K] - \omega_n^2 [m]\right]\phi = 0$$

$$\left[\begin{bmatrix} 3K & -2K \\ -2K & 3K \end{bmatrix} - \omega_n^2 \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\right] \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = 0.$$

$$\begin{bmatrix} 3K - m\omega_n^2 & -2K \\ -2K & 3K - m\omega_n^2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$(3k - m\omega_n^2)\phi_1 - 2k\phi_2 = 0 \rightarrow (a)$$

$$-2k\phi_1 + (3k - m\omega_n^2)\phi_2 = 0 \rightarrow (b)$$

$$\phi \neq 0$$

In dynamics solution is.

$$\left| [k] - \omega_n^2 [m] \right| = 0$$

$$\begin{vmatrix} 3k - m\omega_n^2 & -2K \\ -2k & 3k - m\omega_n^2 \end{vmatrix} = 0$$

$\because \lambda = \omega_n^2.$ (say)

$$\begin{vmatrix} 3K - m\lambda & -2K \\ -2K & 3K - m\lambda \end{vmatrix} = 0.$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = |ad - bc| = 0$$

$$(3k - \lambda m)^2 - 4k^2 = 0.$$

$$9k^2 - 2 \times 3k \times \lambda m + \lambda^2 m^2 - 4k^2 = 0$$

$$5k^2 - 6k\lambda m + \lambda^2 m^2 = 0 \Rightarrow m^2\lambda^2 - 6k\lambda m + 5k^2 = 0$$

$$ax^2 + bx + c$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$\frac{-6Km \pm \sqrt{(6Km)^2 - 4 \times m^2 \times 5K^2}}{2 \times m^2}$$

3.

$$= \frac{6km \pm \sqrt{56\,km}}{2m^2}$$

P_1 or $\lambda_1 = \frac{(6+\sqrt{16})km}{2m^2} = 5\frac{k}{m}$

P_2 or $\lambda_2 = \frac{6-\sqrt{16}}{2m^2} km = \frac{k}{m}$

$\omega_n = \sqrt{\frac{k}{m}}$

$\lambda_1 = \omega_{n_1}^2 = \frac{k}{m} = \omega_{n_1} = \sqrt{\frac{k}{m}}$

$\lambda_2 = \omega_{n_2}^2 = 5\left(\frac{k}{m}\right) = \omega_{n_2} = 2.23\sqrt{\frac{k}{m}}$

Sub λ_1 in Eqn (a)

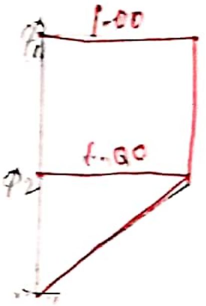
$P_1 = \omega_{n_1}^2$

$$\left[3k - m \times \frac{k}{m}\right]\phi_1 - 2k\phi_2 = 0$$

$$\phi_1 - \phi_2 = 0 \Rightarrow \phi_1 = \phi_2$$

mode shapes for $\omega_{n_1} = \sqrt{\frac{k}{m}}$

$$[\phi] = \begin{bmatrix}\phi_1 \\ \phi_2\end{bmatrix} = \begin{bmatrix}1 \\ 1\end{bmatrix}$$



Mode shape

$(P_2 =)\ \omega_{n_2}^2$ in Eqn (b)

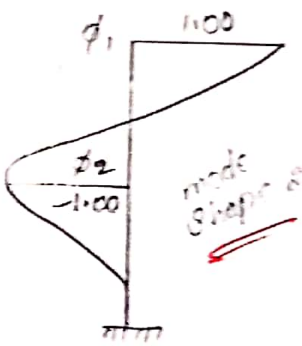
$$-2k\phi + \left(3k - m \times 5\frac{k}{m}\right)\phi_2 = 0$$

$$-2k\phi_1 - 2k\phi_2 = 0$$

$$\phi_2 = -\phi_1$$

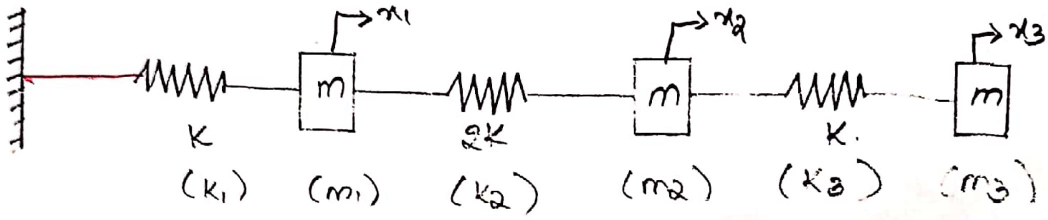
mode shape for $\omega_{n_2} = 5\sqrt{\frac{k}{m}}$

$$\phi = \begin{bmatrix}\phi_1 \\ \phi_2\end{bmatrix} = \begin{bmatrix}1 \\ -1\end{bmatrix}$$

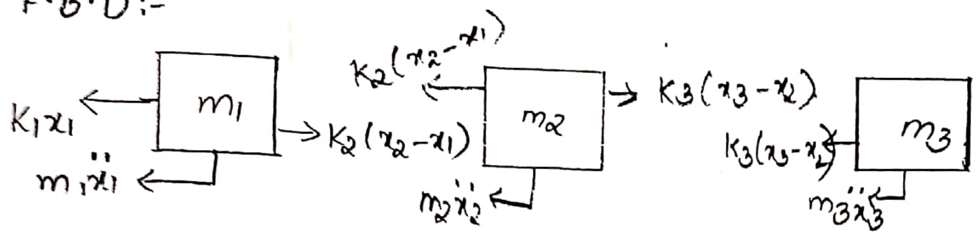


mode shape

3. Write Equations of motion, cal. natural frequency & mode shapes & draw mode shapes.



F.B.D:-



Apply D'Alemberts principle.

for m_1
$$m_1\ddot{x}_1 + Kx_1 + K_2x_1 - K_2x_2 = 0$$

$m = m_1$ $\quad K_2 = 2K.$

$$m\ddot{x}_1 + 3Kx_1 - 2Kx_2 = 0 \rightarrow ①$$

for m_2
$$m_2\ddot{x}_2 + K_2x_2 - K_2x_1 - K_3x_3 + K_3x_2 = 0$$

$$m\ddot{x}_2 - 2Kx_1 + 3KQ - Kx_3 = 0. \rightarrow ②$$

for m_3 $\quad m_3\ddot{x}_3 + K_3(x_3 - x_2) = 0$

$$m\ddot{x}_3 - Kx_2 + Kx_3 = 0 \rightarrow ③$$

Matrix form:-

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} 3K & -2K & 0 \\ -2K & 3K & -K \\ 0 & -K & K \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This can be expressed as eigen value problem

$$\left[[K] - \omega_n^2 [m]\right] [\phi] = 0$$

Sub $\omega_n^2 = \lambda$

$$\left[\begin{bmatrix} 3K & -2K & 0 \\ -2K & 3K & -K \\ 0 & -K & K \end{bmatrix} - \begin{bmatrix} m\omega_n^2 & 0 & 0 \\ 0 & m\omega_n^2 & 0 \\ 0 & 0 & m\omega_n^2 \end{bmatrix}\right] \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = 0$$

$$(3K - \omega_n^2)\phi_1 - 2K\phi_2 = 0$$

$$-2K\phi_1 + (3K - m\omega_n^2)\phi_2 - K\phi_3 = 0$$

$$-K\phi_2 + (K - m\omega_n^2)\phi_3 = 0$$

$$\begin{vmatrix} 3K - \omega_n^2 & -2K & 0 \\ -2K & (3K - m\omega_n^2) & -K \\ 0 & -K & (K - m\omega_n^2 \end{vmatrix} = 0$$

$$\lambda^3 - 7\left(\frac{K}{m}\right)\lambda^2 + 10\left(\frac{K}{m}\right)^2\lambda - 2\left(\frac{K}{m}\right)^3 = 0$$

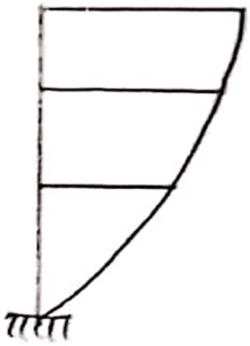
$$\lambda_1^* = 5.13\left(\frac{K}{m}\right) \Rightarrow \omega_{n1} = 2.26\sqrt{\frac{K}{m}}$$

$$\lambda_2 = 0.238\left(\frac{K}{m}\right) \Rightarrow \omega_{n2} = 0.488\sqrt{\frac{K}{m}}$$

$$\lambda_3 = 1.62\left(\frac{K}{m}\right) \Rightarrow \omega_{n3} = 1.279\sqrt{\frac{K}{m}}$$

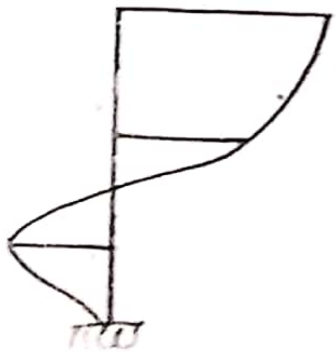
Node Shape 1:-

$$\phi_1 = \begin{bmatrix} 1.00 \\ 1.38 \\ 1.81 \end{bmatrix}$$



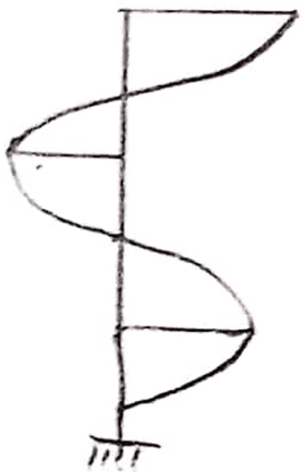
Node Shape 2:-

$$\phi_2 = \begin{bmatrix} 1.00 \\ 0.685 \\ -1.06 \end{bmatrix}$$



Node Shape 3:-

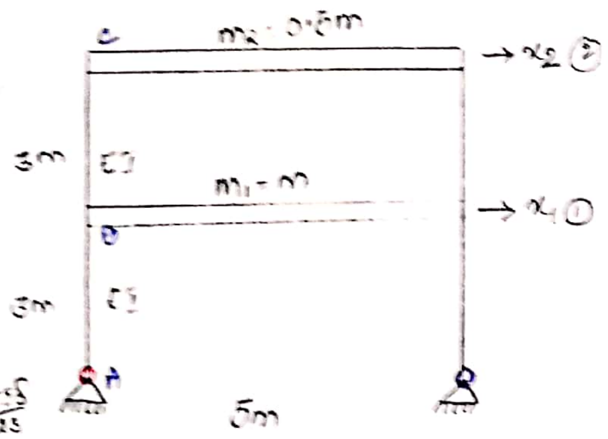
$$\phi_3 = \begin{bmatrix} 1.00 \\ -1.015 \\ 0.268 \end{bmatrix}$$

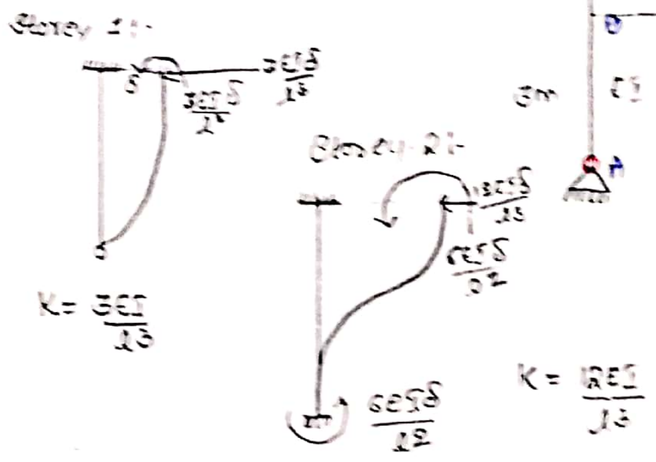


Compute the natural frequency & modes for the given shear frame & prove the orthogonality of modes

$EI = 5 \times 10^6 \, Nm^2$

$m = 500 \times 10^3 \, N \cdot s^2/m$





$K_1 = \Sigma$ all columns

Storey-1 :-

$K_1 = 2\left(\frac{3EI}{L^3}\right) \rightarrow K_{AB} = \frac{3EI}{L^3}$

$K_1 = 2 \times \frac{3 \times 5 \times 10^6}{3^3} = 1.1 \times 10^6 \, N/m$

Storey - 2 :-

$K_2 = 2\left(\frac{12EI}{L^3}\right) \quad K_{BC} = \frac{12EI}{L^3}$

$K_2 = 2 \times \frac{12 \times 5 \times 10^6}{3^3} = 4.4 \times 10^6 \, N/m$

$$[K] = \begin{bmatrix} K_1 + K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} = \begin{bmatrix} 5.55 & -4.44 \\ -4.44 & 4.44 \end{bmatrix} \times 10^6 \, N/m$$

$$[m] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} = \begin{bmatrix} 500 \times 10^3 & 0 \\ 0 & 250 \times 10^3 \end{bmatrix} \frac{Ns^2}{m}$$

$$[m]\ddot{x} + [K]x = 0$$

$$\begin{bmatrix} 500\times10^3 & 0 \\ 0 & 250\times10^3 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 5.55 & -4.44 \\ -4.44 & 4.44 \end{bmatrix}\times10^6 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

This can be expressed as eigen value problem

$$[[k] - \omega_n^2 [m]]\,[\phi] = 0$$

$$\left[\begin{bmatrix} 5.55 & -4.44 \\ -4.44 & 4.44 \end{bmatrix}\times10^6 - \omega_n^2 \begin{bmatrix} 500\times10^3 & 0 \\ 0 & 250\times10^3 \end{bmatrix}\right]\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = 0.$$

$$\begin{bmatrix} 5.55\times10^6 - \omega_n^2\, 500\times10^3 & -4.44\times10^6 \\ (-4.44\times10^6) & [4.44\times10^6 - \omega_n^2 \times 250\times10^3] \end{bmatrix}\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = 0.$$

$\phi \neq 0.$

so $|k - \omega_n^2 m| = 0.$

$$\begin{vmatrix} 5.55 - \omega_n^2\, 0.5 & -4.44 \\ -4.44 & 4.44 - 0.25\omega_n^2 \end{vmatrix} = 0.$$

$$(5.55 - \omega_n^2\, 0.5)\times(4.44 - 0.25\omega_n^2) - 19.7136 = 0$$

$$24.642 - 1.387\omega_n^2 - 2.22\omega_n^2 + 0.125\omega_n^4 - 19.7136 = 0$$

$$0.125\omega_n^4 - 3.6075\omega_n^2 + 4.9284 = 0$$

$$\omega_n^2 = \lambda$$

$$0.125\lambda^4 - 3.6075\lambda^2 + 4.9284 = 0.$$

$\lambda_1 = 1.44$ $\qquad$ $\lambda_2 = 27.04$

$\omega_{n_1} = \sqrt{\lambda_1} = 1.2$ $\qquad$ $\omega_{n_2} = \sqrt{\lambda_2} = 5.2$.

Natural frequencies.

$(5.55 - \omega_n^2\, 0.5)\,\phi_1 - 4.44\,\phi_2 = 0 \rightarrow (1)$

$-4.44\,\phi_1 + (4.44 - 0.25\,\omega_n^2)\,\phi_2 = 0 \rightarrow (2)$.

For node 1 $\quad \omega_{n_1} = 1.2$

$(5.55 - 1.2^2 \times 0.5)\,\phi_1 - 4.44\,\phi_2 = 0$

$4.83\,\phi_1 - 4.44\,\phi_2 = 0$

$4.83\,\phi_1 = 4.44\,\phi_2$

$\phi_1 = 0.91\,\phi_2$.

$\phi_2 = 1.09\,\phi_1$

For Node 2, $\omega_{n_2} = 5.23$ in Eqn (2)

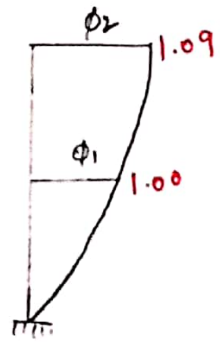
$-4.44\,\phi_1 + (4.44 - 0.25 \times 5.23^2)\,\phi_2 = 0$

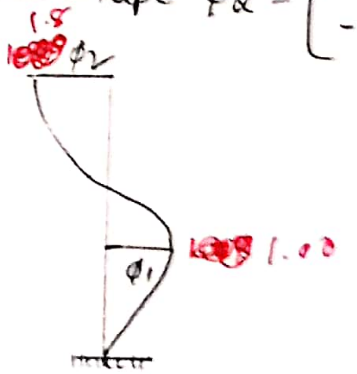
$-4.44\,\phi_1 - 2.39\,\phi_2 = 0$.

$\phi_2 = -1.8\,\phi_1$

For $\omega_{n_1} = 1.2$ mode shape $\phi_1 = \begin{bmatrix} 1.00 \\ 1.09 \end{bmatrix}$ $\begin{bmatrix} \phi_{11} \\ \phi_{12} \end{bmatrix}$

For $\omega_{n_2} = 5.23$ mode shape $\phi_2 = \begin{bmatrix} 1.00 \\ -1.8 \end{bmatrix}$ $\begin{bmatrix} \phi_{21} \\ \phi_{22} \end{bmatrix}$

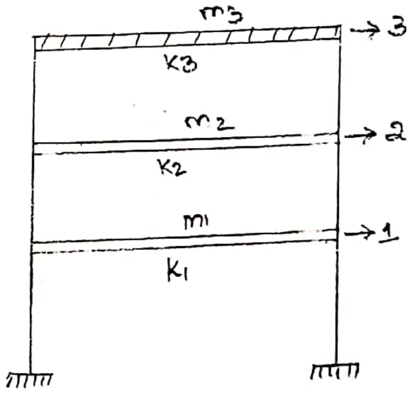




2. For a 3 Storey Sha

$$[\phi_n]^T[m][\phi] = [1.00 \quad 1.09]\begin{bmatrix} 500\times10^3 & 0 \\ 0 & 250\times10^3 \end{bmatrix}\begin{bmatrix} 1.00 \\ -1.80 \end{bmatrix} = 0$$

2. For a 3 Storey Shear building shown in fig (a) natural frequency, Natural Periods & plot mode Shapes.



$K_1 = 40\times10^6 \, N/m$

$K_2 = K_3 = 100\times10^6 \, N/m$

$m_1 = 110\times10^3 \, N\cdot s^2/m \simeq kg.$

$m_2 = 160\times10^3 \, N\cdot s^2/m$

$m_3 = 30\times10^3 \, N\cdot s^2/m$

$$m = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} = \begin{bmatrix} 110\times10^3 & 0 & 0 \\ 0 & 160\times10^3 & 0 \\ 0 & 0 & 30\times10^3 \end{bmatrix}$$

$$K = \begin{bmatrix} K_1+K_2 & -K_2 & 0 \\ -K_2 & K_2+K_3 & -K_3 \\ 0 & -K_3 & K_3 \end{bmatrix} = \begin{bmatrix} 140 & -100 & 0 \\ -100 & 200 & -100 \\ 0 & -100 & 100 \end{bmatrix}\times10^6$$

$$[m][\ddot{x}] + [K][x] = 0$$

$$\begin{bmatrix} 110\times10^3 & 0 & 0 \\ 0 & 160\times10^3 & 0 \\ 0 & 0 & 30\times10^3 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} 140 & -100 & 0 \\ -100 & 200 & -100 \\ 0 & -100 & 100 \end{bmatrix}\times10^6\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

This problem can be expressed as eigen value. Problem

$$[[K] - \omega_n^2[m]][\phi] = 0$$

$$\left[\begin{bmatrix}140 & -100 & 0\\ -100 & 200 & -100\\ 0 & -100 & 100\end{bmatrix}\times 10^6 - \omega_n^2\begin{bmatrix}110\times10^3 & 0 & 0\\ 0 & 160\times10^3 & 0\\ 0 & 0 & 30\times10^3\end{bmatrix}\right]\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\end{bmatrix} = 0$$

$$140\times10^6 - \omega_n^2\, 110\times10^3 \quad -100\overset{\times10^6}{\phi}_2 = 0.$$

$$-100\times10^6 + \omega_n(200^{\times10^6} - \omega_n^2 160\times10^3) - 100\phi_3 = 0.$$

$$-100^{\times10^6}\phi_2 + (100\times10^6 - 0.3\omega_n^2)\,\phi_3 = 0.$$

$[\phi] \neq 0$

$$|[K] - \omega_n^2[m]| = 0$$

let $\omega_n^2 = \lambda$.

$$= \begin{vmatrix}140 - \omega_n^2 0.11 & -100 & 0\\ -100 & 200 - \omega_n^2 0.16 & -100\\ 0 & -100 & 100 - 0.3\omega_n^2\end{vmatrix} = 0.$$

$$\left[(140 - \lambda 0.11)\times\left[\left[(200 - 0.16\lambda)\times(100 - 0.3\lambda)\right] - 10^4\right] + 100\right.$$

$$\left.\left[-100\times(100 - 0.3\lambda)\right]\right] = 0.$$

$$(140 - \lambda 0.11)\left[(2\times10^4) - 60\lambda - 16\lambda + 4.8\times10^{-3}\lambda^2\ ] - 10^4\right] +$$

$$100\left[[-10^4 - 30\lambda]\right] = 0.$$

$$[140 - 0.11\lambda]\left[(2\times10^4) - 22\lambda + 4.8\times10^{-3}\lambda^2\right] - 10^4\ ] - 100[10^4 - 3\lambda] = 0.$$

$$[140 - 0.11\lambda][1\times10^4 - 22\lambda + 4.8\times10^{-3}\lambda^2] - 100(10^4 - 3\lambda] = 0$$

$$14\times10^5 - 3080\lambda + 0.672\lambda^2 - 1100\lambda + 2.42\lambda^2 - 5.28\times10^{-4}\lambda^3 -$$

$$-10^6 + 300\lambda = 0.$$

$$-5.28\times10^{-3}x^3 + 9.032x^2 - 3080\,x + 4\times10^5 = 0$$

$x_1 = 1309.2$ $\quad x_2 = 200.67$ $\quad x_3 = 200.67$

purely

$$(\omega^2-\beta)^2-\alpha^2=0$$

$$\omega^4-2\beta\omega^2+\beta^2-\alpha^2=0$$

$$\omega^4-2\beta\omega^2+(\beta^2-\alpha^2)=0$$

$$\omega^2=\frac{2\beta\pm\sqrt{4\beta^2-4(\beta^2-\alpha^2)}}{2}$$

$$=\frac{2\beta\pm\sqrt{4\beta^2-4\beta^2+4\alpha^2}}{2}$$

$$=\frac{2\beta\pm2\alpha}{2}$$

$\omega^2=\beta\pm\alpha$ (roots)

The ω^2 has 2 roots, sub β & α in roots

$$\omega_1^2=\beta+\alpha=\frac{K_1+2K_2}{m}$$

$$\omega_2^2=\beta-\alpha=\frac{K_1}{m}$$

add ω_1^2 and ω_2^2 to get β.

$$\beta=\frac{\omega_1^2+\omega_2^2}{2}=\frac{4}{2}=2$$

$$\alpha=\frac{3-1}{2}=1$$

ch DOF

Let $K_1=K_2=m=1$

et of $A-\lambda$

Then $\omega_1^2=3$ (λ_1), $\omega_2^2=1$ (λ_2)

to get the

multiply λ

for the 1st eigen vector (or) to find first eigen vector, we consider 1st eigen value (ω_1^2)

$$\left[A+\omega_1^2 I\right][v_1]=0 \quad \leftarrow (A-\lambda)x=0$$

$$\left[\begin{bmatrix}-\beta & \alpha\\ \alpha & -\beta\end{bmatrix}+3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right][v_1]=0 \quad (x)$$

$$\left[\begin{bmatrix}-2 & 1\\ 1 & -2\end{bmatrix}+3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right]v_1=0$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}_{2\times2} \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix}_{2\times1} = 0$$

$$V_{11} + V_{12} = 0$$

$$V_{11} = -V_{12}$$

$V_{11} = 1$ (say)

$V_{12} = -1$

$$V_1 = \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

displacement vectors (eigenvectors) & based on this mode shapes are drawn

To get 2nd eigen vector

$$[A + \omega_2^2 I][V_2] = 0$$

$$\left[\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} = 0$$

$$-V_{21} + V_{22} = 0$$

$$V_{21} = V_{22}$$

Second eigen vector

$$V_2 = \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

V. Imp

Orthogonality of normal modes:- For each eigen value there will be eigen vector $[\omega_n^2]$ & vector (V). Eigen value represent natural frequency & Eigen vector represent mode shapes. (it is deflected shape).

It is to be noted that the modes corresponding

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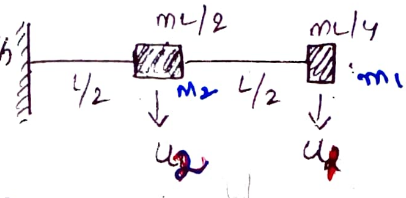
Example

Derive the Equation of motion for free vibration of cantilever beam with DOF as shown

Here m - mass per unit length



l = length of beam

EI - flexural rigidity of beam

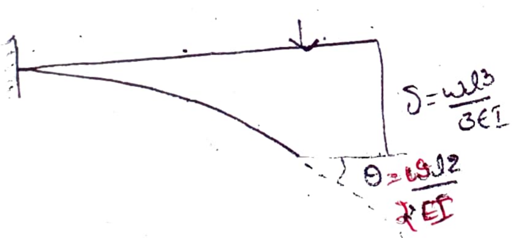
$$[M][\ddot{x}] + [K][x] = 0 \qquad [C] = 0 \qquad [F] = 0$$

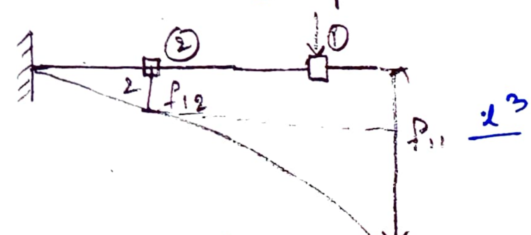
$$[m] = \begin{bmatrix} mL/4 & 0 \\ 0 & mL/2 \end{bmatrix} \qquad \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}$$

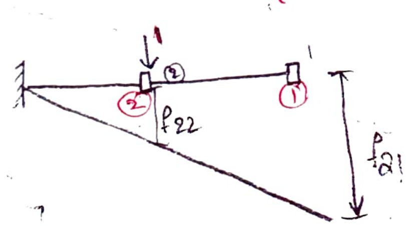
acceleration vector $[\ddot{x}] = \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix}$ $\qquad [x] = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

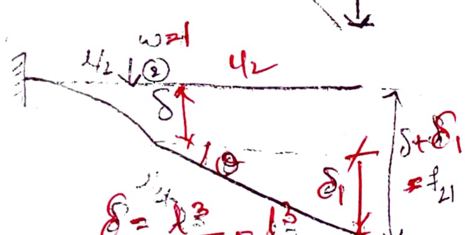
$$[x] = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \text{ amplitude vector}$$

$$\begin{cases} f_{12} = \\ f_{21} = \dfrac{5l^3}{48EI} \end{cases}$$









$$f = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}_{2\times2}$$

$$\delta = \frac{l^3}{3EI} = \frac{l^3}{24EI}$$

$$\tan\theta = \frac{\delta_1}{l/2} \approx \theta$$

$$f_{11} = \frac{wl^3}{3EI} \quad (w = 1)$$

$$\delta_1 = \left(\frac{l}{2}\right)\theta$$

$$= \frac{l^3}{3EI}$$

$$\delta_1 = \left(\frac{l}{2}\right)\left(\frac{wl^2}{2EI}\right) \rightarrow l/2$$

$$\delta_1 = \frac{l^3}{16EI}$$

$$f_{21} = \delta + \delta_1 = \frac{l^3}{24EI} + \frac{l^3}{16EI}$$

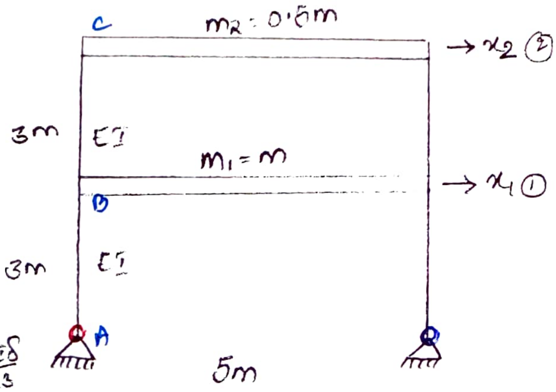
2015

Exam

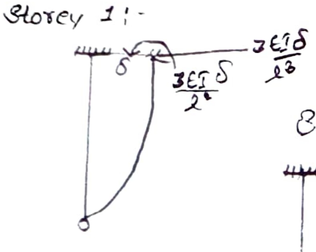
Compute the natural frequency & modes for the given shear frame & prove the orthogonality of modes

$EI = 5\times10^6\ Nm^2$

$m = 500\times10^3\ N\cdot S^2/m$

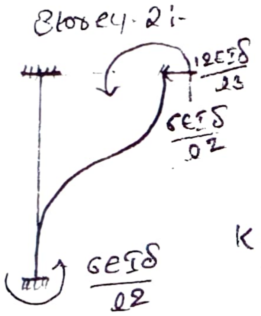


Storey 1 :-



$K = \frac{3EI}{l^3}$

Storey 2 :-



$K = \frac{12EI}{l^3}$

$K_1 = \sum$ all columns

Storey-1 :-

$K_1 = 2\left(\frac{3EI}{l^3}\right) \rightarrow K_{AB} = \frac{3EI}{l^3}$

$K_1 = 2\times\frac{3\times5\times10^6}{3^3} = 1.1\times10^6\ N/m$

Storey-2 :-

$K_2 = 2\left(\frac{12EI}{l^3}\right)$ $\quad K_{BC} = \frac{12EI}{l^3}$

$K_2 = 2\times\frac{12\times5\times10^6}{3^3} = 4.4\times10^6\ N/m$

$$[K] = \begin{bmatrix} K_1+K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} = \begin{bmatrix} 5.55 & -4.44 \\ -4.44 & 4.44 \end{bmatrix}\times10^6\ N/m$$

$$[m] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} = \begin{bmatrix} 500\times10^3 & 0 \\ 0 & 250\times10^3 \end{bmatrix}\ \frac{NS^2}{m}$$

$$[m]\ddot{x} + [K]x = 0$$

Practical Vibration Analysis (Numerical Methods)

Analytical Method / Graphical Method / Numerical Method

There are two numerical methods based on which practical vibration analysis is carried out using which fundamental natural frequency & modal vector of a vibratory system having multi-degrees of freedom are found. Both the methods are based on iterative improvement but fundamentally different in concept.

Numerical methods.

1. Stodola method
2. Holzer method
3. Stodola's method

* The disadvantage of Stodola method is only fundamental natural frequency & modal vector of vibration is found.

In Stodola method an initial assumption is made about vibration mode shape & it is adjusted iteratively till an adequate approximation of the true mode shape is obtained. The frequency of vibration is then determined from the Eqn. of motion. This method is based on finding inertia forces & deflections at various points using flexibility influence coefficients.

Holzer method

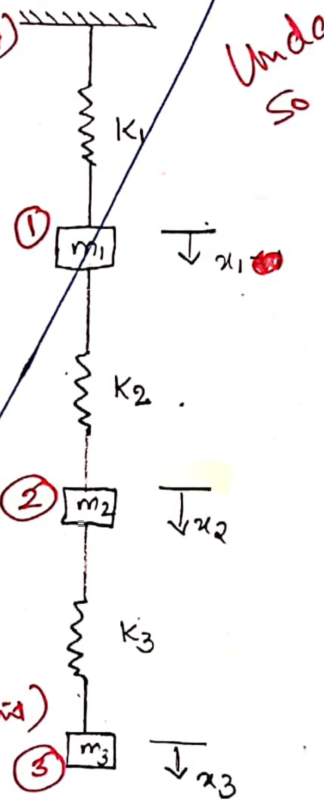
In Holzer method an initial assumption is made about vibration frequency & it is adjusted iteratively until the boundary conditions are satisfied.

The mode shape is obtained in the process of satisfying boundary condition.

1. Obtain the stiffness influence co-efficient for given system

$$K = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}_{3\times 3}$$

$K = \frac{f}{\delta\,(\text{unit})}$



Undamped so $C=0$

equivalent

$$K = \begin{bmatrix} K_1+K_2 & -K_2 & 0 \\ -K_2 & K_2+K_3 & -K_3 \\ 0 & -K_3 & K_3 \end{bmatrix}$$

flexibility co-efficient matrix

$$\alpha = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix}_{3\times 3}$$

$\alpha = \frac{\delta}{F\,(\text{unit})}$

$K = \frac{P}{x}$ $\quad \alpha = \frac{x}{P}$

$K = \frac{1}{x}$ $\quad$ for $P=1$, $\alpha = x$

$x = \frac{1}{K}$

Apply force @ 1 find disp at 1, 2, 3

$\alpha_{11} = x_1 = 1/K_1$

$\alpha_{21} = x_1 = 1/K_1$

$\alpha_{31} = x_1 = 1/K_1$

force @ 2 find disp @ 1, 2, 3

$\alpha_{12} = x_1 = 1/K_1$

$\alpha_{22} = x_1 + x_2 = \frac{1}{K_1} + \frac{1}{K_2}$

$\alpha_{32} = x_1 + x_2 = \frac{1}{K_1} + \frac{1}{K_2}$

Apply force @ 3 find disp @ 1, 2, 3

$\alpha_{13} = x_1 = \frac{1}{K_1}$

$\alpha_{23} = x_1 + x_2 = \frac{1}{K_1} + \frac{1}{K_2}$

$\alpha_{33} = x_1 + x_2 + x_3 = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$

$$\alpha = \begin{bmatrix} \frac{1}{K_1} & \frac{1}{K_1} & \frac{1}{K_1} \\ \frac{1}{K_1} & \frac{1}{K_1}+\frac{1}{K_2} & \frac{1}{K_1}+\frac{1}{K_2} \\ \frac{1}{K_1} & \frac{1}{K_1}+\frac{1}{K_2} & \frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3} \end{bmatrix}$$

$K = [\alpha]^{-1}$

$$[\alpha] = [K]^{-1}$$

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Procedure Steps - Stodola method

1. Assume model vector of system [mode shape]

Assume displacement or modal vector (at each DOF)

Ex:- $\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$

2. Find out Inertia forces of system at each mas

$a = x \cdot \omega^2$

$f_1 = m_1 a_1 = m_1 x_1 \omega^2$

$f_2 = m_2 a_2 = m_2 x_2 \omega^2$

$f_3 = m_3 a_3 = m_3 x_3 \omega^2$

3. Find new deflection vector using flexibility influen co-efficient $[x]_{3\times1} = [\alpha]_{3\times3} [P]_{3\times1}$

$\alpha = \frac{\delta}{P} = \frac{x}{P}$

$x = P\alpha$ (to displacement)

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix}_{3\times1} = \begin{Bmatrix} f_1\alpha_{11} + f_2\alpha_{12} + f_3\alpha_{13} \\ f_1\alpha_{21} + f_2\alpha_{22} + f_3\alpha_{23} \\ f_1\alpha_{31} + f_2\alpha_{32} + f_3\alpha_{33} \end{Bmatrix}$$

$P = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$

flexibility coefficients $\alpha = \begin{bmatrix} \alpha_{11} & \cdots \\ \vdots & \ddots \end{bmatrix}_{3\times3}$

$\alpha = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix}_{3\times}$

$$[x] = [\alpha]_{3\times3}[P]_{3\times1}$$

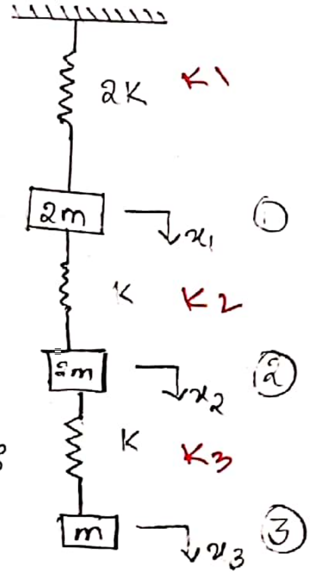
4. If the assumed model vector is equal to mod vector obtained in step 3 then the solution is obtained. The natural frequency can be obtained as follows.

if $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{Bmatrix} x_1' \\ x_2' \\ x_3' \end{Bmatrix}$ stop iterations then the natural frequency is obtained as

$$x_1' = f_1\alpha_{11} + f_2\alpha_{12} + f_3\alpha_{13}$$

5. If assumed mode vector is not equal to modal vector obtained in step-3 the continue iteration.

Example Find fundamental natural frequency & modal vector of given vibratory system using stodola method.



Step-1 :- 1st Iteration

Assume model vector of system

$$[u_1] = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

Step-2 :-

Find inertia force at each mass

$f_1 = m_1 x_1 \omega^2 = 2m\omega^2$ $\quad x_1 = 1$

$f_2 = m_2 x_2 \omega^2 = 2m\omega^2$ $\quad x_2 = 1$ } assumed

$f_3 = m_3 x_3 \omega^2 = m\omega^2$ $\quad x_3 = 1$

$m_1 = 2m$
$m_2 = 2m$
$m_3 = m$

Step-3 :- find new deflection vector using flexibility influence co-efficient

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{Bmatrix} f_1\alpha_{11} + f_2\alpha_{12} + f_3\alpha_{13} \\ f_1\alpha_{21} + f_2\alpha_{22} + f_3\alpha_{23} \\ f_1\alpha_{31} + f_2\alpha_{32} + f_3\alpha_{33} \end{Bmatrix}$$

$$\alpha = \begin{bmatrix} \frac{1}{K_1} & \frac{1}{K_1} & \frac{1}{K_1} \\ \frac{1}{K_1} & \frac{1}{K_1}+\frac{1}{K_2} & \frac{1}{K_1}+\frac{1}{K_2} \\ \frac{1}{K_1} & \frac{1}{K_1}+\frac{1}{K_2} & \frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3} \end{bmatrix}$$

$K_1 = 2K$

$K_2 = K$

$K_3 = K$

$$\alpha = \begin{bmatrix} \frac{1}{2K} & \frac{1}{2K} & \frac{1}{2K} \\ \frac{1}{2K} & \frac{3}{2K} & \frac{3}{2K} \\ \frac{1}{2K} & \frac{3}{2K} & \frac{5}{2K} \end{bmatrix} \qquad \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}$$

$$x_1' = f_1\alpha_{11} + f_2\alpha_{12} + f_3\alpha_{13}$$

$$= 2m\omega^2 \times \frac{1}{2K} + 2m\omega^2 \times \frac{1}{2K} + m\omega^2 \times \frac{1}{2K}$$

$$x_1' = \frac{5}{2K} m\omega^2$$

$$x_2' = f_1\alpha_{21} + f_2\alpha_{22} + f_3\alpha_{23}$$

$$= 2m\omega^2 \times \frac{1}{2K} + 2m\omega^2 \times \frac{3}{2K} + m\omega^2 \times \frac{3}{2K}$$

$$x_2' = \frac{11m\omega^2}{2K}$$

$$x_3' = f_1\alpha_{31} + f_2\alpha_{32} + f_3\alpha_{33}$$

$$= 2m\omega^2 \times \frac{1}{2K} + 2m\omega^2 \times \frac{3}{2K} + \frac{5}{2K} \times m\omega^2$$

$$= \frac{13}{2K} m\omega^2$$

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \frac{m\omega^2}{2K} \begin{bmatrix} 5 \\ 11 \\ 13 \end{bmatrix}$$

New deflection value

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \frac{5m\omega^2}{2K}\begin{bmatrix} 1 \\ 2.2 \\ 2.6 \end{bmatrix} = k[u_2] \qquad k = \frac{5m\omega^2}{2K}$$

K – Some constant

$[x] = k[u_2]$

$[u_2] \neq [u_1]$ ✓

2nd Iteration :-

The initial vector will be $\begin{Bmatrix} 1 \\ 2.2 \\ 2.6 \end{Bmatrix}$

$x_1 = 1$

$x_2 = 2.2$

$x_3 = 2.6$

Step-2 :- $f_1 = m_1 x_1 \omega^2 = 2m\omega^2$

$f_2 = m_2 x_2 \omega^2 = 4.4m\omega^2$

$f_3 = m_3 x_3 \omega^2 = 2.6m\omega^2$

Step-3 :-

$$\begin{bmatrix} x_1'' \\ x_2'' \\ x_3'' \end{bmatrix} = \begin{bmatrix} f_1\alpha_{11} + f_2\alpha_{12} + f_3\alpha_{13} \\ f_1\alpha_{21} + f_2\alpha_{22} + f_3\alpha_{23} \\ f_1\alpha_{31} + f_2\alpha_{32} + f_3\alpha_{33} \end{bmatrix}$$

$$x_1'' = 2m\omega^2 \times \frac{1}{2K} + 4.4m\omega^2 \times \frac{1}{2K} + 2.6m\omega^2 \times \frac{1}{2K}$$

$$x_1'' = \frac{9m\omega^2}{2K}$$

$$x_2'' = 2m\omega^2 \times \frac{1}{2K} + 4.4m\omega^2 \times \frac{3}{2K} + 2.6m\omega^2 \times \frac{3}{2K}$$

$$= \frac{23m\omega^2}{2K}$$

$$x_3'' = 2m\omega^2 \times \frac{1}{2K} + 4.4m\omega^2 \times \frac{3}{2K} + 2.6m\omega^2 \times \frac{5}{2K}$$

$$= \frac{141m\omega^2}{10K}$$

$$\begin{bmatrix} x_1'' \\ x_2'' \\ x_3'' \end{bmatrix} = \frac{9m\omega^2}{2K}\begin{bmatrix} 1 \\ 2.55 \\ 3.13 \end{bmatrix} = k[u_3] \qquad [u_2] \neq [u_3]$$ ✓

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3rd Iteration:-

initial vector will be $\begin{Bmatrix} 1 \\ 2.55 \\ 3.13 \end{Bmatrix}$

2. $f_1 = m_1 x_1 \omega^2 = 2m \times 1 \times \omega^2 = 2m\omega^2$

$f_2 = m_2 x_2 \omega^2 = 5.1 m\omega^2$

$f_3 = m_3 x_3 \omega^2 = 3.13 m\omega^2$

3. $x_1''' = f_1 \alpha_{11} + f_2 \alpha_{12} + f_3 \alpha_{13}$

$x_1''' = 2m\omega^2 \times \frac{1}{2k} + 5.1 m\omega^2 \times \frac{1}{2k} + 3.13 \times \frac{1}{2k}$

$x_1''' = 5.115 \frac{m\omega^2}{k}$

$x_2''' = 2m\omega^2 \times \frac{1}{2k} + 5.1 m\omega^2 \times \frac{3}{2k} + 3.13 \times \frac{3}{2k} m\omega^2$

$= 16.475$

$$\begin{bmatrix} x_1''' \\ x_2''' \\ x_3''' \end{bmatrix} = \frac{m\omega^2}{k} \begin{bmatrix} 5.115 \\ 13.345 \\ 16.475 \end{bmatrix}$$

$$= 5.115 \frac{m\omega^2}{k} \begin{bmatrix} 1 \\ 2.6 \\ 3.22 \end{bmatrix} = [u_4]\, K$$

fundamental mode shape

eigen value or frequency

$K = \frac{P}{\delta}$

$[x] = k \times [u_4]$

$x' = \lambda [x]$ → amplitude

$[u_3] \simeq [u_4]$ ✓

WKT $[x] = [\,I\,][u]$

(x and u are both modal values or displacement values.)

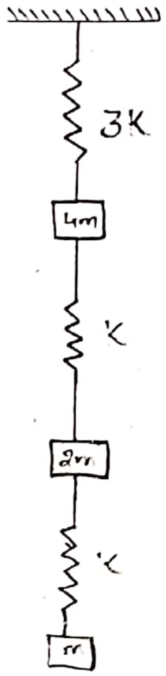
$\therefore 5.115 \frac{m\omega^2}{k} = 1$

$\omega = 0.44 \sqrt{\frac{K}{m}}$ ✓

natural frequency or fundamental frequency

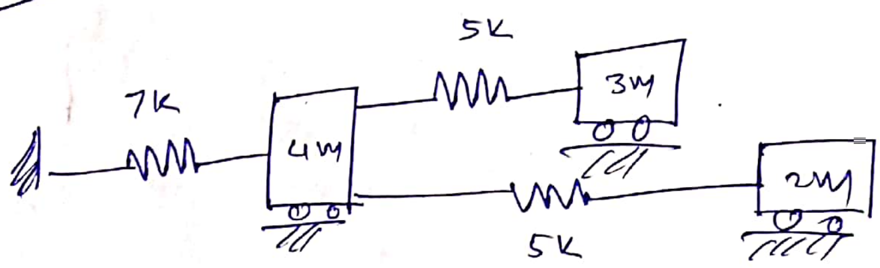
Exercise

2. For a given system find the lowest natural freq. (→ fundamental frequency) by Stodola method (carry out 2 iteration)



Stodola Method — Pg 124

Exercise Problem



Find fundamental mode of vibration and its natural frequency for the system

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* Holzer method:-

→ This method is an iterative method to find natural frequency & modal vector of vib. system having multi DOF

Step-1:-
Assume magnitude of trail frequency (ω) [Cal. Inertia forces, storey shear, storey deformation at each storey level]

Assume frequency

Step-2:-
Assume amplitude for 1st mass

Step-3:-
Cal. amplitude of 2nd mass from Equation of motion

Step-4:- Similarly calculate amplitude of 3rd mass & so on

Step-5:- Substitute all these amplitudes in the basic Equation if the Eqn is satisfied assumed frequency is natural freq. else assume another value for 'ω'

Advantages:-
This method can be applicable to both linear & tortional system.
Similarly both semi definite & definite system.

No fixidity at support (or) elastic systems — for semi definite

fixidity at supports for definite systems, deflection at fixed pt. zero.

{ Inertia forces (linear systems)
{ Inertia torques (torsional system)

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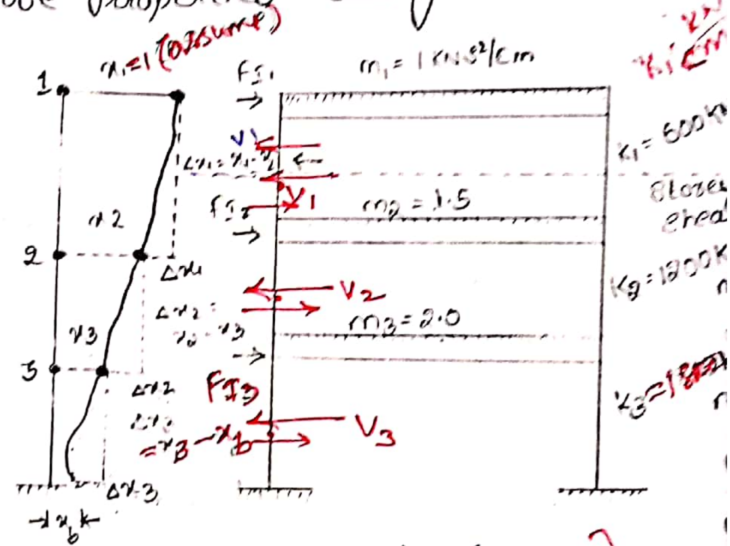
Sample

Demonstrate the holzer method to determine the fundamental frequency & mode shapes of 3 storey shear building whose properties are given below



Here $K = \frac{F}{x}$

$F = K \cdot x$
↓
Spring force

$\omega = \sqrt{\frac{k}{m}}$

$k = \omega^2 m$

$F_I = \omega^2 m x$
↓
Inertia force

Note:

- x_b – base displacement (flexible base) ✓
- If the assumed frequency is correct $x_b = 0$
- if not correct $x_b \neq 0$
- if x_b is +ve then increase 'ω' value
- if x_b is −ve then decrease 'ω' value.

Trial – 1 :–

$F_I = \omega^2 m x$

(I) 1. Assume fundamental freq: $\omega^2 = 100$.

Further assume top displacement $x_1 = 1$

~~No. of~~ Number the masses from top to bottom

(a) Then inertia force at ~~1st~~ storey level ~~Top storey~~

$$F_{I_1} = \omega^2 m_1 x_1$$

$$= 100 \times 1 \times 1 = 100\,kN$$

(b) Top storey shear $V_1 = F_{I_1} = 100$ $V_1 = 100$

(c) Top storey deformation $\Delta x_1 = x_1 - x_2$ $F_{I_1} = V_1$ above the line

$$\Delta x_1 = \frac{F_{I_1}}{k_1} = \frac{V_1}{k_1} = \frac{100}{600} = 0.167\,cm$$

kN/m

$K = \frac{P}{\delta} = \frac{F}{x}$

$x = \frac{F}{K}$

(II) **Displacement of Second Storey** $x_2 = x_1 - \Delta x_1$

$= 1 - 0.167$

$= 0.833\ cm$

(a) **Inertia force @ 2nd Storey level**

$F_{I_2} = \omega^2 m_2 x_2 = 100 \times 1.5 \times 0.833$

$F_{I_2} = 124.95 \simeq 125\ KN$

(b) **Second Storey Shear** $V_2 = F_{I_1} + F_{I_2} = 100 + 125 = 225\ KN$

(c) **Second Storey deformation**

$\Delta x_2 = \frac{V_2}{K_2} = \frac{225}{1200} = 0.187\ cm$

(III) **displacement @ 3rd Storey** $\Rightarrow x_3 = x_2 - \Delta x_2$.

$= 0.833 - 0.187$

$= 0.646\ cm$

(a) **Inertia force @ 3rd Storey** $F_{I_3} = \omega^2 m_3 x_3 = 100 \times 2 \times 0.646 = 129\ KN$

(b) **3rd Storey Shear** $V_3 = 100 + 125 + 129$

$= 354\ KN.$

(c) **3rd Storey deformation.**

$\Delta x_3 = \frac{V_3}{K_3} = \frac{354}{1800} = 0.197\ cm$

(IV) **displacement** $x_b = x_3 - \Delta x_3$

$= 0.646 - 0.197$

$x_b = 0.449\ cm \neq 0$

Here $x_b \neq 0$ $\cdot x_b \rightarrow +ve$

So increase the value of ω

$0.5 \simeq \omega_1^2 + \omega^2$

As x_b reduces to half then ω^2 should be doubled.

Trial-2:-

1. Assume fundamental frequency $= \omega^2 = 200$

I Top 1st Storey

(a) Inertia force $F_{I_1} = \omega^2 m_1 x_1$

$= 200 \times 1 \times 1 = 200\,kN$

(b) Top Storey Shear $V_1 = F_{I_1}$

$= 200\,KN$

(c) Top Storey deformation $\Delta x_1 = x_1 - x_2$

$$\Delta x_1 = \frac{F_{I_1}}{K_1} = \frac{V_1}{K_1} = \frac{200}{600} = 0.333$$

$x_2 = x_1 - \Delta x_1$

$= 1 - 0.333 = 0.667.$

II 2nd Storey

(a) Inertia force $F_{I_2} = \omega^2 m_2 x_2$

$= 200 \times 1.5 \times 0.667$

$= 200\,KN$

(b) Second Storey Shear $V_2 = F_{I_1} + F_{I_2} = 200 + 200$

$V_2 = 400\,KN$

(c) Second Storey deformation

$$\Delta x_2 = \frac{F_{I_2}}{K_2} = \frac{400}{1200} = 0.333.$$

displacement @ 3rd Storey $\cdot\ x_3 = x_2 - \Delta x_2$

$= 0.667 - 0.333$

$= 0.334.$

III Ist 3rd Storey

(a) Inertia force $F_{I_3} = \omega^2 m_3 x_3$

$= 200 \times 2 \times 0.334 = 133.6.$

I sr

(b) 3rd Storey Shear $= V_3 = F_{I_1} + F_{I_2} + F_{I_3}$

$= 533 \text{ KN}$

I sr

(c) 3rd Storey deformation

$$\Delta x_3 = \frac{F_{I_3}}{K_3} = \frac{533}{1800} = 0.296.$$

(IV) displacement @ 3rd I sr

$$x_b = x_3 - \Delta x_3.$$

$x_b = 0.038 \neq 0$ (increase ω value)

Because x_b is +ve

Trial-3:-

→ fundamental frequency $= \omega^2 = 209$

(I) 1st Storey

(a) Inertia force $F_{I_1} = \omega^2 m_1 x_1$

$= 209 \text{ KN}$

$x_1 = 1$

(b) Shear $= V_1 = F_{I_1} = 209 \text{ KN}.$

(c) Storey deformation $\Delta x_1 = 0.348 \text{ cm}$

displacement $x_2 = 0.652.$

(II) 2nd Storey

(a) Inertia force. $F_{I_2} = \omega^2 m_2 x_2$

$= 204.4 \text{ KN}.$

(b) Storey Shear. $V_2 = F_{I_1} + F_{I_2}$

$= 209 + 204.4$

$= 413.4 \text{ KN}$

(c) Storey deformation

$$\Delta x_2 = \frac{F_{I_2}}{K_2} = \frac{413.4}{1200} = 0.3445$$

se ω value)

displacement $x_3 = x_2 - \Delta x_2$.

$x_3 = 0.307$

(III) 3rd storey

(a) Inertia force $FI_3 = \omega^2 m_3 x_3$

$= 209 \times 2 \times 0.307$

$= 128.32$

(b) Storey shear $V_3 = FI_1 + FI_2 + FI_3$

$= 541.72$.

(c) Storey deformation $= \Delta x_3 = \dfrac{FI_3}{K_3} = \dfrac{541.72}{1800}$

$= 0.301$

(IV) displacement $x_b = x_3 - \Delta x_3$

$= 0.307 - 0.301$

$x_b = 0.0065 \simeq 0$. ✓

fundamental or natural frequency

$\omega^2 = 209$ KN

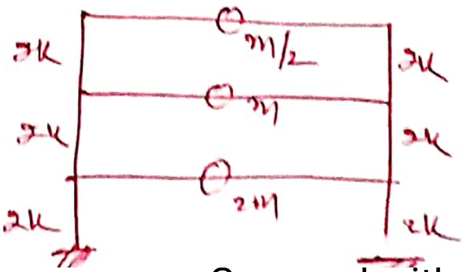
$\omega = 14.45$ rad/sec.

Mode shape - $\begin{bmatrix} 1 \\ 0.652 \\ 0.307 \end{bmatrix}$ → x_1, x_2, x_3.

Assignment: For the multistorey building shown. Obtain frequencies and modes of vibration using Stodola and Holzer method. Assume

$m = 5 \times 10^4$ kg

$k = 5 \times 10^4$ kN/m



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Practical Vibration Analysis (Numerical Methods)

There are two numerical methods based on which practical vibration analysis is carried out using which fundamental natural frequency & modal vector of a vibratory system having multi-degrees of freedom are found. Both the methods are based on iterative improvement but fundamentally different in concept.

Numerical methods

1. Stodola method
2. Holzer method
3. [illegible] method

* The disadvantage of Stodola method is only fundamental natural frequency & modal vector of vibration is found.

In Stodola method an initial assumption is made about vibration mode shape & it is adjusted iteratively till an adequate approximation of the true mode shape is obtained. The frequency of vibration is then determined from the Eqn. of motion. This method is based on finding inertia forces & deflections at various points using flexibility influence coefficient.

Holzer method

In Holzer method an initial assumption is made about vibration frequency & it is adjusted iteratively until the boundary conditions are satisfied.

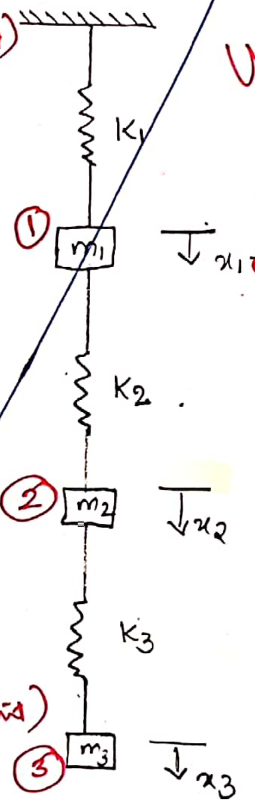
The mode shape is obtained in the process of satisfying boundary condition.

1. obtain the stiffness influence co-efficient for given system

$$K = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}_{3\times3}$$

$K = \dfrac{f}{\delta\,(\text{unit})}$

Undamped so $C = 0$



equivalent

$$K = \begin{bmatrix} K_1+K_2 & -K_2 & 0 \\ -K_2 & K_2+K_3 & -K_3 \\ 0 & -K_3 & K_3 \end{bmatrix}$$

flexibility co-efficient matrix

$$\alpha = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix}_{3\times3}$$

$\alpha = \dfrac{\delta}{F\,(\text{unit})}$

$K = \dfrac{P}{x}$ $\quad \alpha = \dfrac{x}{P}$

$K = \dfrac{1}{x}$ $\quad$ for $P = 1$, $\alpha = x$

$x = \dfrac{1}{K}$

Apply force @ 1 find disp at 1, 2, 3

$\alpha_{11} = x_1 = 1/K_1$

$\alpha_{21} = x_1 = 1/K_1$

$\alpha_{31} = x_1 = 1/K_1$

force @ 2 find disp @ 1, 2, 3

$\alpha_{12} = x_1 = 1/K_1$

$\alpha_{22} = x_1 + x_2 = \dfrac{1}{K_1} + \dfrac{1}{K_2}$

$\alpha_{32} = x_1 + x_2 = \dfrac{1}{K_1} + \dfrac{1}{K_2}$

Apply force @ 3 find disp @ 1, 2, 3

$\alpha_{13} = x_1 = \dfrac{1}{K_1}$

$\alpha_{23} = x_1 + x_2 = \dfrac{1}{K_1} + \dfrac{1}{K_2}$

$\alpha_{33} = x_1 + x_2 + x_3 = \dfrac{1}{K_1} + \dfrac{1}{K_2} + \dfrac{1}{K_3}$

$$\alpha = \begin{bmatrix} \frac{1}{K_1} & \frac{1}{K_1} & \frac{1}{K_1} \\ \frac{1}{K_1} & \frac{1}{K_1}+\frac{1}{K_2} & \frac{1}{K_1}+\frac{1}{K_2} \\ \frac{1}{K_1} & \frac{1}{K_1}+\frac{1}{K_2} & \frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3} \end{bmatrix}$$

$K = [\alpha]^{-1}$

$[\alpha] = [K]^{-1}$

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Procedure Steps – Stodola method

1. Assume model vector of system [mode shape]

Assume displacement or modal vector (at each DOF)

Ex:- $\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$

2. Find out Inertia forces of system at each mas

$a = x \cdot \omega^2$

$f_1 = m_1 a_1 = m_1 x_1 \omega^2$

$f_2 = m_2 a_2 = m_2 x_2 \omega^2$

$f_3 = m_3 a_3 = m_3 x_3 \omega^2$

3. Find new deflection vector using flexibility influen co-efficient $[x]_{3\times1} = [\alpha]_{3\times3}[P]_{3\times1}$

$\alpha = \frac{\delta}{P} = \frac{x}{P}$

$x = P\alpha$ to displacement

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix}_{3\times1} = \begin{Bmatrix} f_1\alpha_{11} + f_2\alpha_{12} + f_3\alpha_{13} \\ f_1\alpha_{21} + f_2\alpha_{22} + f_3\alpha_{23} \\ f_1\alpha_{31} + f_2\alpha_{32} + f_3\alpha_{33} \end{Bmatrix}$$

$P = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$

$\alpha = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix}_{3\times}$

flexibility coefficients $\alpha = \begin{bmatrix} \alpha_{11} & \cdots \\ \vdots & \ddots \end{bmatrix}_{3\times3}$

$[x] = [\alpha]_{3\times3}[P]_{3\times1}$

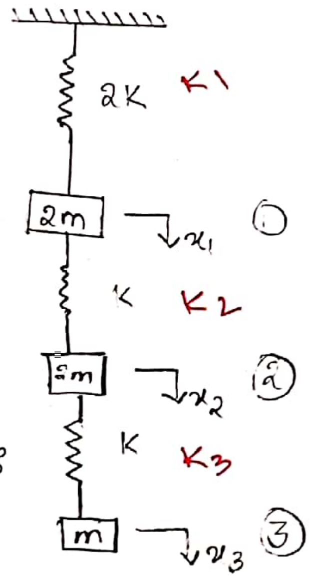
4. If the assumed model vector is equal to mod vector obtained in step 3 then the solution is obtained. The natural frequency can be obtained as follows.

if $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{Bmatrix} x_1' \\ x_2' \\ x_3' \end{Bmatrix}$ stop iterations then the natural frequency is obtained as

$$x_1' = f_1 \alpha_{11} + f_2 \alpha_{12} + f_3 \alpha_{13}$$

5. If assumed mode vector is not equal to modal vector obtained in step-3 the continue iteration.

Example Find fundamental natural frequency & modal vector of given vibratory system using stodola method.



Step-1 :- 1st Iteration

Assume model vector of system

$$[u_1] = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

Step-2 :-

Find inertia force at each mass

$f_1 = m_1 x_1 \omega^2 = 2m\omega^2$ $x_1 = 1$

$f_2 = m_2 x_2 \omega^2 = 2m\omega^2$ $x_2 = 1$ } assumed

$f_3 = m_3 x_3 \omega^2 = m\omega^2$ $x_3 = 1$

$m_1 = 2m$
$m_2 = 2m$
$m_3 = m$

Step-3 :- find new deflection vector using flexibility influence co-efficient

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{Bmatrix} f_1 \alpha_{11} + f_2 \alpha_{12} + f_3 \alpha_{13} \\ f_1 \alpha_{21} + f_2 \alpha_{22} + f_3 \alpha_{23} \\ f_1 \alpha_{31} + f_2 \alpha_{32} + f_3 \alpha_{33} \end{Bmatrix}$$

$$\alpha = \begin{bmatrix} \frac{1}{K_1} & \frac{1}{K_1} & \frac{1}{K_1} \\ \frac{1}{K_1} & \frac{1}{K_1}+\frac{1}{K_2} & \frac{1}{K_1}+\frac{1}{K_2} \\ \frac{1}{K_1} & \frac{1}{K_1}+\frac{1}{K_2} & \frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3} \end{bmatrix}$$

$K_1 = 2K$

$K_2 = K$

$K_3 = K$

$$\alpha = \begin{bmatrix} \frac{1}{2K} & \frac{1}{2K} & \frac{1}{2K} \\ \frac{1}{2K} & \frac{3}{2K} & \frac{3}{2K} \\ \frac{1}{2K} & \frac{3}{2K} & \frac{5}{2K} \end{bmatrix} \qquad \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix}$$

$$x_1' = f_1\alpha_{11} + f_2\alpha_{12} + f_3\alpha_{13}$$

$$= 2m\omega^2 \times \frac{1}{2K} + 2m\omega^2 \times \frac{1}{2K} + m\omega^2 \times \frac{1}{2K}$$

$$x_1' = \frac{5}{2K} m\omega^2$$

$$x_2' = f_1\alpha_{21} + f_2\alpha_{22} + f_3\alpha_{23}$$

$$= 2m\omega^2 \times \frac{1}{2K} + 2m\omega^2 \times \frac{3}{2K} + m\omega^2 \times \frac{3}{2K}$$

$$x_2' = \frac{11m\omega^2}{2K}$$

$$x_3' = f_1\alpha_{31} + f_2\alpha_{32} + f_3\alpha_{33}$$

$$= 2m\omega^2 \times \frac{1}{2K} + 2m\omega^2 \times \frac{3}{2K} + \frac{5}{2K} \times m\omega^2$$

$$= \frac{13}{2K} m\omega^2$$

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \frac{m\omega^2}{2K} \begin{bmatrix} 5 \\ 11 \\ 13 \end{bmatrix}$$

New deflection value

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \frac{5m\omega^2}{2K}\begin{bmatrix} 1 \\ 2.2 \\ 2.6 \end{bmatrix} = k[u_2] \qquad k = \frac{5m\omega^2}{2K}$$

K – Some constant

$$[x] = k[u_2]$$

$$[u_2] \neq [u_1]$$ ✓

2nd Iteration :-

The initial vector will be $\begin{Bmatrix} 1 \\ 2.2 \\ 2.6 \end{Bmatrix}$

$x_1 = 1$

$x_2 = 2.2$

$x_3 = 2.6$

Step-2 :- $f_1 = m_1 x_1 \omega^2 = 2m\omega^2$

$f_2 = m_2 x_2 \omega^2 = 4.4m\omega^2$

$f_3 = m_3 x_3 \omega^2 = 2.6m\omega^2$

Step-3 :-

$$\begin{bmatrix} x_1'' \\ x_2'' \\ x_3'' \end{bmatrix} = \begin{bmatrix} f_1\alpha_{11} + f_2\alpha_{12} + f_3\alpha_{13} \\ f_1\alpha_{21} + f_2\alpha_{22} + f_3\alpha_{23} \\ f_1\alpha_{31} + f_2\alpha_{32} + f_3\alpha_{33} \end{bmatrix}$$

$$x_1'' = 2m\omega^2 \times \frac{1}{2K} + 4.4m\omega^2 \times \frac{1}{2K} + 2.6m\omega^2 \times \frac{1}{2K}$$

$$x_1'' = \frac{9m\omega^2}{2K}$$

$$x_2'' = 2m\omega^2 \times \frac{1}{2K} + 4.4m\omega^2 \times \frac{3}{2K} + 2.6m\omega^2 \times \frac{3}{2K}$$

$$= \frac{23m\omega^2}{2K}$$

$$x_3'' = 2m\omega^2 \times \frac{1}{2K} + 4.4m\omega^2 \times \frac{3}{2K} + 2.6m\omega^2 \times \frac{5}{2K}$$

$$= \frac{141m\omega^2}{10K}$$

$$\begin{bmatrix} x_1'' \\ x_2'' \\ x_3'' \end{bmatrix} = \frac{9m\omega^2}{2K}\begin{bmatrix} 1 \\ 2.55 \\ 3.13 \end{bmatrix} = K[u_3] \qquad [u_2] \neq [u_3]$$ ✓

3rd Iteration:-

initial vector will be $\begin{Bmatrix} 1 \\ 2.55 \\ 3.13 \end{Bmatrix}$

2. $f_1 = m_1 x_1 \omega^2 = 2m \times 1 \times \omega^2 = 2m\omega^2$

$f_2 = m_2 x_2 \omega^2 = 5.1 m\omega^2$

$f_3 = m_3 x_3 \omega^2 = 3.13 m\omega^2$

3. $x_1''' = f_1 \alpha_{11} + f_2 \alpha_{12} + f_3 \alpha_{13}$

$x_1''' = 2m\omega^2 \times \frac{1}{2k} + 5.1 m\omega^2 \times \frac{1}{2k} + 3.13 \times \frac{1}{2k}$

$x_1''' = 5.115 \frac{m\omega^2}{k}$

$x_2''' = 2m\omega^2 \times \frac{1}{2k} + 5.1 m\omega^2 \times \frac{3}{2k} + 3.13 \times \frac{3}{2k} m\omega^2$

$= 16.475$

$$\begin{bmatrix} x_1''' \\ x_2''' \\ x_3''' \end{bmatrix} = \frac{m\omega^2}{k} \begin{bmatrix} 5.115 \\ 13.345 \\ 16.475 \end{bmatrix}$$

$$= 5.115 \frac{m\omega^2}{k} \begin{bmatrix} 1 \\ 2.6 \\ 3.22 \end{bmatrix} = [u_4]\, K$$

fundamental mode shape

eigen value or frequency

$K = \frac{P}{\delta}$

$[x] = k \times [u_4]$

$x' = \lambda [x]$ → amplitude

$[u_3] \simeq [u_4]$ ✓

WKT $[x] = [1][u]$

(x and u are both modal values or displacement values.)

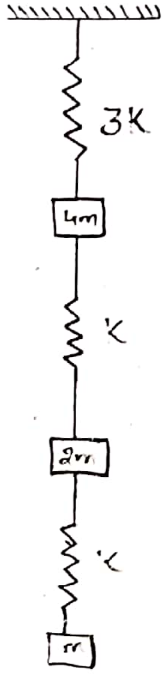
$\therefore 5.115 \frac{m\omega^2}{k} = 1$

$\omega = 0.44 \sqrt{\frac{k}{m}}$ ✓

natural frequency or fundamental frequency

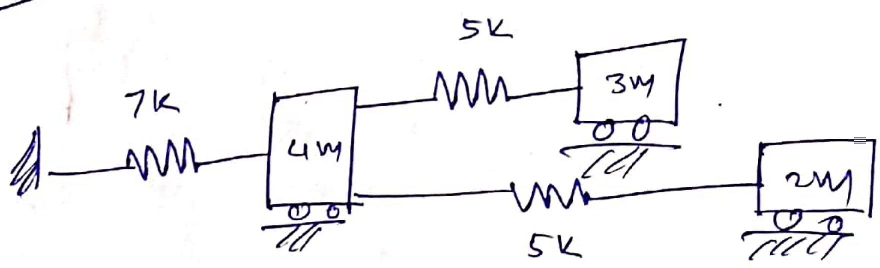
Excercise

2. For a given system find the lowest natural freq. (→ fundamental frequency) by Stodola method (carry out 2 iteration)



Stodola Method — Pg 124

Excercise Problem



Find fundamental mode of vibration and its natural frequency for the system

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* Holzer method:-

→ This method is an Iterative method to find natural frequency & modal vector of vib. system having multi DOF

Step-1:- Assume magnitude of trail frequency (ω) [Cal. Inertia forces, Storey shear, Storey deformation at each Storey level]

Assume frequency

Step-2:- Assume amplitude for 1st mass

Step-3:- Cal. amplitude of 2nd mass from Equation of motion

Step-4:- Similarly calculate amplitude of 3rd mass & so on

Step-5:- Substitute all these amplitudes in the basic Equation if the Eqn is satisfied assumed frequency is natural freq. else assume another value for 'ω'

Advantages:-

This method can be applicable to both linear & tortional system.

Similarly both Semi definite & definite system.

{ Inertia forces (linear systems)
{ Inertia torques (tortional system)

No fixidity at support (or) elastic systems

fixidity at supports for definite systems, deflection at fixed pt zero.

31/8/15

Example

Demonstrate the holzer method to determine the fundamental frequency & mode shapes of 3 storey shear building whose properties are given below

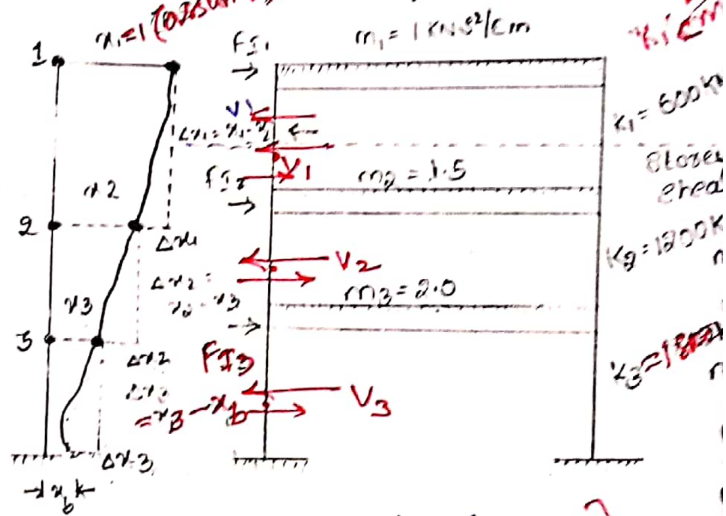
Here $K = \frac{F}{x}$

$F = K \cdot x$ → Spring force

$\omega = \sqrt{\frac{K}{m}}$

$K = \omega^2 m$

$F_I = \omega^2 m x$ → Inertia force



Note :-

- x_b – base displacement (flexible base)
- If the assumed frequency is correct $x_b = 0$
- if not correct $x_b \neq 0$
- if x_b is +ve then increase 'ω' value
- if x_b is −ve then decrease 'ω' value.

Trial – 1 :–

$f_I = \omega^2 m x$

(I) 1. Assume fundamental freq: $\omega^2 = 100$.

Further assume top displacement $x_1 = 1$

Number the masses from top to bottom

(a) Then inertia force at ~~1st~~ storey level (Top storey)

$$F_{I_1} = \omega^2 m_1 x_1$$

$$= 100 \times 1 \times 1 = 100\,kN$$

(b) Top storey shear $V_1 = F_{I_1} = 100$ $V_1 = 100$

(c) Top storey deformation $\Delta x_1 = x_1 - x_2$ $F_{I_1} = V_1$ above the line

$$\Delta x_1 = \frac{F_{I_1}}{K_1} = \frac{V_1}{K_1} = \frac{100}{600} = 0.167\,cm$$

kN/m

$K = \frac{P}{\delta} = \frac{F}{x}$

$x = \frac{F}{K}$

(II) **Displacement of Second Storey** $x_2 = x_1 - \Delta x_1$

$= 1 - 0.167$

$= 0.833\ cm$

(a) **Inertia force @ 2nd Storey level**

$F_{I_2} = \omega^2 m_2 x_2 = 100 \times 1.5 \times 0.833$

$F_{I_2} = 124.95 \simeq 125\ KN$

(b) Second Storey Shear $V_2 = F_{I_1} + F_{I_2} = 100 + 125 = 225\ KN$

(c) Second Storey deformation

$\Delta x_2 = \frac{V_2}{K_2} = \frac{225}{1200} = 0.187\ cm$

(III) displacement @ 3rd Storey $\Rightarrow x_3 = x_2 - \Delta x_2.$

1st

$= 0.833 - 0.187$

1st

$= 0.646\ cm$

(a) Inertia force @ 3rd Storey $F_{I_3} = \omega^2 m_3 x_3 = 100 \times 2 \times 0.646 = 129\ KN$

(b) 3rd Storey Shear $V_3 = 100 + 125 + 129$

1st

$= 354\ KN.$

(c) 3rd Storey deformation.

1st

$\Delta x_3 = \frac{V_3}{K_3} = \frac{354}{1800} = 0.197\ cm$

(IV) displacement $x_b = x_3 - \Delta x_3$

$= 0.646 - 0.197$

$x_b = 0.449\ cm \neq 0$

Here $x_b \neq 0$ $\cdot x_b \to +ve$

So increase the value of ω

$0.5 \simeq \omega_1^2 + \omega_2^2$

As x_b reduces to half then ω^2 should be doubled

Δx_1

167

.3 cm

kN

25 = 225 kN

7 cm

?.

:7

$\times 2 \times 0.646 =$
129 kN

m

Trial-2:-

1. Assume fundamental frequency $= \omega^2 = 200$

(I) Top (1st) storey

(a) Inertia force $F_{I_1} = \omega^2 m_1 x_1$

$= 200 \times 1 \times 1 = 200 \text{ kN}$

(b) Top storey shear $V_1 = F_{I_1}$

$= 200 \text{ KN}$

(c) Top storey deformation $\Delta x_1 = x_1 - x_2$

$$\Delta x_1 = \frac{F_{I_1}}{K_1} = \frac{V_1}{K_1} = \frac{200}{600} = 0.333$$

$x_2 = x_1 - \Delta x_1$

$= 1 - 0.333 = 0.667$

(II) 2nd storey

(a) Inertia force $F_{I_2} = \omega^2 m_2 x_2$

$= 200 \times 1.5 \times 0.667$

$= 200 \text{ KN}$

(b) Second storey shear $V_2 = F_{I_1} + F_{I_2} = 200 + 200$

$V_2 = 400 \text{ KN}$

(c) Second storey deformation

$$\Delta x_2 = \frac{F_{I_2}}{K_2} = \frac{400}{1200} = 0.333$$

displacement @ 3rd storey $\cdot x_3 = x_2 - \Delta x_2$

$= 0.667 - 0.333$

$= 0.334$.

(III) 1st 3rd storey

(a) Inertia force $F_{I_3} = \omega^2 m_3 x_3$

$= 200 \times 2 \times 0.334 = 133.6$.

(b) 3rd storey shear $= V_3 = F_{I_1} + F_{I_2} + F_{I_3}$

$= 533\,kN$

(c) 3rd storey deformation

$$\Delta x_3 = \frac{F_{I_3}}{K_3} = \frac{533}{1800} = 0.296.$$

(IV) displacement @ 3rd

$x_b = x_3 - \Delta x_3.$

$x_b = 0.038 \neq 0$ (increase ω value)

Because x_b is +ve

Trial - 3 :-

→ fundamental frequency $= \omega^2 = 209$

(I) 1st storey

(a) Inertia force $F_{I_1} = \omega^2 m_1 x_1$

$= 209\,kN$ $\quad x_1 = 1$

(b) Shear $= V_1 = F_{I_1} = 209\,KN.$

(c) storey deformation $\Delta x_1 = 0.348\,cm$

displacement $x_2 = 0.652.$

(II) 2nd storey

(a) Inertia force. $F_{I_2} = \omega^2 m_2 x_2$

$= 204.4\,kN.$

(b) storey shear. $V_2 = F_{I_1} + F_{I_2}$

$= 209 + 204.4$

$= 413.4\,KN$

(c) storey deformation

$$\Delta x_2 = \frac{F_{I_2}}{K_2} = \frac{413.4}{1200} = 0.3445$$

displacement $x_3 = x_2 - \Delta x_2$.

$x_3 = 0.307$

(III) 3rd storey

(a) Inertia force $F_{I_3} = \omega^2 m_3 x_3$

$= 209 \times 2 \times 0.307$

$= 128.32$

(b) Storey shear $V_3 = F_{I_1} + F_{I_2} + F_{I_3}$

$= 541.72$.

se ω value)

(c) Storey deformation $= \Delta x_3 = \dfrac{F_{I_3}}{K_3} = \dfrac{541.72}{1800}$

$= 0.301$

(IV) displacement $x_b = x_3 - \Delta x_3$

$= 0.307 - 0.301$

$x_b = 0.0065 \simeq 0$. ✓

Fundamental or Natural frequency

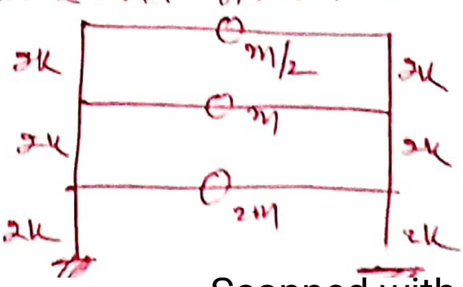
$\omega^2 = 209$ KN

$\omega = 14.45$ rad/sec.

Mode shape - $\begin{bmatrix} 1 \\ 0.652 \\ 0.307 \end{bmatrix}$ → x_1, x_2, x_3.

Assignment: For the multistorey building shown. Obtain frequencies and modes of vibration using Stodola and Holzer method.

Assume
$m = 5 \times 10^4$ kg
$k = 5 \times 10^4$ kN/m



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Derivation for governing differential equation of motion for continuous system

(OR)

Equation of motion for transverse flexural vibration of beam

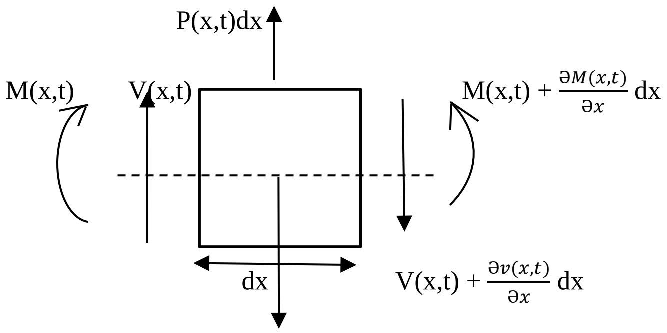


fig - 2

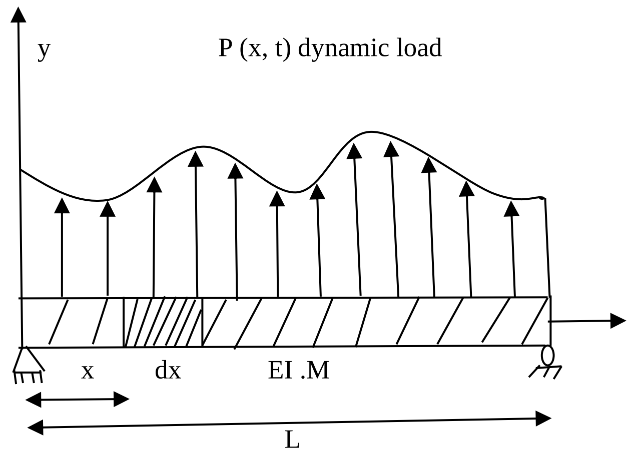
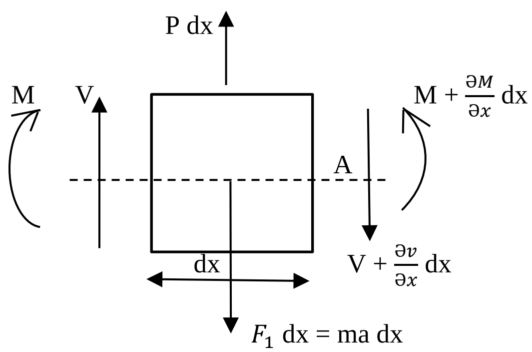


fig - 1



(Load applied and inertia forces are opposite in direction)

F_1 dx = ma dx

F_1 dx = (ma dx) $\frac{\partial^2 y}{\partial t^2}$

Consider a free body diagram of a small segment of beam as shown, whose length is dx

Shear forces : V & V+$\frac{\partial v}{\partial x}$ dx

Bending moments: M & M+ $\frac{\partial M}{\partial x}$ dx

Lateral load : P dx

Inertia force: Mdx.$\frac{\partial^2 y}{\partial t^2}$ are show in figure

Where

M = Mass per unit length

P = Load per unit length

Considering equilibrium of element along Y-axis

$\Sigma F_y = 0$

$$P\,dx + \cancel{V} = \cancel{V} + \frac{\partial v}{\partial x}\,dx + F_1\,dx \qquad \text{———} \qquad (1)$$

$$\cancel{P\,dx} = \frac{\partial v}{\partial x}\cancel{dx} + (m\cancel{dx})\left[\frac{\partial^2 y}{\partial t^2}\right] \qquad \text{———} \qquad (2)$$

$$\frac{\partial v}{\partial x} = P - m\,\frac{\partial^2 y}{\partial t^2} \qquad \text{———} \qquad (3)$$

Taking moment about point 'A' on element & droping 2nd order & common terms

$$\cancel{M} + V\,\cancel{dx} + \left[p\,\cancel{dx}.\frac{dx}{2}\right] = \left[(\cancel{m\,dx})\left(\frac{\partial^2 y}{\partial t^2}\right)\frac{dx}{2}\right] + \left[\cancel{M} + \frac{\partial M}{\partial x}\cancel{dx}\right] \quad \text{here } \frac{\partial^2 y}{\partial t^2} = (F_1 dx)\frac{dx}{2}$$

$$V\,dx = \frac{\partial M}{\partial x}\,dx \qquad \text{here } V = \frac{\partial M}{\partial x}$$

Derive both sides wrt 'x'

$$\frac{\partial v}{\partial x} = \frac{\partial^2 m}{\partial x^2} \qquad \text{———} \qquad (4)$$

From 2nd 3rd and 4th

$$\left[P - m\,\frac{\partial^2 y}{\partial t^2}\right] = \frac{\partial^2 m}{\partial x^2}$$

P is applied on the beam continuously so this is considered as damped case

We know that $\quad M = EI.\,\frac{\partial^2 y}{\partial x^2}$

$$\left[P - m.\,\frac{\partial^2 y}{\partial t^2}\right] = EI.\,\frac{\partial^4 Y}{\partial x^4}$$

$$\boxed{EI\,\frac{\partial^4 Y}{\partial x^4} + m\,\frac{\partial^2 y}{\partial t^2} = P}$$

This is governing differential equation for beam subjected to flexural vibration (damped vibration)

Un damped free vibrations of beams in flexure:

Governing differential equation for undamped free vibration beams in flexure is given by

$$EI \frac{\partial^4 Y}{\partial x^4} + m \frac{\partial^2 y}{\partial t^2} = P$$

For undamped case P = 0 (No load)

$$EI \frac{\partial^4 Y}{\partial x^4} + m \frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{——} \qquad (1)$$

The general solution for this equation is given in the form of

$$y(x, t) = Ø(x)\, Y(t) \qquad \text{——} \qquad (2)$$

Where, Ø (x) = deformed shape of the beam

Y(t) = amplitude of the vibration

Equation 1 is separated in to two terms

$EI \frac{\partial^4 Y}{\partial x^4}$ - Special term (distance based) (x)

$m \frac{\partial^2 y}{\partial t^2}$ – Temporal term (time based) (t)

so the solution-(2) is also assumed as two term
Ø (x) – special term, Y(t) – temporal term.

Equation-(1) is 4th order differential equation hence we need 4 boundary conditions to solve this which will be based on support conditions at the end.

The second term which is 2nd order differential equation need 2 boundary conditions to solve this term (Initial deflection & velocity at any point).

Inserting solution equation-(2) in governing differential equation –(1)

$$EI \frac{\partial^4(\varnothing(x)\,Y(t))}{\partial x^4} + m \frac{\partial^2(\varnothing(x)\,Y(t))}{\partial t^2} = 0$$

Divide the term by m. Y(t). Ø(x)

$$\left(\frac{EI}{m\varnothing(x)}\right) \frac{\partial^4(\varnothing(x))}{\partial x^4} = - \frac{1}{Y(t)} \frac{\partial^2(Y(t))}{\partial t^2}$$

Equate each term to some constant say ω^2

$$EI. \frac{\partial^4(\varnothing(x))}{\partial x^4} = m\, \varnothing(x).\, \omega^2 \qquad \text{——} \qquad 3(a)$$

$$\frac{\partial^2\,(Y(t))}{\partial t^2} + \omega^2\, Y(t) = 0 \qquad \text{——} \qquad 3(b)$$

$$\ddot{Y} + \omega^2\, Y(t) = 0 \qquad \text{——} \qquad 3(b)$$

Equation (3b) is similar to equation of motion for SDOF system so the solution for (3b) is in the form of

$$Y(t) = Y_0 \cos \omega t + \left(\frac{Y_0}{\omega}\right) \sin \omega t \qquad \text{——} \qquad (4)$$

Y_0 = Y at time t = 0

In order to evaluate ω use equation 3 (a) & introduce α term

$$\alpha^4 = \frac{\omega^2 m}{EI} \qquad \text{——} \qquad (5)$$

$$\omega = \sqrt{\frac{\alpha^4 EI}{m}}$$

α = frequency parameter

$\frac{EI}{m}$ = beam property

Assume a solution form

$$\varnothing(x) = Ce^{sx} \qquad \text{here } s = \frac{\partial}{\partial x}$$

Substitute this in equation (3a)

$$EI. \frac{\partial^4(Ce^{sx})}{\partial x^4} = m\, Ce^{sx}.\ \omega^2$$

$$\frac{\partial^4(Ce^{sx})}{\partial x^4} = \frac{m\,\omega^2}{EI}.Ce^{sx} \qquad \text{here } \frac{m\,\omega^2}{EI} = \alpha^4$$

$$(s^4\text{-}\alpha^4)\ Ce^{sx} = 0 \qquad \text{———} \qquad (6)$$

For a solution to be non-trivial we required (s^4-α^4) = 0 the roots of these equation are s_1 = α , s_2 = - α , s_3 = αi , s_4 = -αi

Substitute these roots in equation (6) we give solution to (3a) then the general solution is then given by 4 possible solutions which on adding up these 4 possible solutions get the differential equation for undamped free vibrations for beams as following

$$Ø(x) = C_1e^{\alpha x} + C_2e^{-\alpha x} + C_3e^{\alpha ix} + C_3e^{-\alpha ix} \qquad \text{———} \qquad (7)$$

Where, C_1, C_2, C_3, C_4 are constant of integration

Equation 7 can be expressed in terms of trigonometric hyperbolic functions by means of relations.

$$e^{\pm\theta x} = \cos h\alpha x \pm \sin h\alpha x$$

$$e^{\pm i\theta x} = \cos \alpha x \pm \sin \alpha x$$

Substitute above values in equation (7)

$$Ø(x) = a_1 \sin \alpha x + a_2 \cos \alpha x + a_3 \sin h(\alpha x) + a_4 \cos h(\alpha x) \qquad \text{———} \qquad (8)$$

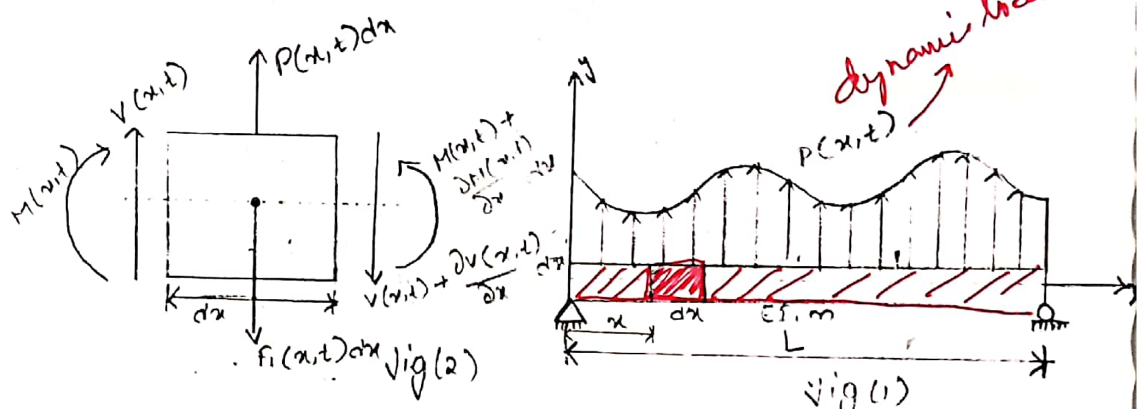
This constants a_1, a_2, a_3, a_4 can be found from boundary conditions of the ends of the beam and these 4 constants represents the shape and amplitude of the beam.

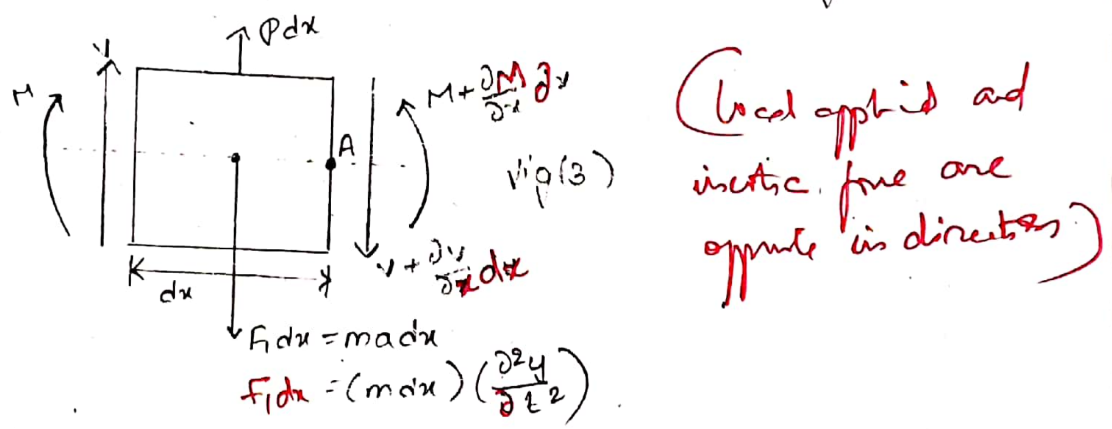
pg 132

Q4 Derivation for Governing Differential Equation of Motion for Continuous System

(or)

Equation of Motion for transverse flexural vibration of beams





Consider a free body diagram of a small segment of beam as shown, whose length is dx

Shear forces : V & $V + \frac{\partial V}{\partial x} dx$

Bending moments : M & $M + \frac{\partial M}{\partial x} dx$

Lateral load : $P\,dx$

Transverse Inertia force : $M\,dx \cdot \frac{\partial^2 y}{\partial t^2}$ are shown in figure

where M = mass per unit length

P = load per unit length

considering Equilibrium of element along y-axis

$\Sigma F_y = 0$

$$P\,dx + V = V + \frac{\partial v}{\partial x}dx + F_i\,dx \quad -(1)$$

$$P\,dx = \frac{\partial v}{\partial x}dx + (m\,dx)\left(\frac{\partial^2 y}{\partial t^2}\right) \quad -(2)$$

$$\frac{\partial v}{\partial x} = P - m\frac{\partial^2 y}{\partial t^2} \quad -(3)$$

HOT

Taking moments about Point 'A' on element & droping 2nd order & common terms

$$M + V\,dx + \left[P\,dx \cdot \frac{dx}{2}\right] = \left[(m\,dx)\left(\frac{\partial^2 y}{\partial t^2}\right)\frac{dx}{2}\right] + \left[M + \frac{\partial M}{\partial x}dx\right]$$

neglect; $(F_i\,dx)\frac{dx}{2}$

3/9/15

$$V\,dx = \frac{\partial M}{\partial x}\cdot dx \Rightarrow V = \frac{\partial M}{\partial x}$$

Derive both sides wrt 'x'

$$\frac{\partial v}{\partial x} = \frac{\partial^2 m}{\partial x^2} \quad -(4)$$

From Eqn 3 & 4.

$$\left[P - m\cdot\frac{\partial^2 y}{\partial t^2}\right] = \frac{\partial^2 m}{\partial x^2}$$

→ P is applied on the beam continuously so this is considered as damped case

W.K.T.; we know that $m = EI \cdot \frac{\partial^2 y}{\partial x^2}$

$$\left[P - m\frac{\partial^2 y}{\partial t^2}\right] = EI\cdot\frac{\partial^4 y}{\partial x^4}$$

$$\boxed{EI\frac{\partial^4 y}{\partial x^4} + m\cdot\frac{\partial^2 y}{\partial t^2} = P}$$

This is governing diff. Eqn for beam subjected to flexural vibration. (Damped vibration) 'order

$y + \frac{\partial M}{\partial x} dx$

Undamped free vibrations of beams in flexure :-

Governing differential Equation for undamped free vibration beams in flexure is given by

$$EI \cdot \frac{\partial^4 y}{\partial x^4} + m \cdot \frac{\partial^2 y}{\partial t^2} = P$$

for Undamped ~~where~~ Case $P = 0$ [No load] (Undamped)

$$\therefore EI \cdot \frac{\partial^4 y}{\partial x^4} + m \cdot \frac{\partial^2 y}{\partial t^2} = 0 \quad -(1)$$

The general solution for this Equation is given in the form of $y(x,t) = \phi(x) Y(t) \quad -(2)$

where, $\phi(x)$ = deformed shape of the beam

$Y(t)$ = amplitude of the vibration

deflection

$\phi(x)$ can be zero but $Y(t)$ cannot be zero in dynamic cases

Equation (1) is seperated into two terms

$EI \frac{\partial^4 y}{\partial x^4}$ - Spacial term → (distance based) (x)

$m \cdot \frac{\partial^2 y}{\partial t^2}$ - Temporal term → (time based) (t)

So the solution (2) is also assumed as two term

$\phi(x)$ - Spacial term, $Y(t)$ - temporal term.

Eqn (1) is 4th order differential Equation hence we need 4 boundary conditions to solve this which will be based on support conditions at end.

The second term which is 2nd order diff. Eqn needs 2 boundary conditions to solve this term i.e. [Initial deflection & velocity at any point].

Inserting Solution (Eqn (2)) in Governing differential Equation - Eqn (1)

'ed to

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$$EI \cdot \frac{\partial^4[\phi(x)\, Y(t)]}{\partial x^4} + m \frac{\partial^2[\phi(x) \cdot Y(t)]}{\partial t^2} = 0$$

divide the term by $m \cdot Y(t) \cdot \phi(x)$

$$\left(\frac{EI}{m\phi(x)}\right) \frac{\partial^4[\phi(x)]}{\partial x^4} = -\frac{1}{Y(t)} \left(\frac{\partial^2 (Y(t))}{\partial t^2}\right)$$

Equate Each terms to some constant say ω^2

$\left(\frac{EI}{m\phi(x)}\right)\left(\frac{\partial^4 \phi(x)}{\partial x^4}\right) = \omega^2$

$$EI \cdot \frac{\partial^4[\phi(x)]}{\partial x^4} = m\phi(x) \cdot \omega^2 \rightarrow 3(a)$$

$-\frac{1}{Y(t)}\left(\frac{\partial^2 y(t)}{\partial t^2}\right) = \omega^2$

$$\frac{\partial^2 [Y(t)]}{\partial t^2} + \omega^2 Y(t) = 0 \rightarrow 3(b)$$

$$\ddot{Y}(t) + \omega^2 Y(t) = 0 \rightarrow 3(b)$$

Eqn 3(b) is similar to Equation of motion for SDOF system so the solution for Eqn 3(b) is in the form of

$Y_0 = Y$ at time $t=0$

SDOF system EOM →
$$Y(t) = Y_0 \cos\omega t + \left(\frac{\dot{Y}_0}{\omega}\right) \sin\omega t \rightarrow (4)$$

In order to evaluate ω use Eqn 3(a) & introduce α term

$\omega = \sqrt{\frac{\alpha^4 EI}{m}}$

α = frequency parameter

$$\alpha^4 = \frac{\omega^2 m}{EI} \rightarrow (5)$$

Assume a solution of form

$s = \frac{\partial}{\partial x}$

$$\phi(x) = Ce^{sx}$$

$\frac{EI}{m}$ = beam property

Sub this in Eqn 3(a)

$$EI \cdot \frac{\partial^4[Ce^{sx}]}{\partial x^4} = m \cdot C e^{sx} \omega^2 \Rightarrow \frac{\partial^4 [Ce^{sx}]}{\partial x^4} = \left(\frac{m\omega^2}{EI}\right) Ce^{sx}$$

$\left(\frac{m\omega^2}{EI}\right) \rightarrow \alpha^4$ (say)

$$(s^4 - \alpha^4) Ce^{sx} = 0 \rightarrow (6) \quad \left(s = \frac{\partial}{\partial x}\right)$$

For a solution to be non-trivial we require $s^4-\alpha^4=0$. The roots of these equation are

$s_1=\alpha$, $s_2=-\alpha$, $s_3=\alpha i$, $s_4=-\alpha i$

($i=\sqrt{-1}$, $i^2=-1$)

Sub. these roots in Eqn (6) we give solution to 3(a) then the general solution is then given by 4 possible solutions which on adding up this 4 possible solutions we get the diff. Eqn for undamped free vibrations for beams as follows

$$\phi(x)=c_1e^{\alpha x}+c_2e^{-\alpha x}+c_3e^{i\alpha x}+c_4e^{-i\alpha x} \rightarrow (7)$$

where, c_1, c_2, c_3, c_4 are constant of integration.

Eqn (7) can be expressed in terms of trignometric hyperbolic functions by means of relations.

$$e^{\pm\alpha x}=\cosh\alpha x\pm\sinh\alpha x$$
$$e^{\pm i\alpha x}=\cos\alpha x\pm\sin\alpha x$$
(Euler's equations)

Sub. above values in Eqn (7)

$$\phi(x)=a_1\sin\alpha x+a_2\cos\alpha x+a_3\sinh(\alpha x)+a_4\cosh(\alpha x) \rightarrow (8)$$

This constants a_1, a_2, a_3, a_4 can be found from boundary conditions at the ends of the beam & these 4 constants represents the mode shape & amplitude of the beam.

$$\alpha^4=\frac{m\omega^2}{EI}$$
$$\alpha=\sqrt[4]{\frac{m\omega^2}{EI}}$$
$$\omega^2=\frac{EI\alpha^4}{m}$$

say ω^2

(a)

3(b)

...tion for ...3(b) is

$\rightarrow(4)$

...n 3(a)

Note:- Solution that involves number zero is called trivial otherwise non-trivial.

Ex:- $x+5y=0$ — trivial solution is (0,0); non trivial solution is (5,−1) or (−2,0.4)

equation (8) can be used to obtain frequencies and mode shapes of the beam using boundary conditions

(Pg 135)

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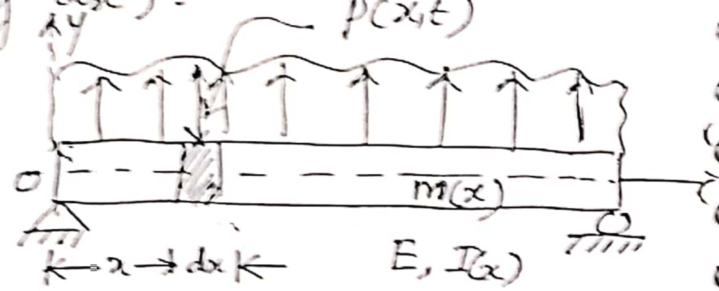
Chapter-7 Continuous systems (Vibration of beams):-

Introduction:- All structures in reality are distributed mass system, since massless springs and dashpots hasve assumed earlier are physically impossible. However some structure like buildings may be approximated as lumped mass systems. For other structures like beams the mass is continuously distributed. Because of this the DOF are infinite. However they can be reduced as many times only the first few modes are important for getting the response.

Derivation of GOE for transverse flexural vibrations of beams:- (Elementary case):-



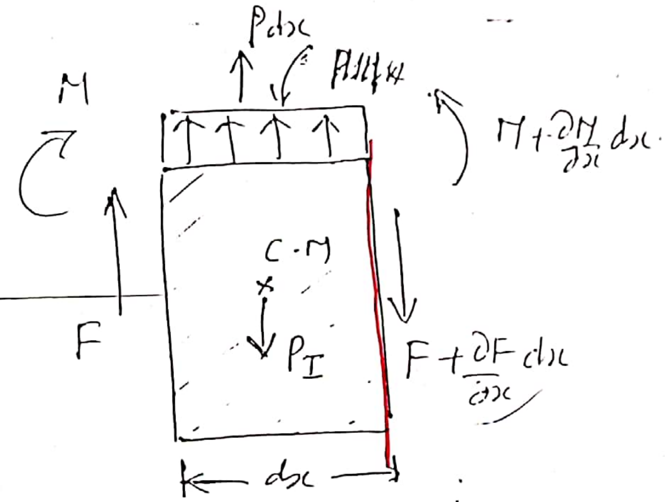
In elementary case, the effects of transverse shear deformations and rotatory inertia are neglected.

Let the beam be of variable section having flexural rigidity $EI(x)$ and mass per unit length $m(x)$. Let it be subjected to an arbitrary transverse loading $p(x,t)$ which varies with both distance x and time t.

The eqn of motion can be derived by considering the forces acting on a small element dx of the beam at a distance x from left support. The forces acting on this element are shown in the fig.

Since $\Sigma V = 0$ ✓

$$F - \left(F + \frac{\partial F}{\partial x} dx\right) + P dx - P_I = 0 \quad \text{--- (1)}$$



In which the inertia force

$$P_I = (m \cdot dx) \cdot \frac{\partial^2 y}{dt^2}$$

$$= (m\, dx)\, \ddot{y} \quad \text{--- (2)}$$

from eqn (1), we get

$$-\frac{\partial F}{\partial x} d.x + P dx - (m\, dx) \frac{\partial^2 y}{dt^2} = 0$$

$$\boxed{\frac{\partial F}{\partial x} = P - m \frac{\partial^2 y}{\partial t^2}} \quad \text{--- (3)}$$

Which is the standard relationship between the SF and loading which now includes the inertia force of the accelerating beam.

$\Sigma M = 0$ ✓

Taking moments @ the right face of the element we get

$$M + F \cdot dx - \left(M + \frac{\partial M}{\partial x} dx\right) + P dx \cdot \frac{dx}{2} = 0 \quad \text{--- (4)}$$

$+ P dx \cdot \frac{dx}{2} \rightarrow$ HOT neglected

neglecting small quantities of higher order and moment of inertia force.

from eqn (4) we set

$$F\,dx - \frac{\partial M}{\partial x}dx = 0$$

$$\therefore \frac{\partial M}{\partial x} = F \quad \text{—— (5)}$$

Which is again the same std relationship between B.M & S.F. No inertia forces come into the moment equilibrium.

differentiating eqn (5) w.r.t x

$$\frac{\partial F}{\partial x} = \frac{\partial^2 M}{\partial x^2} \quad \text{—— (6)}$$

making use of eqn (3) and eqn the moment curvature relationship $EI\frac{\partial^2 y}{\partial x^2} = M$ —— (7)

(3) = (6)

$$p - m\frac{\partial^2 v}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 v}{\partial x^2}\right)$$

therefore which may be written as

$$\frac{\partial^2}{\partial x^2}\left(EI_x\frac{\partial^2 y}{\partial x^2}\right) + m\frac{\partial^2 v}{\partial t^2} = p(x,t) \quad \text{—— (8)}$$

If the beam is of constant section, the above eqn simplifies to

$$EI\frac{\partial^4 v}{\partial x^4} + \bar{m}\frac{\partial^2 v}{\partial t^2} = p \quad \text{—— (9)}$$

- This is the GDE for "transverse flexural vibrations".

or

EOM or GDE for Continuous systems

* To find the natural frequencies of vibration & mode shapes of a simple beam in flexure, we have to consider the case of free vibrations.

FREE VIBRATIONS OF BEAMS:—

~~The governing eqn for which~~ is obtained by taking $P = 0$ for free vibrations

$$EI \frac{\partial^4 y}{\partial x^4} + \bar{m} \frac{\partial^2 y}{\partial t^2} = 0 \quad \text{———} \quad (10)$$

$$\therefore \frac{\partial^4 y}{\partial x^4} + \frac{\bar{m}}{EI} \frac{\partial^2 y}{\partial t^2} = 0 \quad \text{———} \quad (10)a$$

$$y^{IV} + \frac{\bar{m}}{EI} \ddot{y} = 0 \quad \text{———} \quad (11)$$

One of the solutions of the above eqn is obtained by seperating the variables i.e putting

$$y(x,t) = \phi(x) \cdot Y(t) \quad \text{———} \quad (12)$$

differentiating eqn (12) 4 times wrt x and 2 times wrt t, substituting in eqn (10)a

pg (75) pls check

we get two eqns

$$\phi^{IV}(x) - a_n^4 \phi(x) = 0 \quad \text{and} \quad \text{———} \quad 13$$

$$\ddot{Y}(t) + \omega_n^2 Y(t) = 0 \quad \text{———} \quad 14$$

Eqn (14) is the usual eqn. already solved in chapter (1). Eqn (13) is solved using the function

$$\phi(x) = C e^{sx} \quad (15)$$

the solution of which gives ϕ in terms of e to the power $\pm i a_n x$ and $\pm a_n x$

making use of Eulers formula and hyperbolic functions, the solution for the mode shape $\phi(x)$ can be expressed in the following form

$$\phi(x) = A_1 \sin a_n x + A_2 \cos a_n x + A_3 \sinh a_n x + A_4 \cosh a_n x \quad (16)$$

where the frequency parameter

$$a_n = \sqrt[4]{\frac{\bar{m}\omega_n^2}{EI}} \quad 16(a)$$

or $$a_n^4 = \frac{\bar{m}\omega_n^2}{EI} \quad 16(b)$$

or $$\omega_n^2 = \frac{EI a_n^4}{\bar{m}} \quad 16(c)$$

$a_n \rightarrow$ frequency parameter

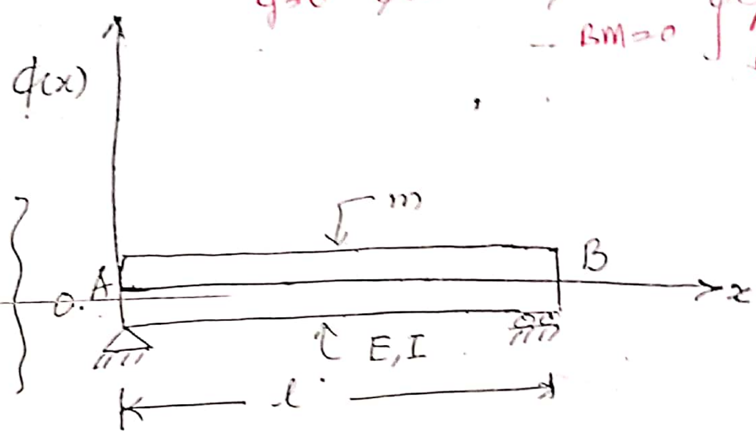
$\frac{EI}{\bar{m}} \rightarrow$ beam property.

the frequencies and mode shapes of the beam can be obtained using 16(c) and the boundary conditions.

*Natural frequencies and Mode shapes of simple beams with different load conditions:-

Ex:1 (a) The simply supported end conditions:-

$y=0 \quad \phi(x)=0 \quad \phi''(x)=0$ } @ A & B
BM = 0



The deflection is

$y = 0$ means ? deflection zero.

$\phi(x) = 0$ deflection ($\because y(x,t) = \phi(x).Y(t)$)

$B.M = 0$

$M = EI \frac{\partial^2 y}{dx^2}$ ie $\phi''(x) = 0$

We have, the mode shape $\phi(x)$ —(16) $a_n = \alpha$

$\phi(x) = A_1 \sin a_n x + A_2 \cos a_n x + A_3 \sinh a_n x + A_4 \cosh a_n x$

BC's are

@ the left end A, the deflection $= 0$ ($y=0$ @ $x=0$), which means

$\phi(0) = 0$ - - - (i) deflection

and also $B.M = 0$ ie

$\phi''(0) = 0$ - - - (ii) Bending Moment @ B

at the right end B, deflection & $B.M = 0$

$\phi(l) = 0$ - - - (iii)

$\phi''(l) = 0$ - - - (iv)

→ Applying the B.C (i) at left end A

$$\phi(x) = A_1 \sin a_n x + A_2 \cos a_n x + A_3 \sinh a_n x + A_4 \cosh a_n x \quad \text{--- (16)}$$

$$\phi(0) = A_1 \cdot 0 + A_2 \cdot 1 + A_3 \cdot 0 + A_4 \cdot 1$$

$$\Rightarrow \quad A_2 + A_4 = 0 \quad \text{--- (17)}$$

For applying the 2^{nd} B.C we have to differentiate eqn (16) twice

$$\phi''(x) = a_n^2(- A_1 \sin a_n x - A_2 \cos a_n x + A_3 \sinh a_n x + A_4 \cosh a_n x) \quad \text{--- (15)}$$

at $x = 0$; $\phi''(x) = 0$ --- (ii) B C

$$\phi''(0) = a_n^2(- A_1 * 0 - A_2 * 1 + A_3 * 0 + A_4 * 1)$$

$$- A_2 + A_4 = 0 \quad \text{--- (18)}$$

hence From (17) & (18) $A_2 = A_4 = 0$

$\therefore$ Eqn (16) becomes

$$\phi(x) = A_1 \sin a_n x + A_3 \sinh a_n x \quad \text{--- (20)}$$

$$\therefore \phi''(x) = a_n^2(- A_1 \sin a_n x + A_3 \sinh a_n x) \quad \text{--- (21)}$$

apply B.C's @ end B

@ $x = l$; $\phi(l) = 0$ $\quad \phi''(l) = 0$

$\therefore$ from eqn (20) we get

$$0 = A_1 \sin a_n l + A_3 \sinh a_n l \quad \text{--- (22)}$$

from eqn (21)

$0 = a_n^2 \left(-A_1 \sin a_n l + A_3 \sinh a_n l\right)$ —— (23)a

$\therefore 0 = -A_1 \sin a_n l + A_3 \sinh a_n l$ —— (23)

$\sinh x = 0$ if $x = 0$

$\sinh 0 = 0$

$\sinh x = \frac{e^x}{2}$

it becomes large

$\sinh x = 0$ then

$x = 0, \pi, 2\pi, 3\pi$

$\sin x = 0$

then $x = 0, \pi, 2\pi, 3\pi$

adding (22) & (23)

$2A_3 \sinh a_n l = 0$

$\because \sinh a_n l \neq 0$ — l cannot be zero — since $A_3 = 0$

$\therefore \phi(x) = A_1 \sin a_n x$

hence A_1 is not equal

Subtracting eqn (22) & (23) we get

$2A_1 \sin a_n l = 0$

Since $A_1 \neq 0$

$\sin a_n l = 0$, this is the characteristic eqn or frequency eqn

i.e. $\sin a_n l = \sin 0$ —— (25)

Which is a trigonometric eqn, whose solution is

$a_n l = n\pi$ —— (26) $n \to$ an integer

for $n = 1, 2, 3 \ldots$

$\therefore a_n = \frac{n\pi}{l}$ —— 26(a)

The frequency parameter a_n can be written as from eqn (16) c pg 134

$$\omega_n^2 = \frac{EI}{\bar{m}} a_n^4$$

$$\therefore \omega_n = \sqrt{\frac{EI}{\bar{m}}}\, a_n^2 \quad \text{—— 27}$$

Substitute $a_n = \frac{n\pi}{l}$ from eqn 26 (a)

$$\therefore \omega_n = \frac{n^2\pi^2}{l^2}\sqrt{\frac{EI}{\bar{m}}}$$

$$\therefore \omega_1 = \frac{\pi^2}{l^2}\sqrt{\frac{EI}{\bar{m}}}$$

$$\omega_2 = \frac{2^2\pi^2}{l^2}\sqrt{\frac{EI}{\bar{m}}}$$

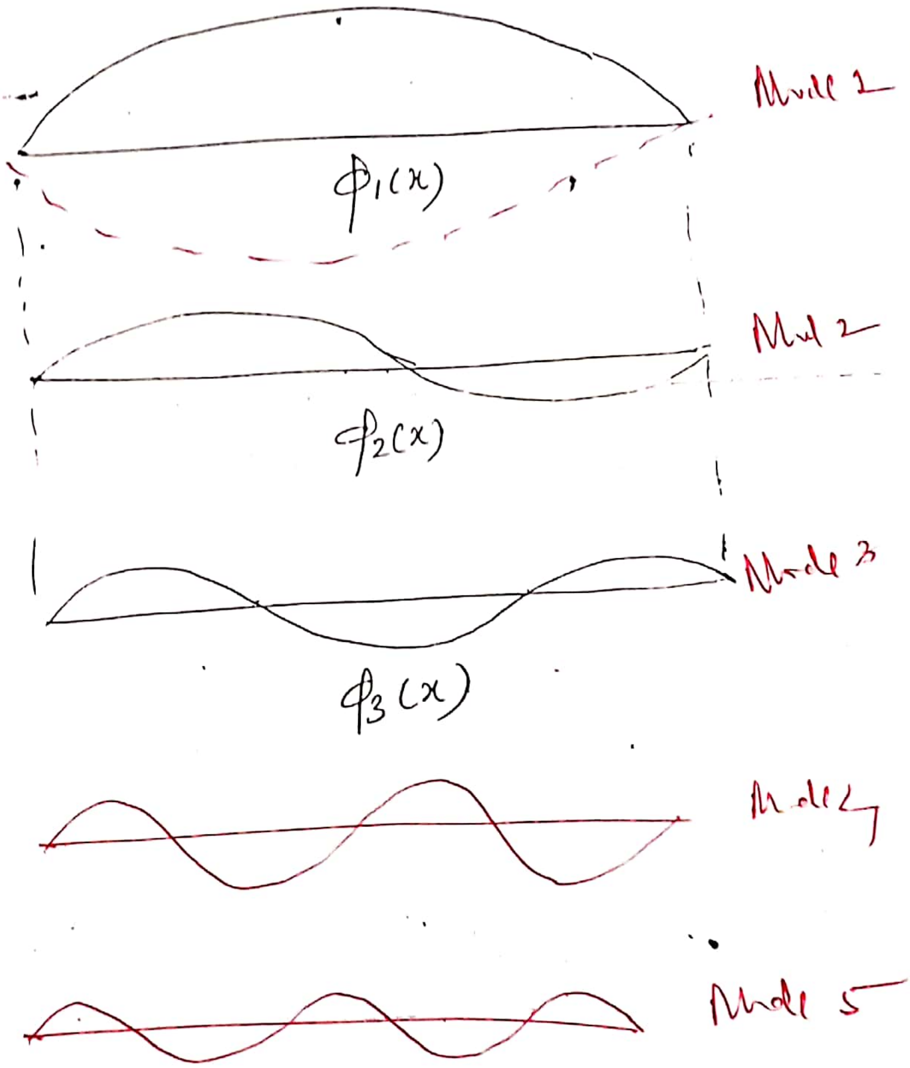
$$\omega_3 = \frac{3^2\pi^2}{l^2}\sqrt{\frac{EI}{\bar{m}}}$$

We have $\phi(x) = A_1 \sin a_n x = A_1 \sin \frac{n\pi}{l} x$

$$\phi_1(x) = A_1 \sin \frac{\pi}{l} x$$

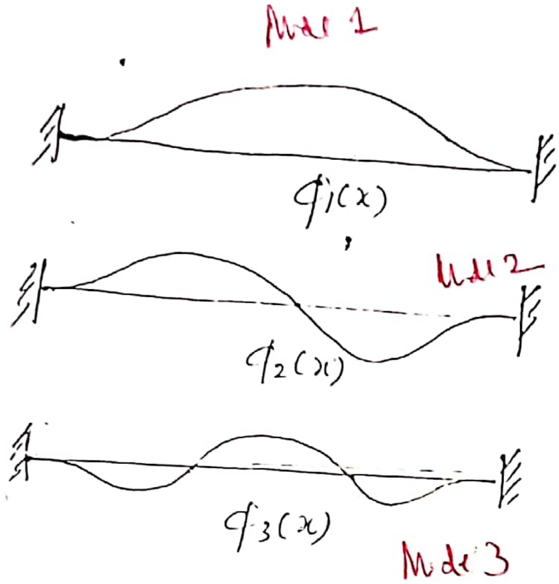
$$\phi_2(x) = A_1 \sin \frac{2\pi}{l} x$$

$$\phi_3(x) = A_1 \sin \frac{3\pi}{l} x$$


Mode 1
$\phi_1(x)$
Mode 2
$\phi_2(x)$
Mode 3
$\phi_3(x)$
Mode 4
Mode 5

(B) Fixed beam:-

$$\phi(x) = A_1 \sin a_n x + A_2 \cos a_n x + A_3 \sinh a_n x + A_4 \cosh a_n x \quad \text{--- (16)}$$



B.C's are :-

At A @ $x = 0$

$\phi(0) = 0$ --- (i)

$\phi'(0) = 0$ --- (ii) slope

@ B @ $x = l$

$\phi(l) = 0$ --- (iii)

$\phi'(l) = 0$ --- (iv) slope

→ applying the 1st B.C, we get

$$A_2 + A_4 = 0 \Rightarrow \therefore A_4 = -A_2$$

applying the 2nd B.C, we get

$$a_n A_1 + a_n A_3 = 0 \qquad \because a_n \neq 0$$

$$\therefore A_3 = -A_1$$

from eqn (16), we get

$$\phi(x) = A_1 \sin a_n x + A_2 \cos a_n x - A_1 \sinh a_n x - A_2 \cosh a_n x \quad \text{--- (30)}$$

$$\phi(x) = A_1(\sin a_n x - \sinh a_n x) + A_2(\cos a_n x - \cosh a_n x) \quad \text{--- (31)}$$

$$\phi'(x) = a_n A_1(\cos a_n x - \cosh a_n x) + a_n A_2(-\sin a_n x - \sinh a_n x) \quad \text{--- (32)}$$

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→ Applying Condition (iii) & (iv)

$$0 = A_1(\sin a_n l - \sinh a_n l) + A_2(\cos a_n l - \cosh a_n l) \quad \text{--- (33)}$$

$$0 = a_n A_1(\cos a_n l - \cosh a_n l) + a_n A_2(\sin a_n l - \sinh a_n l)$$

$$A_1(\cos a_n l - \cosh a_n l) + A_2(-\sin a_n l - \sinh a_n l) = 0 \quad \text{--- (34)}$$

This can be written as

$$\begin{bmatrix} (\sin a_n l - \sinh a_n l) & (\cos a_n l - \cosh a_n l) \\ (\cos a_n l - \cosh a_n l) & (-\sin a_n l - \sinh a_n l) \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \text{--- (35)}$$

For non-trivial solution the determinant of coefficient matrix is zero

$$\begin{vmatrix} (\sin a_n l - \sinh a_n l) & (\cos a_n l - \cosh a_n l) \\ (\cos a_n l - \cosh a_n l) & (-\sin a_n l - \sinh a_n l) \end{vmatrix} = 0 \quad \text{--- (36)}$$

$$(\sin a_n l - \sinh a_n l)(-\sin a_n l - \sinh a_n l) - (\cos a_n l - \cosh a_n l) * (\cos a_n l - \cosh a_n l) = 0$$

$\sin^2\theta + \cos^2\theta = 1$ $\cosh^2\theta - \sinh^2\theta = 1$

Simplifying

$$\cos a_n l \cdot \cosh a_n l = 1 \quad \text{--- 37}$$

Which is the frequency eqn for fixed beam. Whose solution is

$a_n l = (n+\frac{1}{2})\pi$ for $n = 1, 2, 3$

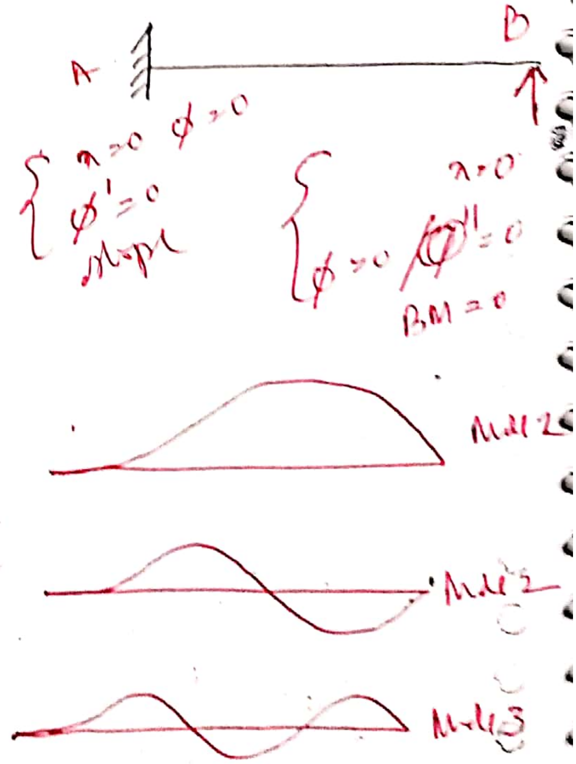
$\therefore a_n = (n+\frac{1}{2})\frac{\pi}{l}$ —— (35)

$\therefore$ We have

$$\omega_n = a_n^2 \sqrt{\frac{EI}{m}}$$

$$\omega_n = (n+\frac{1}{2})^2 \frac{\pi^2}{l^2}\sqrt{\frac{EI}{m}} \qquad n = 1, 2, 3$$

ϕ For a beam fixed at left end and propped at right e



frequency eqn is

$$\tan a_n l = \tanh a_n l$$

Solution is

$$a_n l = (n+\frac{1}{4})\pi$$

$$\therefore a_n = (n+\frac{1}{4})\frac{\pi}{l}$$

$$\therefore \omega_n = (n+\frac{1}{4})^2 \frac{\pi^2}{l^2}\sqrt{\frac{EI}{m}}$$

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Q. For a Cantilever beam

The frequency eqn is

$\phi(x) = 0$, $\phi'(x) = 0$ slope } @ A $(x = 0)$

$\phi''(l) = 0$ @ B $(x = l)$

$\phi'''(l) = 0$, Shear

$$\cos a_n l \cdot \cosh a_n l = -1$$

$$a_n l = \left(n - \tfrac{1}{2}\right)\pi \quad , \; n = 2, 3, 4$$

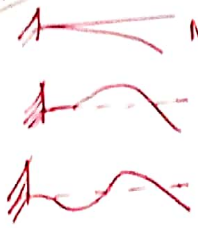
$\phi(0) = 0$ deflection

$\phi'(0) = 0$ slope

$$a_n = \left(n - \tfrac{1}{2}\right)\frac{\pi}{l}$$

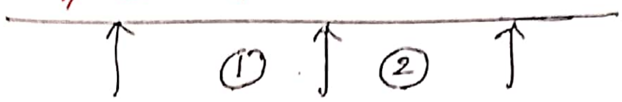
$$\therefore \omega_n = \left(n - \tfrac{1}{2}\right)^2 \frac{\pi^2}{l^2}\sqrt{\frac{EI}{m}}$$

for $n = 1$ { includes π val.

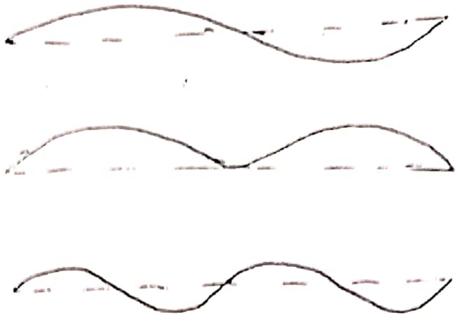


$$a_1 = \frac{1.875}{l}$$

Q. Continuous beams.



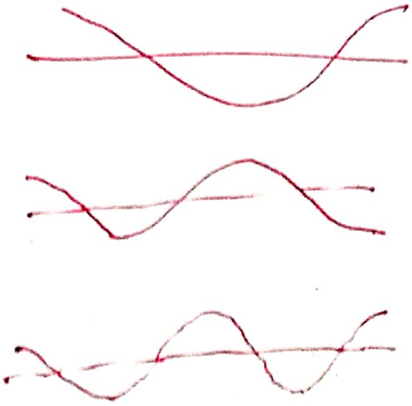
We get 3-Moment eqn



Q. Both end free

Moment ← $\phi'' = 0$ } $x = 0$ @ A

shear ← $\phi''' = 0$ } $x = l$ @ B.

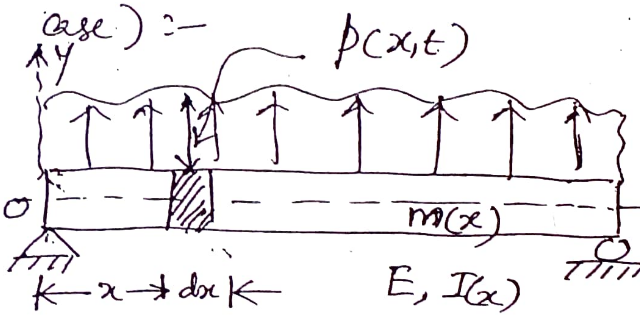


08/06/2006

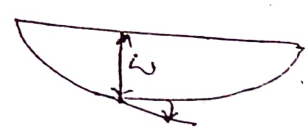
Chapter 7 Continuous systems (Vibration of beams) :-

Introduction:- All structures in reality are distributed mass system, since massless springs and dashpots assumed earlier are physically impossible. However some structure like buildings may be approximated as lumped mass systems. For other ~~structures~~ like beams the mass is continuously distributed. Because of this the DOF are infinite. However they can be reduced as many times only the first few modes are important for getting the response.

* Derivation of GOE for transverse flexural vibrations of beams:- (Elementary case):-



In elementary case, the effects of transverse shear deformations and rotatory inertia are neglected.



Let the beam be of variable section having flexural rigidity $EI(x)$ and mass per unit length $m(x)$. Let it be subjected to an arbitrary transverse loading $p(x,t)$ which varies with both distance x and time t.

The eqn of motion can be derived by considering the forces acting on a small element dx of the

Eqn (14) is the usual eqn, already solved in chapter (1). Eqn (13) is solved using the function

$$\phi(x) = c\, e^{sx} \quad \text{--- (15)}$$

the solution of which gives ϕ in terms of e to the power $\pm i a_n x$ and $\pm a_n x$

making use of Eulers formula and hyperbolic functions, the solution for the mode shape $\phi(x)$ can be expressed in the following form

$$\phi(x) = A_1 \sin a_n x + A_2 \cos a_n x + A_3 \sinh a_n x + A_4 \cosh a_n x \quad \text{--- (16)}$$

where the frequency parameter is

$$a_n = \sqrt[4]{\frac{\bar{m}\, \omega_n^2}{EI}} \quad \text{--- 16 (a)}$$

or

$$a_n^4 = \frac{\bar{m}\, \omega_n^2}{EI} \quad \text{--- 16 (b)}$$

or

$$\omega_n^2 = \frac{EI\, a_n^4}{\bar{m}} \quad \text{--- 16 (c)}$$

$a_n \rightarrow$ frequency parameter

$\frac{EI}{\bar{m}} \rightarrow$ beam property

The frequencies and mode shapes of the beam can be obtained using 16 (c) and the boundary conditions.

from eqn (21)

$$0 = a_n^2(-A_1 \sin a_n l + A_3 \sinh a_n l) \quad \text{—— (23)}$$

$$\therefore 0 = -A_1 \sin a_n l + A_3 \sinh a_n l \quad \text{—— (23)}$$

adding (22) & (23)

$$2A_3 \sinh a_n l = 0$$

$\because \sinh a_n l \neq 0$ (l cannot be zero) since $A_3 = 0$

$\sinh x = 0$ if $x = 0$

$\sinh 0 = 0$

$\sinh x = \frac{e^x}{2}$

if x becomes large

$\sinh x = 0$ then

$x = 0, \pi, 2\pi, 3\pi$

$\sin x = 0$

then $x = 0, \pi, 2\pi, 3\pi$..

~~$\therefore \phi(x) = A_1 \sin a_n x$~~

~~hence A_1 is not equal~~

Subtracting eqn (22) & (23) we get

$$2A_1 \sin a_n l = 0$$

Since $A_1 \neq 0$

$\therefore \sin a_n l = 0$, this is the characteristic eqn or frequency eqn

ie $\sin a_n l = \sin 0$ —— (25)

which is a trigonometric eqn, whose solution is

$a_n l = n\pi$ —— (26) $\quad n \rightarrow$ an integer

for $n = 1, 2, 3 \ldots$

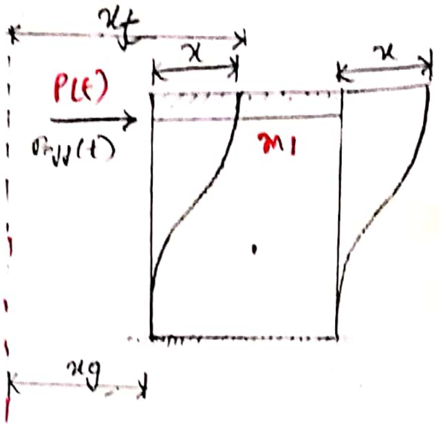
$$\therefore a_n = \frac{n\pi}{l} \quad \text{—— 26(a)}$$

Introduction to Earthquake Analysis.

** Equation of Motion for SDOF System Subjected to Seismic excitation [rigid base excitation] pg (163)

If 'x_g' is displacement of the ground due to EQ

x - relative displacement due to mass m_1 at floor level then the total displacement.



$$x_t = x + x_g \rightarrow (1)$$

The equation of motion for SDOF system is

$$F_I + F_D + F_S = p(t) \rightarrow (2)$$

Inertia Damping Spring force

In case of earthquakes there is a ground acceleration, (no loading in that case — continuous force) $p(t) = 0$. further the elastic forces are proportional to relative displacements & the damping force is proportional to relative velocity

$$F_I = m\ddot{x}(t)$$

$$\left.\begin{array}{l} F_S = Kx \\ F_D = c\dot{x} \end{array}\right\} \rightarrow (3)$$

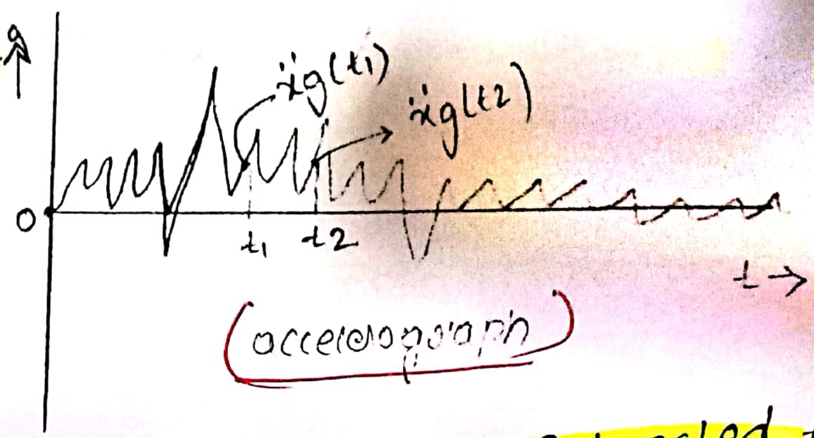
but the inertia forces are proportional to total or absolute acceleration. Hence for seismic analysis for Eqn of motion for seismic SDOF system is

$$m\ddot{x}(t) + c\dot{x} + Kx = 0$$

$$m(\ddot{x} + \ddot{x}_g) + c\dot{x} + Kx = 0$$

$$\boxed{m\ddot{x} + c\dot{x} + Kx = -m\ddot{x}_g = P_{eff}(t)}$$

~~Equation of motion for MDOF system subjected~~



Equation of Motion for MDOF System Subjected to Seismic analysis:-

For MDOF System the Eqn of motion remains the same except that the individual terms m, k, c are replaced by matrices

$$[m]\ddot{x} + [c]\dot{x} + [k]x = -[m]\ddot{x}_g$$

2 marks questions

UNIT – I

THEORY OF VIBRATION

Two Marks Questions and Answers

1. **What is mean by Frequency?**

Frequency is number of times the motion repeated in the same sense or alternatively. It is the number of cycles made in one second (cps). It is also expressed as Hertz (H_Z) named after the inventor of the term. The circular frequency ω in units of sec^{-1} is given by 2π f.

2. **What is the formula for free vibration response?**

The vibration which persists in a structure after the force causing the motion has been removed. The corresponding equation under free vibration is $mx + kx = 0$

3. **What are the effects of vibration?**
   i. Effect on Human Sensitivity.
   ii. Effect on Structural Damage
4. **Define damping.**

Damping is the resistance to the motion of a vibrating body. Damping is a measure of energy dissipation in a vibrating system. The dissipating mechanism may be of the frictional form or viscous form. Unit of damping is N/m/s.

5. **What do you mean by Dynamic Response?**

The Dynamic may be defined simply as time varying. Dynamic load is therefore any load which varies in its magnitude, direction or both, with time. The structural response (i.e., resulting displacements and stresses) to a dynamic load is also time varying or dynamic in nature. Hence it is called dynamic response.

6. **What is mean by free vibration?**

The vibration which persists in a structure after the force causing the motion has been removed is known free vibration. **Example:** oscillation of a simple pendulum.

7. **What is meant by Forced vibrations?**

The vibration which is maintained in a structure by steady periodic force acting on the structure is known as forced vibration.

8. **Write a short note on Amplitude.**

It is the maximum response of the vibrating body from its mean position. Amplitude is generally associated with direction – vertical, horizontal, etc. It can be expressed in the form of displacement (u), velocity ($\dot{u}$) or acceleration ($\ddot{u}$).

9. **Define Resonance.**

When the frequency of external force is equal to or matches with one of the natural frequencies of the vibrating system, the amplitude of vibration becomes excessively large. This phenomenon is called resonance.

10. **What is mean by Degrees of freedom?**

The number of degrees of freedom of system equals the minimum number of independent co-ordinates necessary to define the configuration of the system.

11. **Define static force.**

A push or pull or a load or many loads on any system creates static displacement or deflection depending on whether it is a lumped system or a continues system; there is no excitation and hence there is no vibration.

12. **Write a short note on simple Harmonic motion.**

Vibration is periodic motion; the simplest form of periodic motion is simple harmonic. More complex forms of periodic motion may be considered to be composed of a number of simple harmonics of various amplitudes and frequencies as specified in Fourier series

13. **What is the response for impulsive load or Shock loads?**

Impulsive load is that which acts for a relatively short duration. Examples are impact of a hammer on its foundation. Damping is not important in computing response to impulsive loads since the maximum response occurs in a very short time before damping forces can absorb much energy from the structure. Therefore, only the undamped response to impulsive loads will be considered.

14. Write a short note on single degree of freedom (SDOF) systems.

At any instant of time, the motion of this system can be denoted by single co-ordinate (x in this case). It is represented by a rigid mass, resting on a spring of stiffness 'k' and coupled through a viscous dashpot (representing damping) having constant 'C'. Here, the mass 'm' represents the inertial effects of damping (or energy dissipation) in the system. Using the dynamic equilibrium relation with the inertial force included, according to D'Alembert's principle, it can be written as

$$F_I \quad + \quad F_D + \quad F_S \quad = \quad P(t)$$

(Inertia Force)	(Damping force)	(Elastic force)	(Applied force)

This gives mx + Cx +Kx = P

x, x, x respectively denote the displacement, velocity and acceleration of the system. P (t) is the time dependent force acting on the mass. The above equation represents the equation of motion of the single degree freedom system subjected to forced vibrations**.**

15. Define Cycle.

The movement of a particle or body from the mean to its extreme position in the direction, then to the mean and then another extreme position and back to the mean is called a Cycle of vibration. Cycles per second are the unit Hz.

16. Write short notes on D'Alembert's principle.

According to Newton's law, **F = ma**

The above equation is in the form of an equation of motion of force equilibrium in which the sum of the number of force terms equal zero. Hence if an imaginary force which is equal to **ma** were applied to system in the direction opposite to the acceleration, the system could then be considered to be in equilibrium under the action of real force **F** and the imaginary force **ma**. This imaginary force **ma** is known as **inertia force** and the position of equilibrium is called **dynamic equilibrium**.

D'Alembert's principle which state that a system may be in dynamic equilibrium by adding to the external forces, an imaginary force, which is commonly known as the inertia force

17. Write the mathematical equation for springs in parallel and springs in series

Springs in parallel

$$k_e = k_1 \; k_2$$

k_e is called equivalent stiffness of the system

Springs in series

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

18. Define logarithmic decrement method.

Logarithmic decrement is defined as the natural logarithmic value of the ratio of two adjacent peak values of displacement in free vibration. It is a dimensionless parameter. It is denoted by a symbol δ.

19. Write short notes on Half-power Bandwidth method.

Bandwidth is the difference between two frequencies corresponding to the same amplitude. Frequency response curve is used to define the half-power bandwidth. In which, the damping ratio is determined from the frequencies at which the response amplitude is reduced $1/\sqrt{2}$ times the maximum amplitude or resonant amplitude.

20. Define critical damping.

Critical damping is defined as the minimum amount of damping for which the system will not vibrate when disturbed initially, but it will return tot the equilibrium position. This will result in non-periodic motion that is simple decay. The displacement decays to a negligible level after one nature period T.

21. List out the types of damping.

(1) Viscous Damping **(2)** Coulomb Damping, **(3)** Structural Damping, **(4)** Active Damping, **(5)** Passive Damping.

22. What is meant by damping ratio?

The ratio of the actual damping to the critical damping coefficient is called as damping ratio. It is denoted by a symbol ρ and it is dimensionless quantity. It ca be written as

$$\rho = c/c_c$$

23. Define vibration.

Vibration is the motion of a particle or a body or a system of concentrated bodies having been displaced from a position of equilibrium, appearing as an oscillation.

24. What are the types of vibration?

1. Free and forced vibration
2. Linear and non-linear vibration
3. Damped and undamped vibration
4. Deterministic and random vibration
5. Longitudinal, Transverse and torsional vibration.

25. What are the methods to derive equation of motion?

1. Simple harmonic motion method
2. Newton's method
3. Energy method
4. Rayleigh's method
5. D'Alembert's principle.

26. State Newton's second law.

"The rate of change of momentum is proportional of momentum is proportional to the impressed forces and takes place in the direction in which the force acts".

27. What are the methods to measure damping?

1. Logarithmic decrement method.
2. Half power bandwidth method.

28. Define Dynamic Load Factor (DLF) (or) magnification factor.

It is defined as the ratio of dynamic displacement at any time to the displacement produced by static application of load.

$DLF = X_{dyn} / X_{static}$

29. What is meant by frequency response curve?

A curve plotted between frequency ratio and magnification factor for a few values of ρ is known as frequency response curve.

30. What is meant by phase lag?

The steady state response is expected to occur at the forcing period of $T = 2\pi/\omega$ but it occurs with a delay (or) with a time difference of $(\Phi/2\pi)$. Thus $(\Phi/2\pi)$ is called phase lag.

31. What are the instruments used for measuring vibration?

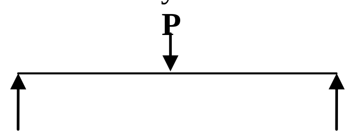
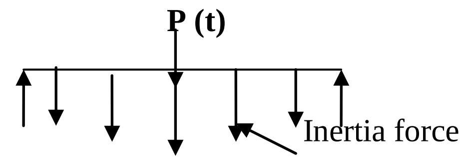
1. Seismometer
2. Accelerometer.

32. Define Transmissibility ratio. (T_R)

Force transmitted to the foundation to the amplitude of the applied forces is known as transmissibility of the support system.

$T_R = F_T/F$

33. Differentiate between static and dynamic loading.

Static Loading	Dynamic loading
1. Load is constant with respect to time. 2. Static problem has only one response i.e., displacement. 3. It has only one solution. 	1. Load and its responses vary with respect to time. 2. It has three responses i.e., displacement, velocity and acceleration. 3. It has an infinite number of solutions. 

34. What are the causes of dynamic effects?

1. Initial conditions
2. Applied forces
3. Support motions.

35. Define mass(m), stiffness(k) and natural period(T)

1. **Mass** is obtained by dividing the weight of the body by the acceleration of gravity. M = w/g. unit is kilograms (kgs).
2. **Stiffness** is defined as the force required to produce unit deformation. Its unit is N/m.
3. **Natural period** is defined as the time required to complete one cycle of free vibration. It is expressed in seconds.

UNIT – II

MULTIPLE DEGREE OF FREEDOM SYSTEM

1. Define degrees of freedom.

The no. of independent displacements required to define the displaced positions of all the masses relative to their original position is called the no. of degrees of freedom for dynamic analysis.

2. Define the fundamental frequency.

The lowest frequency of the vibration is called the fundamental frequency and the corresponding displacement shape of vibration is called first mode of vibration.

3. What is mean by flexibility matrix?

Corresponding to the stiffness (k), there is another structural property known as flexibility which is nothing but the reciprocal of stiffness. The flexibility matrix F is thus the inverse of the stiffness matrix, $[F] = [K]^{-1}$.

4. What is meant by decoupling of equations?

To simplify the response of MDOF system the coupled differential equations must be transformed to independent or uncoupled equations which contain only one dependent variable. This process of uncoupling the coupled differential equation is called decoupling of equations.

5. What are the effects of Damping?

The presence of damping in the system affects the natural frequencies only to a marginal extent. It is conventional therefore to ignore damping in the computations for natural frequencies and mode shapes

6. Write a short note on damping force – Damping force matrix.

If damping is assuming to be of the viscous type, the damping forces may likewise be represented by means of a general damping influence co efficient, C_{ij}. In matrix form this can be represented as

$$\{fD\} = [C]\ \{Y\}$$

7. What are the steps to be followed to the dynamic analysis of structure?

The dynamic analysis of any structure basically consists of the following steps.

1. Idealize the structure for the purpose of analysis, as an assemblage of discreet

elements which are interconnected at the nodal points.

2. Evaluate the stiffness, inertia and damping property matrices of the elements chosen.
3. By supporting the element property matrices appropriately, formulate the corresponding matrices representing the stiffness, inertia and damping of the whole structure.

8. What are normal modes of vibration?

If in the principal mode of vibration, the amplitude of one of the masses is unity, it is known as normal modes of vibration.

9. Define Shear building.

Shear building is defined as a structure in which no rotation of a horizontal member at the floor level. Since all the horizontal members are restrained against rotation, the structure behaves like a cantilever beam which is deflected only by shear force.

10. What are the assumptions made in shear building idealization?

a. Deformation of structures is not affected by the axial force in the columns.
b. Stiffness due to columns and inertia due to slabs are considered.
c. No joint rotations in the structure.
d. Loads act horizontally so that slabs move parallel to each other.

11. What is mass matrix?

The matrix $\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$ is called mass matrix and it can also be represented as [m]

12. What is stiffness matrix?

The matrix $\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$ is called stiffness matrix and it also denoted by [k]

13. Write short notes on orthogonality principles.

The mode shapes or Eigen vectors are mutually orthogonal with respect to the mass and stiffness matrices. Orthogonality is the important property of the normal modes or Eigen vectors and it used to uncouple the modal mass and stiffness matrices.

$\therefore \{\phi\}_i^T [k]\{\phi\}_j = 0$, this condition is called orthogonality principles.

14. Explain Damped MDOF system.

The response to the damped MDOF system subjected to free vibration is governed by

$$[M]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = 0$$

In which [c] is damping matrix and $\{\dot{u}\}$ is velocity vector.

Generally small amount of damping is always present in real structure and it does not have much influence on the determination of natural frequencies and mode shapes of the system. ∴ The naturally frequencies and mode shapes for the damped system are calculated by using the same procedure adopted for undamped system

15. What is meant by first and second mode of vibration?

The lowest frequency of the vibration is called fundamental frequency and the corresponding displacement shape of the vibration is called **first mode or fundamental mode of vibration**. The displacement shape corresponding to second higher natural frequency is called **second mode of vibration.**

16. Define principal mode of vibration.

When both the masses vibrate at same frequency and in phase, a definite relationship exists between the amplitudes of the two coordinates and the displacement configuration which is called principal mode of vibration.

17. Define normal mode of vibration.

If in the principal mode of vibration the amplitude of one of the masses is unity, it is known as normal mode of vibration.

18. Write the equation of motion for an undamped two degree of freedom system.

$$[m]\{\ddot{u}\} + [k]\{u\} = 0$$

This is called equation of motion for an undamped two degree of freedom system subjected to free vibration.

19. What is meant by two degree of freedom and multi degree of freedom system?

The system which requires two independent coordinates to describe the motion is completely is called **two degree of freedom system**.

In general, a system requires n number of independent coordinates to describe it motion is called **multi degree of freedom system.**

20. Define mode shape and node.

Mode shape is a graphical representation of the relative amplitudes of the two coordinates and their phase angle relationship. The point at which the amplitude changes its sign from positive to negative (or) vice versa is called node.

21. Define normalization.

The normal modes are completely determined by assuming unit values for the amplitude of motion at the first degree of freedom. So that the normal modes of the remaining coordinate are computed relatively. This process of normalizing each mode is called normalization.

22. Write the concept of modal superposition method.

It is used to uncouple the coupled differential equations by means of transformation of coordinates which incorporates the orthogonality principles of the mode shapes.

While decoupling, the response of a MDOF system is reduced to the SDOF system.

23. Write the characteristic equation for free vibration of undamped system.

$$|[k] - \omega^2[m]| = 0$$

This equation is called as characteristic equation or frequency equation.

UNIT – III

ELEMENTS OF SEISMOLOGY

1. Define Seismology and Earthquake

Seismology is the study of the generation, propagation generation and recording of elastic waves in the earth and the sources that produce them.

An **Earthquake** is a sudden tremor or movement of the earth's crust, which originates naturally at or below the surface. About 90% of all earthquakes results from tectonic events, primarily movements on the faults.

2. What are the causes of Earthquake?

Earthquake originates due to various reasons, which may be classified into three categories. Decking waves of seashores, running water descending down waterfalls and movement of heavy vehicles and locomotives, causes feeble tremors these earthquakes are feeble tremors, which don't have disastrous effects.

Contrary to the volcanic earthquake and those due to superficial causes, which can be severe, only locally, the more disastrous earthquakes affecting extensive region are associated with movements of layers or masses of rocks forming the crust of the earth. Such seismic shocks, which originate due to crustal movements, are termed as tectonic earthquakes.

3. **What is mean by Epicenter and focus?**

The point at which the rupture begins and the first seismic wave originates is called **focus** or **hypocenter**. The point on the ground directly above the focus is called **epicenter.**

4. **What are the sources of earthquakes?**

Earthquake

Natural sources

1. Tectonic earthquakes
2. Volcanic earthquakes
3. Rock faults

Manmade sources

1. Controlled sources
2. Reserved induced earthquakes
3. Mining induced earthquakes

5. **How earth is divided?**

Earth is divided into four main layers

1. The inner core (solid)
2. Outer core (liquid)
3. Mantle (solid but can deform slowly in a plastic manner)
4. Crust

6. Write a short note on Plate Tectonic Theory

Tectonic is the study of deformations of earth materials that result from deformation. Plate tectonics refers to deformation on a global scale. The basic hypothesis of plate tectonics is that the surface of the earth consists of a number of large plates. These plates move relative to one another. The present six important plates are namely

1. African plate
2. American plate
3. Antarctic plate
4. Australian – Indian plate
5. Eurasian plate
6. Pacific plate

7. Write a short note on Seismic waves.

Large strain energy released during an earthquake travel as seismic waves in all directions through the earth's layers, reflecting at each interface. These waves are of two types,

1. Body waves
    a) primary waves (p-waves)
    b) secondary waves (s-waves)
2. Surface waves.
    a) Love waves
    b) Rayleigh waves

8. Define rock fault.

A fault is a break or fracture in the materials of the earth along which there has been displacement. Fault is a fracture having appreciable movement parallel to the plane of the fracture.

9. What are the types of fault?

1. Dip-slip fault
    a. Normal fault
    b. Reverse fault
2. Strike slip fault
3. Oblique fault.

10. What is meant by Richter scale?

Charles F.Richter, an American seismologist developed a scale of magnitude which is called Richter scale. The logarithmic form of Richter scale is

$$M = \log_{10}A - \log_{10}A_0$$

In Richter scale, the scale number ranges from 0 to 9.

11. Write a short note on Magnitude.

The magnitude of an earthquake is the quantitative measure of the amount of strain energy released during an earthquake. The earthquake scale is devised by Charles F. Richter, an American seismologist be based on the total amount of energy released during an earthquake be called magnitude.

12. What is mean by seismogram?

A seismogram is the graph output by a seismograph. It is a record of ground motion at a measuring station. The energy measured in a seismogram may result from earthquake or from some other source.

13. Write a note on Intensity.

Intensity indicates the intensity of shaking or extent of damage at a given location due to particular earthquake. Thus the intensity of some earthquake will be different at different places. Intensity is a measure earthquake in qualitative way by judging what actually happens on the ground, the damage to the buildings and other structures caused by earthquake waves.

14. What are the commonly used intensity scales?

1. Ten point Rossi Forrel scale
2. Twelve point Modified Mercalli scale.

15. What is Elastic rebound theory?

The concept of possible mode of origin of tectonic earthquakes is known as Elastic Rebound theory.

16. Compare: Magnitude and Intensity of an earthquake.

Sl.No	Magnitude	Intensity
1.	Magnitude measures the energy release at the source of the earthquake. It is determined from measurements on seismographs.	Intensity measures the strength of shaking produced by the earthquake at a certain location. It is determined from the effects on people, structure and natural environment.
2.	Magnitude of an earthquake is a quantitative measure of its size. Thus the magnitude of the earthquake is a single number which does not vary from place to place.	Intensity is a qualitative measure of an earthquake, based on the damage caused by them.
3.	Bhuj earthquake of January 2001 had a magnitude of 7.7 on Richter scale. The earthquake was felt over a large part of the state such as Bhuj, Ahmedabad etc. Magnitude of the earthquake of all the places remains same, i.e. 7.7.	The intensity of the same earthquake at Bhuj is different from the intensity at Ahmedabad vice-versa.

17. How the earthquakes are classified?

Earthquake can be classified into the following types.

(a) According to plate boundaries

(b) According to its depth of focus

(c) According to its origin of the earthquakes

(d) Based on magnitude (M).

18. What is the difference between Inter plate earthquakes and Intra plate earthquakes?

(i) **Inter plate earthquakes:** The earthquake occurring along the boundaries of the tectonic plates are called as inter plate earthquakes.

Example: 1987, Assam Earthquake

(ii) **Intra plate earthquakes:** The earthquakes occurring within a plate are called as intra plate earthquakes.

Example: 1993, Latur Earthquake

19. What are the factors influences the ground motion?

The factors which influence the ground motion are:

(i) Magnitude of earthquake
(ii) Epicentral distance
(iii) Local soil conditions

20. What is the difference between shallow, intermediate and deep focus earthquake?

(i) **Shallow-focus earthquake:** In this case, the seismic shocks originate at a depth of about less than 70 km. Nearly 80% of the world's earthquakes are shallow-focus earthquakes.

(ii) **Intermediate-focus earthquake:** In this case, the seismic waves originate at a depth between 70 km to 300 km.

(iii) **Deep-focus earthquake:** Here, the point of origin of the seismic wave is at a depth of greater than 300 km.

21. What is Seismograph?

Seismograph is an instrument used to recording motions of the earth's surface caused by seismic waves, as a function of time. A modern seismograph includes five basic parts: a clock, a sensor called a seismometer that measures intensity of shaking at the instruments location, a recorder that traces a chart or seismogram, of the seismic arrivals, an electronic amplifier and a data recorder that stores the information for later analysis.

22. What is meant by seismogram?

Seismograms are the records produced by seismographs used to calculate the location and magnitude of an earthquake.

23. Explain volcanic Earthquake?

Earthquakes associated with volcanoes are more localized both in the extent of damage and in the intensity of the waves produced in comparison to those which are associated with faulting motions. Deep below the centre of volcano, hot magma moves slowly through underground passages under pressure, as its makes it ways towards the earth's surface. As this happens, the surrounding rock is put under pressure as the magma pushes against it. This causes the rock to fracture and small earthquakes to occur.

24. What are the basic difference between Focus and Epicentre?

Focus is the location within the earth where fault rupture actually occurs whereas the epicentre is the location on the surface above the focus.

25. What is Focus (or) hypocenter?

Focus is an exact location within the earth where seismic waves are generated by sudden release of stored elastic building. It is called as hypocenter. The place or the point of origin of an earthquake below the surface of the earth is called focus.

26. Define epicenter.

The place or point on the surface vertically above the focus of a particular earthquake is termed as its epicenter.

27. What is meant by isoseismal line or isoseists?

It is a line joining all points at which the intensity of the earthquake is the same.

28. What are the factors influencing ground motion?

1. Magnitude
2. Epicentral distance
3. Local soil conditions

29. What is accelerogram?

A graph plotted between acceleration of ground and time is called accelerogram. The nature of accelerogram's may vary depending on energy released at focus, type of faults, geology along the fault plane and local soil.

30. Explain Uttarkashi earthquake of 1991?

An earthquake of magnitude 6.6 struck the districts of Uttarkashi, Tehri and Chamoli in the state of Uttar Pradesh on October 20, 1991. About 768 persons lost their lives, with about 5,066 injured. Maximum peak ground acceleration of about 0.31g was record at Uttarkashi. Many four-storey buildings in Uttarkashi with RC frame and infill walls sustained the earthquake. Howe ever, some of the ordinary RC buildings collapsed.

31. Write any four recent Indian earthquakes.

1. Bihar-Nepal earthquake, 1988
2. Bhuj earthquake, 2001
3. Sumatra earthquake, 2004
4. Sikkim earthquake, 2006

32. Enumerate TSUNAMI.

A tsunami is a wave train or series of waves, generated in a body of water by an impulsive disturbance that vertically displaces the water column. Tsunami is a Japanese word with the English translation, "harbour wave". The term "tsu" means harbour and "nami" means wave. Tsunami can be generated when the sea floor abruptly deforms and vertically displaces the overlaying water. Tectonic earthquakes are a particular kind of earthquakes that are associated with the earth's crustal deformation; when these earthquakes occur beneath the sea, the water above the deformed area is displaced from the equilibrium position. Waves are formed as the displaced water mass, which acts influence of gravity. When large areas of the sea floor elevate or subside, a tsunami can be created.

33. What is Focal depth and Epicentral distance?

(i) **Focal depth:** The distance between the epicentre and the focus is called focal depth.

(ii) **Epicentral distance:** The distance from epicentre to any point of interest is called epicentral distance.

UNIT – IV

RESPONSE OF STRUCTURES TO EARTHQUAKE

1. What do you understand by response spectrum?

A Response spectrum is the plot of the maximum response (maximum displacement, velocity, acceleration or any other quantity of interest) to a specified load function X_a (t) for all possible SDOF systems (having different natural frequencies or time periods T and a constant damping ratio).

2. What are the factors influencing response spectra?

1. Energy release mechanism
2. Epicentral distance
3. Focal depth
4. Soil condition
5. Richter magnitude
6. Damping
7. Time period

3. Define SEDRS.

A smooth curve is plotted by considering the average of a number of elastic response spectrums corresponding to various possible earthquakes at a particular site. It is known as the Smooth Elastic Design Response Spectrum (SEDRS).

4. What are the factors accompanied in design spectrum?

1. Load Factors
2. Damping to be used in the design
3. Method of calculating of natural period.
4. Type of detailing of ductility.

5. Define Liquefaction.

Liquefaction is a phenomenon in which the strength and stiffness of a soil is reduced by earthquake shaking or other rapid loading. The types of liquefaction includes

1. Flow Liquefaction.
2. Cyclic mobility.

6. What is meant by ductility ratio?

The ratio of the ultimate deformation to the deformation at the initial yielding can give the measure of ductility and it is also called ductility ratio.

7. What is meant by local or element ductility and global or structure ductility?

Introducing ductility in individual element is called **local or element ductility.**

If the ductility is referred with respect to the whole building or entire structure, it is called **global or structure ductility.**

8. What is mean by soil liquefaction?

Soil liquefaction during an earthquake is a process that leads to loss of strength or stiffness of the soil. This could result in the settlement of structures, cause landslides, precipitates failures of earth dams or cause other types of hazards. Soil liquefaction has been observed to occur most often in loose saturated sand deposits.

9. What do you understand by lateral spreading?

Lateral spreading is the movement of surfacial soil layers, which occur there is a loss of shear strength in a subsurface layer due to liquefaction. Lateral spreading usually occurs on very gentle slopes (< 6%). If there is differential lateral under a structure, there could be sufficient tensile stresses developed in the structures that it could literally tear apart. Flexible buildings have been observed to better withstand extensional displacement than more stiff or brittle buildings.

10. What are the methods available on site Modification?

Several site modification methods have been devised and adopted to reduce the potential or susceptibility of the soils beneath a site to liquefy. Some of them include

i. Excavation and Replacement of liquefiable soils
ii. Densification of in – situ soils
iii. In –stu improvement of soils by alteration
iv. Grouting or chemical Stabilization.

11. Write a short note on Soil Alteration?

The third major category of site improvement methods is alteration of the soil to reduce the potential for liquefaction. The soil may be made more resistant by the construction of mixed – in place solidified piles or walls to provide shear resistance which would confine an area of liquefiable soils to prevent flow.

12. What is mean by Grouting?

The fourth category of soil improvement methods is soil grouting or chemical stabilization. These would improve the shear resistance of the soils by injection of particulate matter, resins or chemicals into the voids. Common applications are jet grouting and deep soil mixing.

13. What is mean by Structural Damping?

Damping of structural systems plays a major role in determining the response of the structure for ground motions induced by earthquakes. The actual stiffness of foundation and damping co – efficient are dependent on the frequency of vibration.

14. What are the effects of Damping on soil – structure interaction?

Simple single degree of freedom (SDOF) system is considered for the analysis. The system is mounted on a rigid, mass-less and L-Shaped foundation which in turn is supported on an elastic foundation.

15. Define Ductility.

The ability of a structure or its components or of the materials used to offer resistance in the inelastic domain of response in described by the term ' Ductility". It includes the ability to sustain large deformations, and a capacity to absorb energy hysteretic behavior.

16. What are the basic concepts for ductile performance structures?

i. Selection of sound structural configuration with a well defined lateral load resisting system.

ii. Systematic placement of stiff elements with a view to minimize increase in member forces due to torsion.

iii. Availability of direct load path for force transfer from superstructure to soil medium.

iv. Proper detailing of members and joints is very much necessary

17. Write a short note on Push over analysis.

Pushover analysis is a static analysis procedure for assessing the capacity of structural members against seismic forces. A number of widely used procedures (FEMA 273, ATC – 40) compare these demands with the recommended values of member capacities varying with the level of the performance objectives employed. Each member is classified as either force based or displacement based, depending on its mode of behavior.

18. Mention the different Variable affecting sectional ductility.

The variables that affect sectional ductility include,

i. Material variables such as the maximum usable compressive strain in concrete and grade of reinforcement.

ii. Geometric variables such as the amount of tension and compression reinforcement and the shape of the section.

iii. Loading variables such as the level of axial load accompanying shear.

19. What do you understand by Response reduction factor (R)?

It is the factor by which the actual base shear force, that would be generated if the structure were to remain elastic during its response to design basis Earthquake shaking, shall be reduced to obtain the design lateral force. Ductile buildings are designed for seismic forces that are R times lower than the elastic behavior would require.

20. Write a Short notes on the Analysis of structural Response Based on Soil properties.

Analysis of soil structure interaction can be either using the direct method or the multiple – step method. In the direct method, finite element model of the soil – foundation system is generated and solved in a single step. Multi – step method of analysis uses the principle of superposition to isolate the two primary causes of soil – structure interaction, a) the inability of the foundation to match free field deformation; b) the effect of dynamic response of foundation – structure system on the movement of the supporting soil.

21. What is zero period acceleration?

Zero period acceleration implies maximum acceleration experienced by a structure having zero natural period (T =0). An infinitely rigid structure has zero natural period (T = 0). It doesn't deform. Thus relative motion between its mass and its base, Mass has same acceleration as of the ground. Hence ZPA is the same as peak ground acceleration.

22. What is a design spectrum?

Response spectrum developed for displacement, pseudo-velocity and pseudo acceleration in a combined manner for elcentro earthquake (1940) for various damping ratios. This type of spectrum called tripartite response spectrum. For design purpose, local peaks and valleys should be ignored, since natural period can't be calculated with accuracy. Hence smooth cure plotted by considering the average number of elastic response spectrums corresponding to various possible earthquakes at particular site. It is known as design spectrum.

23. What is peak ground acceleration (PGA)?

PGA is a measure of earthquake acceleration. Unlike Richter scale, it is not a measure of the total size of the earthquake, but rather how hard the earth shakes in a given geographical area. PGA is what is experienced by a particle on the ground.

24. Enumerate site specific response spectrum.

A site specific response spectrum is plotted by taking the average of each record of site specific ground motions. This results in smooth means spectrum. The recorded earthquake motions clearly show that response spectrum shape differs for different types of soil profile at the site. Seed, Ugas and Lysmer (1985) plotted the average shape of response of spectrum.

25. What are the methods to reduce liquefaction?

(a) Avoid liquefaction-susceptible soils

(b) Build liquefaction-resistant structures

(c) Shallow foundation aspects

(d) Deep foundation aspect

(e) Improve the soil

(f) Drainage techniques

(g) Verification of improvement

26. List out the effects of liquefaction.

(a) Loss of bearing strength

(b) Lateral spreading

(c) Sand boils

(d) Flow failures

(e) Ground oscillation

(f) Flotation

(g) Settlement.

27. What is pounding?

Pounding is another important issue in the construction of multistory frame in urban areas. That is when two multistory frames are constructed too close to each other; they may pound on each other during strong ground motion which leads to collision. To avoid collection, adjacent buildings should be separated by minimum gab. These factors imply that nowadays there is a need of earthquake resistance architecture in highly seismic areas.

28. Name the four techniques of aseismic design.

The following four techniques of aseismic design or earthquake resistant building are:

(a) Structural configuration

(b) Lateral strength

(c) Good ductility

(d) Light weight mass.

29. Define inertia force.

Inertia forces are equal to the product of mass and acceleration as per the Newton's Second Law F = ma. Where 'a' is the acceleration and 'm' is the mass.

30. What are the causes of failure of RC frame buildings?

The failure are due to mainly lack of good design of beams/columns frame action and foundation. Poor quality of construction inadequate detailing or laying of reinforcement in various components particularly at joints and in columns/ beams for ductility. Inadequate diaphragm action of roof and floors. Inadequate treatment of masonry walls.

31. What are the advantages of steel and reinforced concrete composite structures?

Steel and RC composite structures are composed of steel skeleton and RC and have the dynamic characteristics of both. It is better with respect to fire resistance and safety against buckling as compared to steel skeleton. Whereas compared to RC structures it has better ductility after yielding. As these features are the properties, which are effective for making a building earthquake resistant and are, found to perform better during earthquakes.

32. Write the behavior of prestressed concrete under earthquake loading.

Prestressed concrete has long been accepted in statically loaded structures. In recent years prestressed concrete has been used in seismic resistant structures. Many thousands of structures have been constructed in which prestressed concrete has been incorporated. Large frame structures are constructed in prestressed concrete and these have performed satisfactorily under normal static and wind loading.

33. Define plain concrete.

Plain concrete is a brittle material. During the first cycle the stress strain curve is the same as that obtained from static tests. The decrease in stiffness and strength of plain concrete is due to the formation of cracks. The compressive strength of concrete depends on the rate of loading. As the rate of loading increases, the compressive strength of concrete increases but the strain at the maximum stress decreases. Plain concrete cannot be subjected to repeated tensile loads since its tensile strength is practically zero.

34. Define Bouchinger effect.

If a specimen is highly deformed in one direction and then immediately reloaded in the opposite direction, it began to flow in this direction at a reduced stress. This is called as

Bouchinger effect. This reflects the material deviation from ideal plastic behavior.

35. What are the two stages of Bouchinger effect?

- ✓ The **first stage** consists of the transient Bouchinger deformation, composed by early re-yielding and work hardening stagnation. The work hardening stagnation appears at a certain range of pre strain.
- ✓ The **second stage** is the permanent softening defined by stress offset in a region after transient period.

36. What is pinching effect?

"pinching effect" is the result of steel bar direction deviating from that of the principal stresses. When the steel bars are oriented in the directions of the applied principal stresses, there was no pinching effect. When the steel bars are oriented at an angle of 45° to the applied principal stresses, there is severe pinching effect. It is obvious that the pinching effect is caused by the orientation of the steel bars, rather than the bond slips between the steel bars and the concrete.

UNIT – V

DESIGN METHODOLOGY

1. What is the formula to find the load factors for plastic design of steel structures?

In plastic design of steel structures, the following load combinations shall be accounted for

1. 1.7(DL+IL)
2. 1.7(DL+EL)
3. 1.3(DL+IL+EL)

When Earthquake forces are considered on a structure, these shall be combined as per Load combination for plastic design of steel structures and partial safety factor for limit state deign of RC and PSC structures.

2. What are the methods of improving element level Ductility?

Ductility in element level is generally with reference to the displacement and moment curvature relationship of a section. This can be generally improved by

i. Decreasing the tension steel area, yield stress and strain of the tension steel increasing the ultimate compressive strain of concrete.
ii. Increasing the area of compression steel.
iii. Reduction in the axial compression on the section.
iv. Provision of effective confinement stirrups, hoops or ties such that compressive steel does not buckle and concrete is led into three dimensional state of stress such that its ultimate compressive strain increases.

3. Write the IS 13920 provisions for flexural members.

The provisions apply to frame members resisting earthquake induced forces and designed to resist flexure. These members shall satisfy the following provisions

(a) The factored axial stress on the member under earthquake loading shall not exceed 0.1fck.
(b) The member shall preferable have a width to depth ratio more than 0.3
(c) Width of the member shall not be less than 200mm.
(d) The depth D of the member shall preferably be not more than ¼ of clear span.

4. What is the formula for finding out the Base shear using seismic co efficient method?

$$V_B = A_h W$$

Where, V_B = is base shear, Ah = Design seismic horizontal seismic coefficient

W = Total weight of building

5. Write the methods of dynamic analysis of multistoreyed structure as per Indian Code IS 1893 (1984)

IS 1893 (1984) gives the Necessary criteria for the earthquake resistant design of structures. This code states that structures should withstand without structural damage, moderate earthquakes and withstand without total collapse, heavy earthquakes.

This code specifies two methods of analysis

i. Seismic co-efficient method
ii. Modal analysis or Response Spectrum method.

6. What are the structural protective systems?

Modern protective system is based on (i) Seismic base isolation (ii) Passive energy dissipaters (iii) Semi active and active systems. Passive energy dissipaters are classified as hysteric, design seismic co – efficient design seismic co – efficient Visco – elastic and others based on the devices used. Eg yielding of metals through sliding friction

7. Write a short note on Mechanism of Base isolation.

The Mechanism of base isolation subjected to ground motion. The isolation reduces the fundamental lateral frequency of the structure from its fixed base frequency and thus shifts the position of structure in the spectrum from peak plateau region. Also it brings forth additional damping due to the increased damping introduced at the base level and thus reduction in the spectral acceleration is achieved.

8. Write down the steps to improve Global level Ductility?

(a) Increasing the redundancy of the structure
(b) Weak bam and strong column approach.
(c) Avoiding soft first storey effects
(d) Avoiding Non ductile failure modes like shear, bond & axial compression at the element level.

9. Define lateral load analysis of building system.

Earthquake force is an inertia force which is equal to mass times acceleration. Mass of the building is mainly located at its floors. Transferring the horizontal component of seismic force safely to the ground is the major task in seismic design. The floors should transfer the horizontal force to vertical seismic elements viz., columns, frames, walls and subsequently to the foundation finally to the soil.

10. Write a short note on Indian seismic codes.

The codes ensure safety of buildings under earthquake excitation IS 1893 – 1962, recommendations for earthquake resistant design of structures. IS 1893 – 1984 the country has divided into five zones in which one can reasonably forecast the intensity of earthquake shock which will occur in the event of future earthquake.

11. Define the term DBE, MCE and MMI.

DBE: Design Basics Earthquake

MCE: Maximum Considered Earthquake

MMI: Mercalli Intensity Scale

12. What is the design philosophy adopted for earthquake resistant structure?

The extreme loading condition caused by an earthquake and also the low probability of such an event occurring within the expected life of a structure, the following dual design philosophy is usually adopted

i. The structure is designed to resist the expected intensity of ground motion due to a moderate earthquake so that no significant damage is caused to the basic structure and

ii. The structure should also be able to withstand and resist total collapse in the unlikely event of a severe earthquake occurring during its lifetime. The designer is economically justified in this case to allow some marginal damage but total collapse and loss of life must be avoided.

13. Write down the formula to find out the Magnitude as per the IS code.

The amount of strain energy released at the source is indicated by the magnitude of the earthquake. Magnitude = Log_{10} (A_{max})

Where A is the maximum aptitude in microns (10^{-3}m) recorded by Wood – Anderson seismograph. If E is the energy released, then

$$\text{Log } E = 11.8 + 1.5 M$$

14. Why is base isolation effective?

The base isolation systems reduce the base shear primarily because the natural vibration period of the isolation mode, providing most of the response, is much longer than the fundamental period of the fixed base structure, leading to a much smaller spectral ordinate. The higher modes are essentially not excited by the ground motion; although their pseudo acceleration is large their modal static responses are very small.

The primary reason for effectiveness of base isolation in reducing earthquake induced forces in a building is the lengthening of the first mode period. The damping is the isolation system and associated energy dissipation is only a secondary factor in reducing structural response.

15. Explain two cases of design horizontal earthquake load.

(a) When the lateral resisting elements are oriented along orthogonal horizontal direction, the structure shall be designed for the effects due to full design earthquake load in one horizontal direction at time.

(b) When the lateral load resisting elements are not oriented along the orthogonal horizontal directions, the structure shall be designed for the effect due to full design earthquake load in one horizontal direction plus 30% of the design earthquake load in the other direction.

16. Write the formula for modal mass (M_k).

The modal mass $\mathbf{M_k}$ of mode k is given by:

$$M_k = \frac{[\sum_{i=1}^{n} W_i \, \phi_{ik}]^2}{g \sum_{i=1}^{n} W_i \, (\phi_{ik})^2}$$

17. Explain design eccentricity.

The design eccentricity, e_{di} to be used at floor I shall be taken as:

$$e_{di} = \begin{Bmatrix} 1.5 \, e_{si} + 0.05 b_i \\ or \; e_{si} - 0.05 b_i \end{Bmatrix}$$

Whichever of these gives the more severe effect in the shear of any frame

Where e_{di} = Static eccentricity

e_{si} = defined as the distance between centre of mass and centre of rigidity

b_i = floor plan dimension of floor

18. What is additive shear?

Additive shear will be super-imposed for a statically applied eccentricity of $\pm 0.05 b_i$ with respect to centre of rigidity.

19. Name types of damper's.

(i) Metallic dampers or yielding dampers

(ii) Friction dampers

(iii) Viscous dampers

20. What is meant by Base isolation technique?

Seismic base isolation is a technique that mitigates the effects of an earthquake by essentially isolating the structure and its contents from potentially dangerous ground motion, especially in the frequency range where the building is most affected.

21. What are the needs for base isolation?

1. Building is located in a high seismic intensity zone.
2. Existing structure is unsafe.
3. Minimize the damage to primary and secondary structural members.
4. Cost economics of the structure with and without isolators.

22. What is meant by seismic dampers?

Dampers can be installed in the structural frame of a building to absorb some of the energy going into the building from the shaking ground during an earthquake.

23. What are the types of seismic dampers?

1. Metallic dampers or yielding dampers.
2. Friction dampers.
3. Viscous dampers.

24. What are the types of earthquake resistant design methods?

1. Lateral strength design
2. Ductility based design
3. Capacity design method
4. Energy based design

25. What are the types of seismic control systems?

1. Passive control system
2. Active control system
3. Hybrid control system
4. Semi-active control system

26. Write short notes on passive control system.

- ✓ It works on energy transformation principle.
- ✓ It is passive in the sense that it does not require any additional energy source to operate and is activated by the earthquake input motion only.
- ✓ **Example:** Isolators, dampers and oscillators.

27. Write the examples of active control system.

Active TMDs, Tuned liquid dampers, active braces systems and active tendon system.

28. What are the factors that contribute the over strength factor?

- ✓ Load factor on seismic and gravity load
- ✓ Material factors
- ✓ Member sizes/reinforcement more than required
- ✓ Special ductile detailing
- ✓ Redundancy
- ✓ Strain hardening in materials
- ✓ Higher material strength under cyclic loads
- ✓ Strength contribution of non structural members

29. Define over strength factor.

It is ratio of yield force or maximum inelastic force in an elasto plastic system and design force.

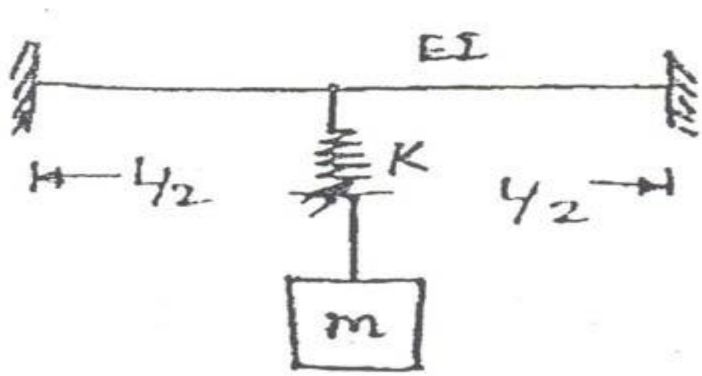
30. What are the factors that govern the architectural configurations?

- ✓ Architectural design
- ✓ Functional requirements
- ✓ Urban design parameters
- ✓ Planning considerations
- ✓ Aesthetic appearance

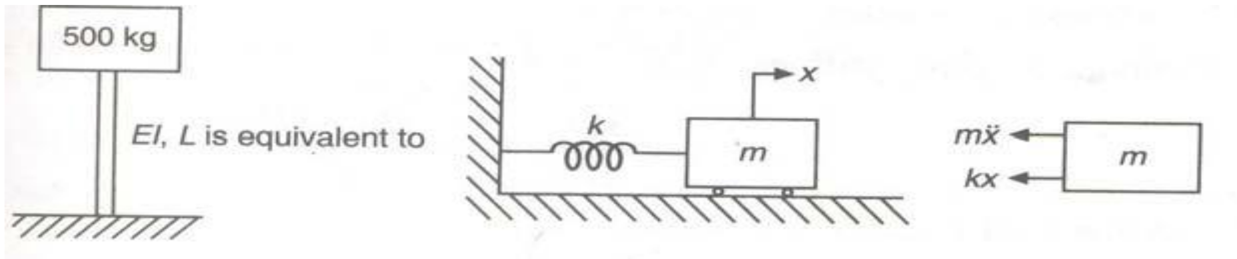
PART-B QUESTIONS (16 MARKS)

UNIT I THEORY OF VIBRATIONS

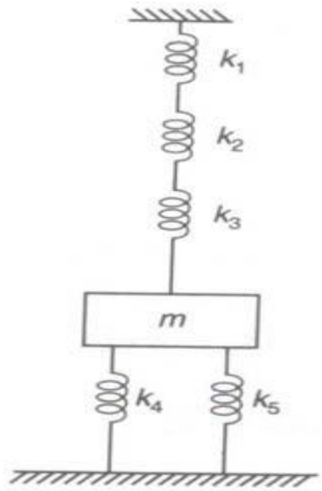
1. Show that the log – decrement is also given by the equation U = 1/n log (U0/ Un) represents the amplitude after n cycles have elapsed.
2. A machine foundation weighs 60 KN. The spring constant is 11000KN/m and dash pot constant (C) = 200KN/s/m. Determine
    a. Whether the system is over damped, under tamped or critically damped.
    b. Logarithmic decrement
    c. Ratio of two successive amplitudes
    d. If the initial displacement is 10mm and initial velocity is zero displacement at t = 0.1s
3. A mass ‘m’ is suspended from a beam shown in figure. The beam is of negligible mass and has a uniform flexural rigidity ‘EI’. Find the natural frequency of the system.



4. A mass of 10kg is supported by a steel wire 1m in dia and 3m long. The system is made to move upwards with a uniform velocity of 10 cm/sec when the upper end is suddenly stopped. Determine the frequency and the amplitude of the resulting vibrations of the mass and the maximum stress on the wire.
5. A vibrating system consists of a mass of 5kg, spring of stiffness 120 N/m and a damper with a damping co-efficient of 5 N/s/m. determine
   a. Damping factor
   b. Natural frequency of the system
   c. Logarithmic decrement
   d. The ratio of two successive amplitude
   e. The number of cycles after which the initial amplitude reduces to 25%
6. A single degree of freedom system having a mass of 2.5m is set into motion with a viscous damping and allowed to oscillate freely. The frequency of oscillation is found to be 20 Hz, and measure of the amplitude of vibration shows two successive amplitude to be 6mm and 5.5mm. Determine the viscous damping co-efficient.
7. A damper offers resistance 0.08 N at a constant velocity 0.06m/s. the damper is used with a spring of stiffness equal to 12 N/m. Determine the damping ratio and frequency of the system when the mass of the system is 0.3 kg.
8. A harmonic motion has a maximum velocity of 6 m/s and it has a frequency of 12 cps. Determine its period, amplitude and maximum acceleration.
9. A cantilever beam 3m long supports a mass of 500 kg at its upper end. Find the natural period and the frequency. E = 2.1 x 106 kg/cm2 and I = 1300 cm4



10. Consider the system shown in figure. If k1 = 2000 N/m, k2 = 1500 N/m, k3 = 3000 N/m, k4 = k5 = 500 N/m, find the mass if the natural frequency of the system is 10 Hz.



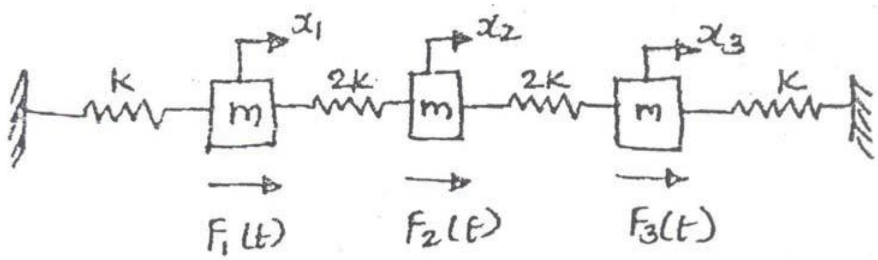
11. Explain duhamel's integral in detail with neat sketches.

UNIT –II MULTIPLE DEGREE OF FREEDOM SYSTEM

1. Determine the natural frequency and mode shapes of a MDF system. The mass and the stiffness matrix of a MDF system is given by

$$[M]=m\begin{bmatrix}1&0&0\\0&1&0\\0&0&2\end{bmatrix},\ [K]=K\begin{bmatrix}2&-1&0\\-1&3&-2\\0&-2&2\end{bmatrix}.$$

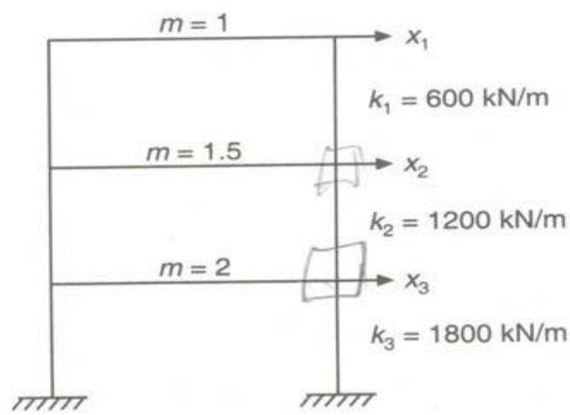
2. A three spring mass system is shown on figure. All the masses are subjected to dynamic forces. Derive the equation of motion in terms of displacements x1, x2, x3 of the masses along the axis of the springs.



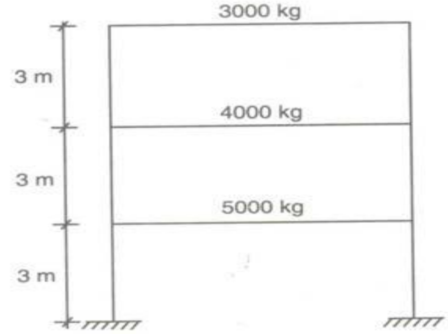
3. State and prove orthogonality property of mode shapes.
4. In a two storey building frame, the mass M1 = M2 = 1000 Kg and stiffness are k1 = k2 = 1 MN/m. If a horizontal force of 20kN is applied ath eht top of ground storey level, determine the displacement of the masses M1, M2. The stiffness and the mass matrix of two degree of freedom system are given.

   K = [200 200 and mass m = [2 0

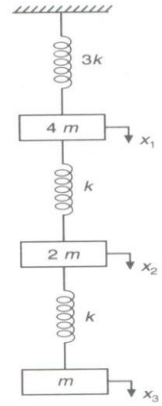
   200 500] 0 3] Determine the natural frequency of the system.
5. Calculate natural frequency and draw the mode shape for the shear building.



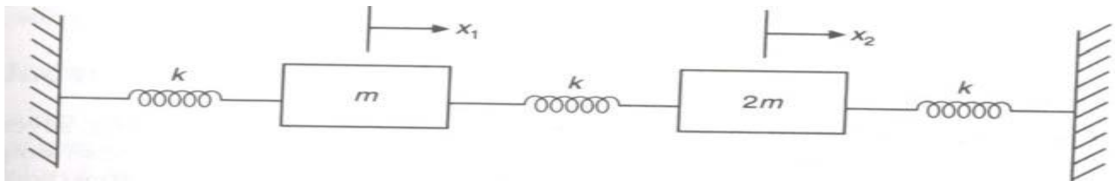
6. Determine the natural frequency and mode shape for the MDOF system. EI = 4.5 x 10^6 N-m^2 for all columns.



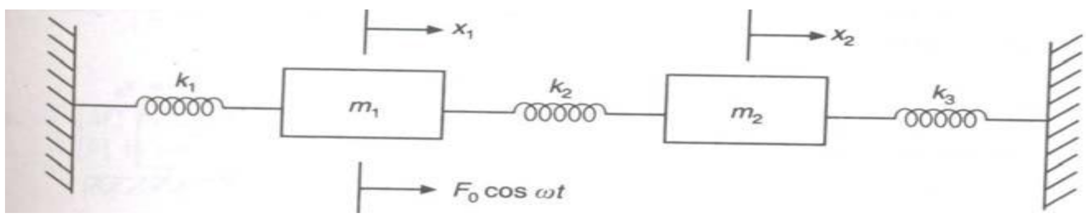
7. Find the natural frequency and mode of the system.



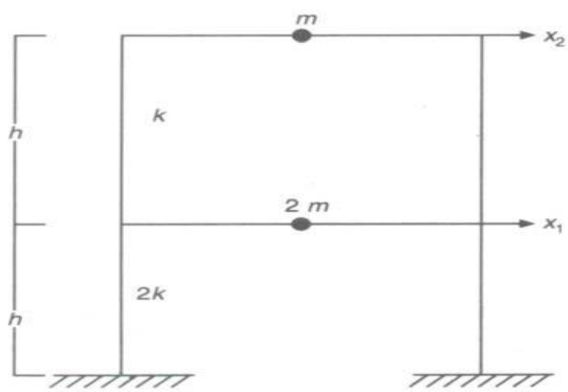
8. Find the natural frequency and mode of vibration for the system shown.



9. Determine the steady state response of the system



10. Determine the natural frequency and mode of vibration of the system



UNIT –III ELEMENTS OF SEISMOLOGY

1. Explain the causes of earthquake and geological faults
2. List out some past disastrous earthquakes
3. Explain the seismic waves with neat sketch

4. Briefly explain plate tectonics and elastic rebound theory.
5. Explain the seismograph and seismogram in detail.
6. Describe the two approaches followed for the prediction of earthquakes.name the major plates of the earth.
7. On what is the assignment of an earthquake magnitude based? is magnitude the same as intensity? Explain.
8. Differentiate magnitude and intensity. How will you measure magnitude and intensity? Explain the methods briefly.
9. Explain about some recent earthquakes and give information on some disastrous earthquakes.
10. What are the types of earthquake explain briefly and discuss about elastic rebound theory.

UNIT –IV RESPONSE OF STRUCTURES TO EARTHQUAKE

1. Explain the effect of soil properties and liquefaction of soils.
2. Explain response behavior and ductility demand in multistoried building with neat sketch.
3. Explain the factors affecting ductility.
4. Define liquefaction of soils. Explain in detail about the factors affecting liquefaction.
5. Briefly explain smooth spectrum and seismic demand diagrams.
6. Explain the Pinching and Bouchinger effect in detail.
7. Explain the behavior of RC steel and prestressed concrete structure under earthquake loading.
8. Define response spectra. Explain the concept and types of response spectra with neat sketch.
9. Why ductility consideration is very important in earthquake resistant design of RC building? Explain the ductile detailing considerations in flexural members as per IS 13920-1993.
10. Define design spectra. Write the concepts of PGA and ZPA.

UNIT –V DESIGN METHODOLOGY

1. What are the effects of base isolation? Explain with suitable examples.
2. What the causes of damage in earthquakes?
3. Explain the planning considerations/architectural concepts as per IS 4326-1993.
4. Briefly explain the salient feature of earthquake resistant provisions as per IS 4326-1993 for RC and masonry structures.

5. What are the methods used to analyses earthquake resistant structures? Explain the procedure of each method as per IS-1893:2002.
6. Why base isolation is effective in earthquake resistant design? Explain the effectiveness in multistory buildings.
7. In what manner is the behavior of soft storey construction likely to be different from a regular construction in the event of an earthquake?
8. Explain about Earthquake design philosophy and also give the main criteria for earthquake resistant measures.
9. Explain in detail about the types of dampers.
10. Write the design procedure for seismic analysis of RC buildings.
11. A three storeyed, symmetrical reinforced concrete school building in zone V with plan dimensions 7m, storey height of 3.5m. Total weight of beams in a storey is 130 kN and total weight of slab in a storey 250kN. Total weight of columns in a storey is 50kN and total weight of walls in a storey 530kN. Live load = 130kN, Weight of terrace floor is 655kN resting on hard rock. Damping = 5%. Determine the base shear and lateral loads at each floor by seismic coefficient method.

CE6702 – PRESTRESSED CONCRETE STRUCTURES

UNIT 1- INTRODUCTION – THEORY AND BEHAVIOUR

1. List out the advantages of prestressed concrete. (AUC Nov/Dec 2011 & 2012)

In case of fully prestressed member, which are free from tensile stresses under working loads, the cross section is more efficiently utilized when compared with a reinforced concrete section which is cracked under working loads.

The flexural member is stiffer under working loads than a reinforced concrete member of the same length.

2. What is meant by pretensioned and post tensioned concrete?

(AUC Nov/Dec 2010 & 2011)

Pre tensioning: A method of Pre stressing concrete in which the tendons are tensioned before the concrete is placed. In this method, the prestress is imparted to concrete by bond between steel and concrete.

Post tensioning: A method of pre stressing concrete by tensioning the tendons against hardened concrete. In this method, the prestress is imparted to concrete by bearing.

3. Why is high tensile steel needed for prestressed concrete construction?

(AUC Nov/Dec 2012)

High strength concrete is necessary for prestress concrete as the material offers highly resistance in tension, shear bond and bearing. In the zone of anchorage the bearing stresses being hired; high strength concrete is invariably preferred to minimizing the cost. High strength concrete is less liable to shrinkage cracks and has lighter modulus of elasticity and smaller ultimate creep strain resulting in a smaller loss of prestress in steel. The use of high strength concrete results in a reduction in a cross sectional dimensions of prestress concrete structural element with a reduced dead weight of the material longer span become technically and economically practicable.

- Tensile strength of high tensile steel is in the range of 1400 to 2000 N/mm^2 and if initially stress upto 1400 N/mm^2 their will be still large stress in the high tensile reinforcement after making deduction for loss of prestress. Therefore high tensile steel is made for prestress concrete.

4. What are the various methods of prestressing? (May/June 2013, Apr/May 2010)

- Pre-tensioning
- Post-tensioning

5. What are the systems of prestressing? (AUC May/June 2013)

- Pretensioning systems

- Abutment method, Strut and Mould Method

Post tensioning systems

- Freyssinet systems
- Gifford Udall
- Lee-Mccall systems
- Magnel blaton systems

6. List the loss of prestress. (AUC Nov/Dec 2010 & 2013)

- Nature of losses of prestress.
- Loss due to elastic deformation of concrete.
- Loss due to shrinkage of concrete.
- Loss due to creep of concrete.
- Loss due to relaxation of stress in steel.
- Loss of stress due to friction.
- Loss due to anchorage slip.

7. What are the classifications of prestressed concrete structures? (AUC Nov/Dec 2013)

- Externally or internally prestressed
- Pretensioning and post tensioning
- End-Anchored or Non-End Anchored Tendons
- Bonded or unbonded tendons
- Precast, cast-in-place, composite construction
- Partial or full prestressing

8. Define load balancing concept. UC Apr/May 2011 & 2012)

It is possible to select cable profiles in a prestressed concrete member such that the traverse component of the cable force balances the given type of external loads. This can be readily illustrated by considering the free body of concrete with the tendon replaced by forces acting on the concrete beam.

9. What are the factors influencing deflections? (AUC Apr/May 2011)

- Imposed load and self weight
- Magnitude of the prestressing force
- Cable profile
- Span of the member
- Length of the deflection field
- Spacing between the deflection plate
- Difference of potential between the plates
- Accelerating voltage of the second anode.

10. What are the sources of prestress force? (AUC Apr/May 2012)

- Mechanical
- Hydraulic
- Electrical
- Chemical

11. Define kern distance. (AUC Apr/May 2010)

Kern is the core area of the section in which if the load applied tension will not be induced in the section Kt = Zb / A, Kb = Zt / A,

If the load applied at K_t compressive stress will be the maximum at the top most fiber and zero stress will be at the bottom most fiber. If the load applied at K_b compressive stress will be the maximum at the bottom most fiber and zero stress will be at the top most fiber.

12. What is Relaxation of steel?

When a high tensile steel wire is stretch and maintained at a constant strain the initially force in the wire does not remain constant but decrease with time. The decrease of stress in steel at constant strain is termed relaxation of steel.

13. What is concordant prestressing?

Pre stressing of members in which the cable follow a concordant profile. In case of statically indeterminate structures. It does not cause any changes in support reaction.

14. Define bonded and non bonded prestressing concrete.

Bonded prestressing: Concrete in which prestress is imparted to concrete through bond between the tendons and surrounding concrete. Pre tensioned members belong to this group.

Non-bonded prestressing: A method of construction in which the tendons are not bonded to the surrounding concrete. The tendons may be placed in ducts formed in the concrete members or they may be placed outside the concrete section.

15. Define axial prestressing.

Members in which the entire cross-section of concrete has a uniform compressive prestress. In this type of prestressing, the centroid, of the tendons coincides with that of the concrete section.

16. Define prestressed concrete.

It is basically concrete in which internal stresses of a suitable magnitude and distribution are introduced so that the stresses resulting from external loads (or) counteracted to a desire degree in reinforced concrete member the prestress is commonly introduced by tensioning the steel reinforcement.

17. Define anchorage.

A device generally used to enable the tendon to impart and maintain prestress to the concrete is called anchorage. E.g. Fressinet, BBRV systems, etc.,

18. What are the main factors for concrete used in PSC?

- Ordinary Portland cement-based concrete is used but strength usually greater than 50 N/mm^2;
- A high early strength is required to enable quicker application of prestress; A larger elastic modulus is needed to reduce the shortening of
- the member; A mix that reduces creep of the concrete to minimize
- losses of prestress;

19. What are the uses of prestressed concrete?

- Railway Sleepers;
- Communications poles;
- Pre-tensioned precast “hollow core” slabs;
- Pre-tensioned Precast Double T units - for very long spans (e.g., 16 m span for car parks);
- Pre-tensioned precast inverted T beam for short-span bridges;
- Post-tensioned ribbed slab;
- This is “glued segmental” construction;

20. Define Magnel diagram.

A Magnel Diagram is a plot of the four lines associated with the limits on stress. As can be seen, when these four equations are plotted, a feasible region is found in which points of 1 *P* and *e* simultaneously satisfy all four equations. Any such point then satisfies all four stress limits.

21. What are the advantages of PSC construction?

- In case of fully prestressed member, which are free from tensile stresses under working loads, the cross section is more efficiently utilized when compared with a reinforced concrete section which is cracked under working loads.
- The flexural member is stiffer under working loads than a reinforced concrete member of the same length.

22. What is the principle of post tensioning?

In post tensioning the concrete units are first cast by incorporating ducts or grooves to house the tendons. When the concrete attains sufficient strength, the high tensile wires are tensioned by means of jack bearing on the end face of the members and anchored by wedges or nuts. The forces are transmitted to the concrete by means of the end anchorages and when the cable is curved, through the radial pressure between the cable and the duct. The space between the tendons and the duct is generally grouted after the tensioning operations.

23. What is chemical prestressing?

It is also called as self stressing of concrete and it was made possible by the development of expanding cement. Generally expanding cements consist of 75 percent Portland cement, 15 percent high alumina cement and 10 percent gypsum, which result in the formation of calcium sulphoaluminate. The linear expansion of cement is about 3 to 4 percent. It is mainly applicable to structural systems and elements in which optimum amount of prestress is relatively low.

24. Enumerate the effect on tendon profile on deflections.

In most of the cases of prestressed beams, tendons are located with eccentricities towards the soffit of beams to counteract the sagging bending moments due to transverse loads. Consequently, the concrete beam deflects upwards on the application or transfer of prestress. Since the bending moment at every section is the product of the prestressing force and eccentricity, the tendon profile itself will represent the shape of the BMD.

25. Write the need for using M35-M40 grade of concrete in prestressed concrete.

Low shrinkage, minimum creep characteristics and a high value of young modulus are necessary for prestressed members. Many desirable properties such as durability, impermeability and abrasion resistance are highly influenced by the strength of concrete like M35 and M40.

26. What are the ways of improving the shear resistance of prestressed concrete beam?

- Horizontal or axial prestressing
- Prestressing by inclined or sloping cables
- Vertical or transverse prestressing

UNIT II DESIGN FOR FLEXURE AND SHEAR

1. What are the different types of flexural failure modes observed in PSC beams?

- Failure of under reinforced sections
- Failure of over reinforced sections
- Failure of sections by other modes
- Fracture of steel in tension

2. What is strain compatibility method?

The method of estimating the flexural strength of prestressed concrete sections is based on the compatibility of strains and equilibrium of forces acting on the section at the stage of failure.

3. What are the assumptions made for strain compatibility method?

- The stress distribution in the compression zone of concrete can be defined by means of coefficients applied to the characteristic compressive strength and the average compressive stress and the position of the centre of compression can be assessed.
- Plane sections normal to the axis remain plane after bending.
- The resistance of concrete in tension is neglected.
- The maximum compressive strain in concrete at failure reaches a particular value.

4. What are the basic assumptions for calculating flexural stressed?

- Plane section remains plane after bending.
- Perfect bond between concrete and prestressing steel for bonded tendons.
- The prestressed concrete section will also behave like a reinforced concrete section only after cracking.
- The tensile stressed are resisted by the steel components namely untensioned reinforcement if and the high tensile steel.
- The theories used for estimating the flexural strength of reinforced concrete section may also use for prestressed concrete sections.

5. List out five factors which influence the ultimate flexural strength of PSC beams.

- The failure mainly a flexural failure, which no effect of shear, bond or anchorage, which might decrease the strength of the section.
- The beams are bonded. Unbounded beams have different ultimate strength than for bonded beams.
- Beams are statically determinate.
- Ultimate loads, obtained are the result of short time static loading. No effect of impact fatigue or long time loading is considered.

6. Explain conventional failure of an over reinforced prestressed concrete beam.

An Over reinforced members fail by the sudden crushing of concrete. The failure being reinforced members fail by the sudden crushing of concrete. The failure being characterized by small deflections and narrow cracks, the area of steel being comparatively large, the stresses developed in steel at failure of the member may not reach the tensile strength.

7. Define degree of prestressing.

A measure of the magnitude of the prestressing force related to the resultant stress occurring in the structural member at working load.

8. Compare the flexure failure of conventional RC beam with PSC beam.

There is no difference in flexural failure of a conventional RC beam and a prestressed concrete beam. Because the prestressed concrete beam will behave as an RC beam, after flexural cracks are formed.

9. What is horizontal shear?

The horizontal shear stress is (normally) maximum at the neutral axis of the beam. This is opposite of the behavior of the bending stress which is maximum at the other edge of the beam, and zero at the neutral axis.

10. What are the three ways of improving the shear resistance of a prestressed concrete beams?

- Horizontal or axial prestressing
- Prestressing by inclined or sloping cables and
- Vertical or transverse prestressing

11. How will you carry out the analysis of the reinforced and prestressed concrete members under shear?

The analysis for axial load and flexure are based on the following principles of mechanics

- Equilibrium of internal and external forces
- Compatibility of strains in concrete and steel
- Constitutive relationship of materials

12. Mention the different types of cracks in a simply supported beam under uniformly distributed load without prestressing.

- Flexural cracks – These cracks form at the bottom near the midspan and propagate upwards.
- Web shear cracks – These cracks form near the neutral axis close to the support and propagate inclined to the beam axis.
- Flexure shear cracks – These cracks form at the bottom due to flexure and propagate due to both flexure and shear.

13. Write the functions of stirrups.

- Stirrups resist part of the applied shear.
- They restrict the growth of diagonal cracks
- The stirrups counteract widening of the diagonal cracks, thus maintaining aggregate interlock to a certain extent
- The splitting of concrete cover is restrained by the stirrups, by reducing dowel forces in the longitudinal bars.

14. Why the stirrups are anchored?

- The stirrups should be anchored to develop the yield stress in the vertical legs.

- The stirrups should be bent close to the compression and tension surfaces, satisfying the minimum cover.
- Each bend of the stirrups should be around a longitudinal bar. The diameter of the longitudinal bar should not be less than the diameter of stirrups.
- The ends of the stirrups should be anchored by standard hooks.
- There should not be any bend in a re – entrant corner. In a re – entrant corner, the stirrups under tension has the possibility to straighten, thus breaking the cover concrete.

15. Differentiate type I, type II and type III structures.

- In a type I member, no tensile stress is allowed under service loads or at transfer.
- In a type II member, tensile stresses are allowed but they should be within the cracking stress.
- For a type III member, the tensile stress can exceed the cracking stress, but still it is limited to a certain value which limits the crack width.

16. What are the variables which influence the moment and shear along the length of the beam?

The types and formation of cracks that depends on the span to depth ratio of the beam and loading influences the moment and shear along the length of the beam.

17. What is meant by unbounded tendons?

For members with unbounded tendons and with the span/depth ratio not exceeding 35, the stress n the tendons is computed by the relation,

$Fps = (fps+70+(fc/100\rho_P))$

18. What are the stages of loading to be considered in design of prestressed concrete section for flexure?

Two stages of loading are to be considered in design of prestressed concrete section for flexure are as,1)Transfer of prestressing force, 2)At working load (service stage)

19. Write the principles of mechanics for the analysis of axial load and flexure in PSC structures.

The analysis for axial load and flexure are based on the flowing principles of mechanics.

- Equilibrium of internal and external forces
- Compatibility of strains in concrete and steel
- Constitutive relationships of materials.

20. List out five factors which influence the ultimate flexural strength of PSC beams.

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- Beams are statically determinate.
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(AUC Apr/May 2010)

An Over reinforced members fail by the sudden crushing of concrete. The failure being reinforced members fail by the sudden crushing of concrete. The failure being characterized by small deflections and narrow cracks, the area of steel being comparatively large, the stresses developed in steel at failure of the member may not reach the tensile strength.

22. Define degree of prestressing.

A measure of the magnitude of the prestressing force related to the resultant stress occurring in the structural member at working load.

23. Define Proof stress.

The tensile stress in steel which produces a residual strain of 0.2 percent of the original gauge length on unloading.

24. Define cracking load.

The load on the structural element corresponding to the first visible crack.

25. Define Debonding.

Prevention of bond between the steel wire and the surrounding concrete.

26. Write formula for Moment of resistance in BIS code.

$Mu = A_{pb} A_{ps} (d - d_n)$

UNIT III DEFLECTION AND DESIGN OF ANCHORAGE ZONE

1. **Define partial prestressing.** **(AUC May/June 2013, Nov/Dec 2011)**

The degree of prestress applied to concrete in which tensile stresses to a limited degree are permitted in concrete under working load. In this case, in addition to tensioned steel, a considerable proportion of untensioned reinforcement is generally used to limit the width of cracks developed under service load.

2. **Mention any two functions of end blocks.** **(AUC May/June 2013, Nov/Dec 2013)**

16. Provide Lateral (horizontal) stability from wind and other horizontal (Racking) loads.
17. Provide additional vertical load capacity for the ends of the joists from point loads above.

3. Define anchorage zone. **(AUC Nov/Dec 2011)**

Prestressed concrete contains tendons which are typically stressed to about 1000 MPa. These tendons need to be anchored at their ends in order to transfer (compressive) force to the concrete. The zone of region is called Anchorage zone.

4. How can PSC beam be considered to carry its own weight? **(AUC Nov/Dec 2012)**

By providing an external initial stress (the prestress) which compresses the beam. Now they can only separate if the tensile stress induced by the self weight of the beam is greater than the compressive prestress introduced.

5. What is effective reinforcement ratio? **(AUC Apr/May 2012)**

Ratio of effective area of reinforcement to the effective area of concrete at any section of a structural member is known as effective reinforcement ratio.

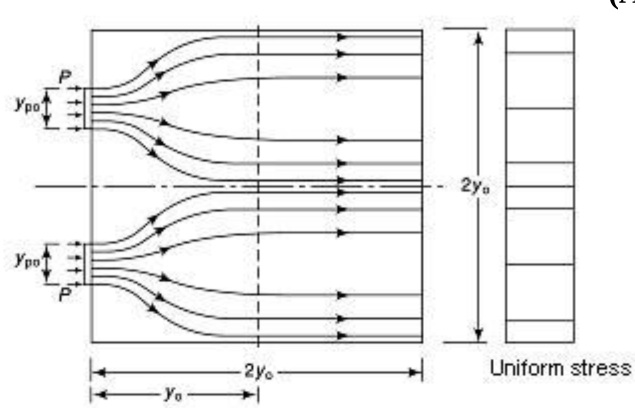
6. At initial stage what forces are considered in prestressed concrete design? **(AUC Apr/May 2011)**

Prestressing force is considered in prestressed concrete design at initial stage.

7. Why anchorage zone has to be given special attention in design? (AUC Apr/May 2011)

Because the main reinforcement in the anchorage zone should be designed to withstand the bursting tension, which is determined by the traverse stress distribution on the critical axis, usually coinciding with the line of action of the largest individual force.

8. Draw a sketch showing the stress distribution in end block by double anchor plate. **(AUC Apr/May 2010)**



Double anchor plate

9. What is meant by end block in a post tensioned member? **J(Nov/Dec 2010)**

The zone between the end of the beam and the section where only longitudinal stress exists is generally referred to as the anchorage zone or end block.

10. List any two applications of partial prestressing. **C(Nov/Dec 2010)**

- Used in large diameter concrete pipes

- Used in railway sleepers
- Water tanks
- Precast concrete piles to counter tensile stress during transport and erection.
- Used in bridges construction

11. Define Bursting tension.

The effect of transverse tensile stress is to develop a zone of bursting tension in a direction perpendicular to the anchorage force resulting in horizontal cracking.

12. Mention any four factors affecting the deflection of the prestressed concrete beam?

(i) Imposed load & self weight
(ii) Magnitude of the prestressing force.
(iii) Second moments of area of cross section
(iv) Shrinkage, creep & relaxation of steel stress.
(v) Modulus of elasticity of concrete
(vi) Cable profile
(vii) Span of the member
(viii) Rigidity condition

13. What are the permissible limits for deflection?

(i) The final deflection due to all loads including the effects of temperature, creep & shrinkage should not normally exceed span / 250.

(ii) Deflections including the effects of temperature, creep & span / 350 or 20mm whichever is less

(iii) If finishes are to be applied to prestressed concrete members the total upward deflections should not exceed span.

14. What is meant by primary moment, secondary moment?

- **Primary moment:** The primary moment is the apparent bending moment at a section in a statically indeterminate structure due to the ahead eccentricity of the tendons from the additional moments.
- **Secondary moment**: Secondary moments are additional moments induced at a section due to the redundant reactions developed as a consequence of prestressing the structure.
- **Resultant moment:** The resultant moment at a section of an indeterminate prestressed structure is the sum of primary & secondary moments (i.e) R.M = (P.M + S.M)

15. What is concordant cable?

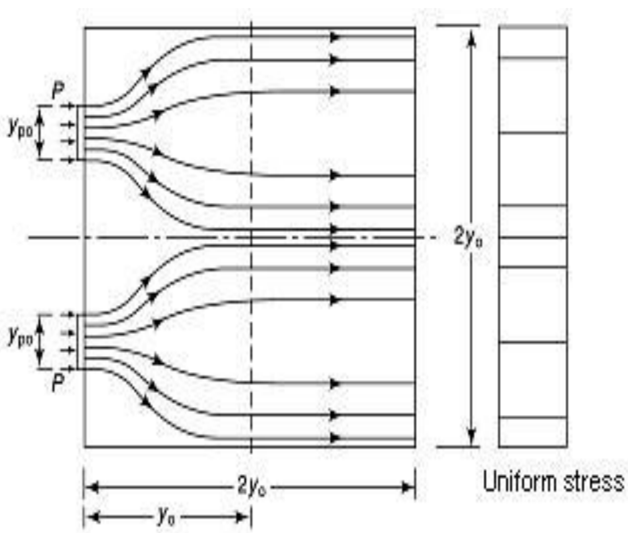
A tendon profile in which the eccentricity is proportional at all cross sections to the bending moment caused by any loading on a rigidity supported statically indeterminate structure is a concordant profile.

16. What is meant by vertical or transverse prestresisng?

Besides the longitudinal prestressing sometimes it may be desirable to provide vertical prestresisng to reduce or eliminate the principal tensile stress. Vertical prestressing is done by providing high tension vertical steel wires of small diameter at suitable pitch & stressed adequately.

17. Enumerate stress distribution in end block.

A physical concept of the state of stress in the transverse direction, that is normal to planes parallel with the top and bottom surfaces of the beam, may be obtained by considering these lines of force as individual fibres acting as curved stuts between end force 2P and the main body of the beam.



Double anchor plate

18. Why reinforcement is necessary in anchorage zone or end block?

In post tensioned members, the prestressing wires are introduced in cables holes or ducts. Pre- formed in the members and then stressed and anchored at the end forces. Large forces concentrated over relatively small areas are applied on the end blocks. It is linearly distributed and develop transverse and shear stressed so that an adequate amount of steel is properly distributed to sustain the transverse tensile stressed.

19. What is meant by transmission length?

In a pretensioned system, when a wire is released, the transmission of prestressing force from steel to concrete is though a bond comprising of adhesion, friction and shearing resistance. At intermediate points along the length of the beam, the bond stress due to friction and shearing resistance is developed. When the bond stress is zero, and uniform stress distribution is prevalent from the section then the length for achieving this is termed as transmission length.

20. Why anchor block is used in prestressed concrete?

Anchor block having higher discontinuous force applied at the end develop transverse and shear stresses. The distribution of stressed in the anchorage zone, can provide an adequate amount of steel, properly distributed to sustain the transverse tensile stresses.

21. Why anchorage zone has to be given special attention in design?

Large amount of prestressing forces, concentrated over relatively small areas are applied on the end blocks through bearing plates. These forces develop transverse and shear stressed. Generally bursting tensile forces or splitting tensile force and spalling tensile forces are developed while transmitting the prestress to the concrete. These forces are resisted by providing suitable arrangements of reinforcement in the end blocks.

UNIT 4 – COMPOSITE CONSTRUCTION AND CONTINUOUS BEAMS

1. Define propped construction. (AUC May/June 2013, Nov/Dec 2013)

The dead load stress developed in the precast prestressed units can be minimized by propping them while casting the concrete in situ. This method of construction is termed as propped construction.

2. How to achieve compositeness between precast and cast in situ part and show the sketches? (AUC May/June 2013, Nov/Dec 2013)

The composite action between the two components is achieved by roughening the surface of the prestressed unit on to which the concrete is cast in situ, thus giving a better frictional resistance or by stirrups protruding from the prestressed unit into the added concrete or by castellations on the surface of the prestressed unit adjoining the concrete which is cast in situ.

3. What is meant by composite construction of prestressed and in situ concrete? (AUC Nov/Dec & Apr/May 2011)

In a composite construction, precast prestressed members are used in conjunction with the concrete cast in situ, so that the members behave as monolithic unit under service loads. The high strength prestressed units are used in the tension zone while the concrete, which is the cast in situ of relatively lower compressive strength is used in the compression zone of the composite members.

4. How deflections in composite members are computed? (AUC Nov/Dec 2011)

In the case of composite members, deflections are computed by taking into account the different stages of loading as well as the differences in the modulus of elasticity of concrete
in the precast prestressed unit and the in situ cast element.

5. What do you mean by unpropped construction? (AUC Nov/Dec 2012)

If the precast units are not propped while placing the in situ concrete, stresses are developed in the unit due to the self weight of the member and the dead weight of the in situ concrete. This method of construction is referred to as unpropped construction.

6. What are the forces considered in the calculation of deflection of prestressed concrete beams? (AUC Apr/May 2010)

Prestressing force
Self weight of the beam
Dead load of the concrete
Live load acting on the concrete

7. What are the roles played by shear connectors in composite construction? (AUC Apr/May 2010)

It is generally assumed that the natural bond at the interface contributes a part of the required shear resistance depending upon the strength of the in situ cast concrete and the roughness of the precast element. Any extra shear resistance over and above this should be provided by shear connectors.

8. What are the advantages in using precast prestressed units? (AUC Apr/May 2011, Nov/Dec 2010 & 2012)

- Saving in the cost of steel in a composite member compared with a reinforced or prestressed concrete member.
- Sizes of precast prestressed units can be reduced due to the effect of composite action.
- Low ratio of size of the precast unit to that of the whole composite member.
- Composite members are ideally suited for construction bridge decks without the disruption of normal traffic.

9. Name the loadings to be considered for computing initial deflection. (AUC Nov/Dec 2010)

- Prestress
- Self weight of the beam
- Weight of the in situ cast concrete

10. How do you compute the shrinkage and resultant stresses in composite member? (AUC Nov/Dec

2012)

The magnitude of differential shrinkage is influenced by the composition of concrete and the environmental conditions to which the composite member is exposed. In the absence of exact data, a general value of 100 micro strains is provided for computing shrinkage stresses.

11. Distinguish between propped and unpropped construction methods.

(AUC Nov/Dec 2012)

S.No	Propped construction	Unpropped construction
1	The dead load stress developed in the precast prestressed units can be minimized by propping them while casting the concrete in situ. This method of construction is termed as propped construction.	If the precast units are not propped while placing them in situ concrete, stresses are developed in the unit due to the self weight of the member and the dead weight of the in situ concrete. This method of construction is referred to as unpropped construction.
2	If the pretensioned beam supports the weight of the slab while casting.	If the slab is externally supported while casting.

12. What are the assumptions made in stresses developed due to differential shrinkage?

- The shrinkage is uniform over the in situ part of the section.
- Effect of creep and increase in modulus of elasticity with age and the component of shrinkage, which is common to both the units are negligible.

18. Name the loadings to be considered for computing deflection if the beam is propped section.

- Prestress
- Self weight of the beam
- Dead weight of the in situ cast concrete
- Live load of the in situ cast concrete

21. Name the loadings to be considered for computing deflection if the beam is unpropped section.

- Prestress
- Self weight of the beam
- Live load of the in situ cast concrete

15. Sketch the typical cross section of precast prestressed concrete beam.

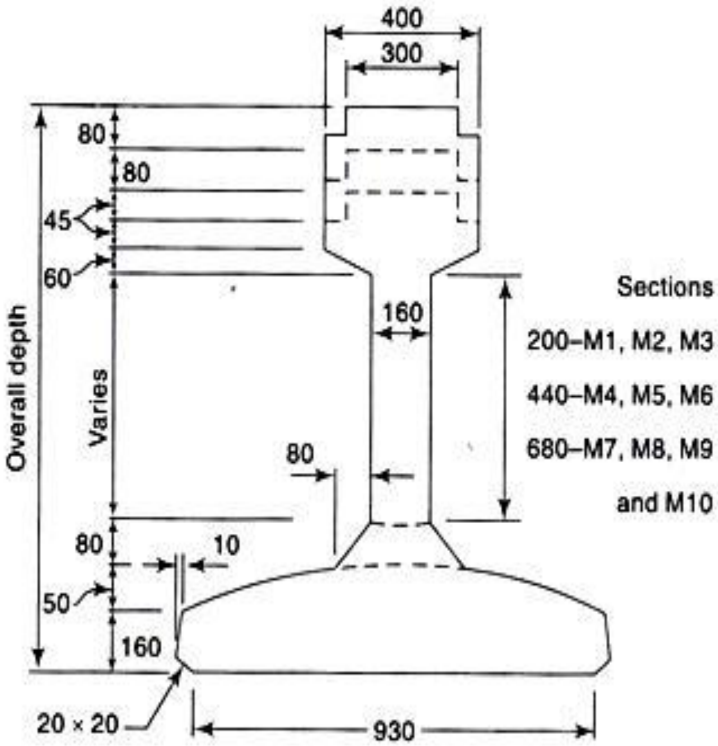


Fig. *Cross-section of Standard C and C.A. Beams*

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16. What are the advantages of statically indeterminate prestressed concrete structures?

(i)The bending moments are more evenly distributed between the centre of span and the supports of members.

(ii) Reduction in the sizes of members results in lighter structure.

(iii) The ultimate load carrying capacity in higher than the 01statically determinate structure due to the redistribution of moment

(iv) Continuity of the members in framed by segmental construction using precast units connected by prestressed cables.

(v) In continuous post tensioned guides, the curved cables can be suitably positioned to resist the span & support moments.

17. What are the disadvantages of prestressed continuous beams?

(i)Loss of prestress due to friction is more in long cables.

(ii) Secondary stresses due to prestressing, creep, shrinkage, temperature & settlement of supports may induce very high stresses unless they are controlled.

(iii)Cable positioned to cater for secondary moments are generally no suitable to provide the required ultimate moment under a given system of loads.

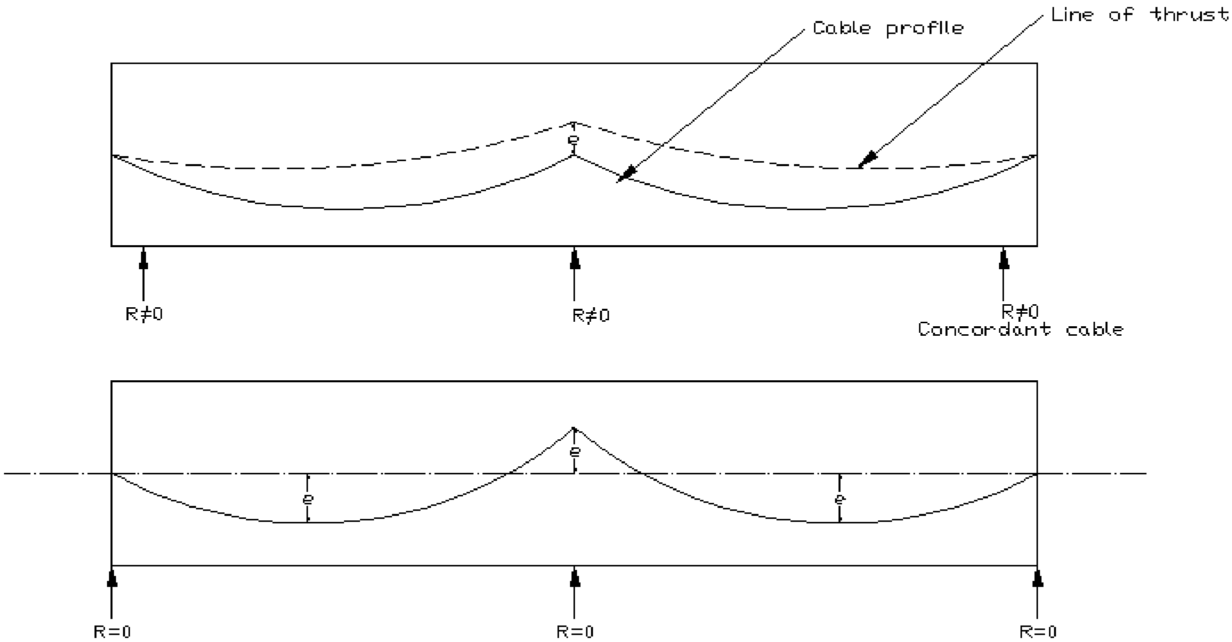
(iv)The computation of collapse (or) ultimate load is influenced by the degree of redistribution of moments.

18. What are the assumptions made for the analysis of secondary moment

(i) The effect of change in length of members due to the prestressing force & external loading in neligible.

(ii) The cable friction is considered to be negligible so that the prestressing force is constant at all points of the cable.

19. Show the resultant thrust line in a two span continuous beam. Prestressed by a parabolic cable with zero eccentricity at all the support



20. What is meant by composite construction is prestressed concrete?

In a composite construction, precast prestressed members are used in conjunction with the concrete cast in site. So that the members behave as a monolithic unit under service loads.

21. What are the advantages of composite constructions?

(i) Appreciable savings in the cost of steel in a composite member compared with a R.C.C. or prestressed concrete members.

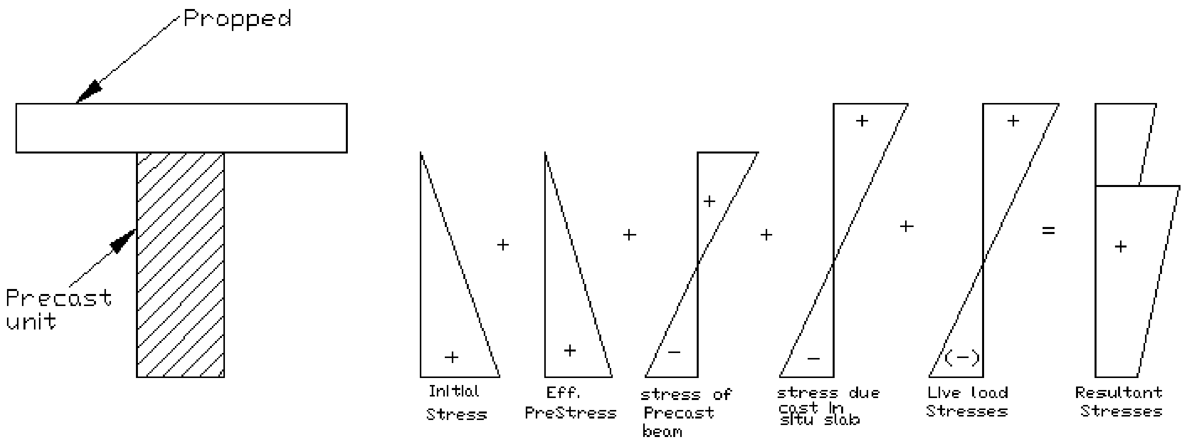
(ii) Sizes of the precast prestressed units can be reduced due to the effect of composite action.

(iii)Low ratio of size of precast unit to that of the composite member & in many cases precast prestressed units serves as supports & dispenses with the form work for the placing of in situ concrete.

(iv) Composite constructions are indically suited for the constructions of bridge decks without disturb the normal traffic.

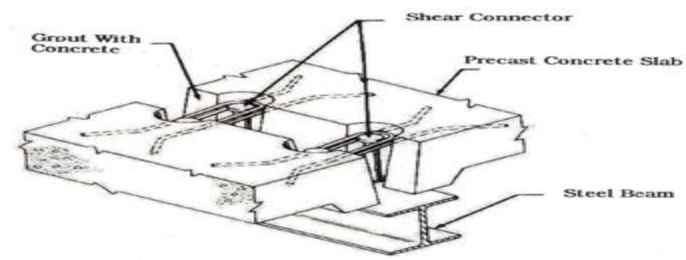
(v) Efficient utilization of materials in a composite section results in reduced dead loads & leading to overall economy.

22. Sketch the stress across the mid span section as various stages of loading in propped composite flexure members.



23. What is meant by shear connectors?

Effective bonding between the two parts of a composite beam may be developed by providing castellation in the precast unit or by projecting reinforcements from the precast unit is known as shear connectors.



UNIT 5 - MISCELLANEOUS STRUCTURES

1. What are the functions of water stopper (water bar) in water tank construction?
(AUC May/June 2013)

19. The base slab is subdivided by joints which are sealed by water stops.
20. The reinforcement in the slab should be well distributed to control the cracking of the slab due to shrinkage and temperature.

2. Differentiate prestressed cylinder and non-cylinder pipe. (AUC May/June 2013)

Prestressed cylinder pipe:

22. It is developed by the Lock Joint Company.
23. A welded cylinder of 16 gauge steel is lined with concrete inside and steel pipe wrapped with a highly stressed wire.
24. Tubular fasteners are used for the splices and for end fixing of the wire and pipe is finished with a coating of rich mortar.
25. It is suitable upto 1.2 m diameter.

Prestressed non-cylinder pipe:

- It is developed by Lewiston Pipe Corporation.
- At first concrete is cast over a tensioned longitudinal reinforcement.
- A concrete pipes after curing are circumferentially stressed by means of a spiral wire wound under tension and protected by a coat of mortar.
- The main function of longitudinal prestress is to prevent cracking in concrete during circumferential winding and cracking due to the bending stresses developed during the handling and installation of pipes.

3. Define circular prestressing. (AUC Nov/Dec 2011, 2012, 2013, 2010)

When the prestressed members are curved in the direction of prestressing, the prestressing is called circular prestressing.

For example, circumferential prestressing in pipes, tanks, silos, containment structures and similar structures is a type of circular prestressing.

4. What are the design criteria for prestressed concrete tanks? (AUC Nov/Dec 2011)

14. It is to resist the hoop tension and moments developed are based on the considerations of desirable load factors against cracking and collapse.
15. It is desirable to have at least a minimum load factor of 1.2 against cracking and 2 against ultimate collapse as per IS code.
16. It is desirable to have at least a minimum load factor of 1.25 against cracking and 2.5 against ultimate collapse as per BS code.
17. The principal compressive stress in concrete should not exceed one-third of the characteristic cube strength.
18. When the tank is full, there should be a residual compressive stress of at least 0.7 N/mm^2.

- When the tank is empty, the allowable tensile stress at any point is limited to 1 N/mm^2.
- The maximum flexural stress in the tank walls should be assumed to be numerically equal to 0.3 times the hoop compression.

5. What are the design criteria for prestressed concrete pipes? (AUC Nov/Dec 2012)

- Circumferential prestressing, winding with or without longitudinal prestressing.
- Handling stresses with or without longitudinal prestressing.
- Condition in which a pipe is supported by saddles at extreme points with full water load but zero hydrostatic pressure.
- Full working pressure conforming to the limit state of serviceability.
- The first crack stage corresponding to the limit state of local damage.

6. How are the tanks classified based on the joint? (AUC Nov/Dec 2013)

- Tank wall with fixed base.

- Tank wall with hinged base.
- Tank wall with sliding base.

7. Define two stage constructions. (Apr/May 2012)

In the first the concrete is cast over a tensioned longitudinal reinforcement. In the second stage the concrete pipes after curing are circumferentially stressed by means of a spiral wire wound under tension and protected by a coat of mortar.

8. Write any two general failures of prestressed concrete tanks. (AUC Apr/May 2012)

- Deformation of the pre-cast concrete units during construction.
- Manufacturing inaccuracies led to out of tolerance units being delivered to the site under investigation.
- It May have affected the ability to achieve a good seal.

9. What is the stress induced in concrete due to circular prestressing? (AUC Apr/May 2010)

The circumferential hoop compression stress is induced in concrete by prestressing counterbalances the hoop tension developed due to the internal fluid pressure.

10. Explain the effect of prestressing force in concrete poles. (AUC Apr/May 2010)

It should be reduced in proportion to the cross section by the techniques of debonding or dead ending or looping some of the tendons at mid height.

11. Write the various types of loadings that act on prestressed concrete poles. (AUC Nov/Dec 2010)

- Bending due to wind load on the cable and on the exposed face.
- Combined bending and torsion due to eccentric snapping of wires.
- Maximum torsion due to skew snapping of wires.
- Bending due to failure of all the wires on one side of the pole.
- Handing and erection stresses.

12. What are the advantages of prestressing water tanks? (AUC Apr/May 2011)

- Water storage tanks of large capacity are invariably made of prestressed concrete.
- Square tanks are used for storage in congested urban and industrial sites where land space is a major constraint.
- This shape is considerable reduction in the thickness of concrete shell.
- The efficiency of the shell action of the concrete is combined with the prestressing at the edges.

3. How are sleepers prestressed? (AUC Apr/May 2011)

- Two block sleepers

- Longitudinal sleepers
- Beam type single piece prestressed concrete sleepers.

14. Mention the importance of shrinkage in composite construction?

The time dependent behavior of composite prestressed concrete beams depends upon the presence of differential shrinkage and creep of the concretes of web and deck, in addition to other parameters, such as relaxation of steel, presence of untensioned steel, and compression steel etc.

15. What are the different types of joints used between the slabs of prestressed concrete tanks?

- Movement joint
- Expansion joint
- Construction Joint
- Temporary Open Joints.

16. What are the advantages of partially prestressed concrete poles?

- Resistance to corrosion in humid and temperature climate and to erosion in desert areas.
- Easy handling due to less weight than other poles.
- Easily installed in drilled holes in ground with or without concrete fill.
- Lighter because of reduced cross section when compared with reinforced concrete poles.
- Fire resisting, particularly grassing and pushing fire near ground line.

17. What are the types of prestressed concrete pipes?

- Monolyte construction
- Two stage construction

18. Distinguish between non-cylinder and cylinder pipes. Non-cylinder pipes:

The design principles are used for determining the minimum thickness of concrete required and the pitch of circumferential wire winding on the pipe.

Cylinder pipes:

The design principles of cylinder pipes are similar to those of the non-cylinder pipes except that the required thickness of concrete is computed by considering the equivalent area of the light gauge steel pipe embedded in the concrete.

19. Define the losses of prestress.

Due to elastic deformation of concrete during circumferential wire winding, there is a loss of prestress which depends upon the modular ratio and the reinforcement ratio.

20. What are the advantages of prestressed concrete piles?

- High load and moment carrying capacity.
- Standardization in design for mass production.
- Excellent durability under adverse environmental conditions.
- Crack free characteristics under handling and driving.
- Resistance to tensile loads due to uplift.
- Combined load moment capacity.

21. What are the advantages of prestressed concrete over R.C.C concrete?

(i) The use of high strength concrete and steel in prestressed members results in lighter and slender members than is possible with reinforced concrete.

(ii) The effectiveness of carrying external loads is only by the section above the neutral axis is reinforced concrete but the entire cross section is effective is prestressed concrete.

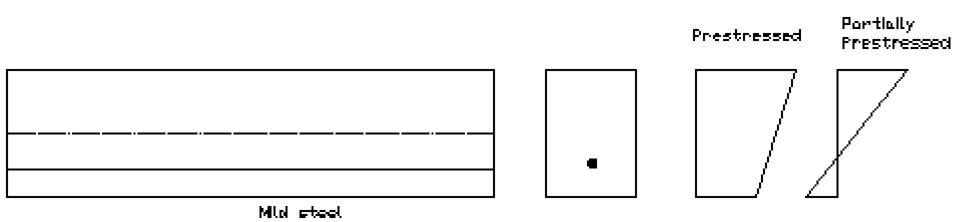
(iii)The reinforced concrete sections are heavy and shear reinforcement is essential where as in prestressed concrete the section is smaller & curved tendons helps to resistance.

(iv) Do the long span structures the prestressed concrete is generally more economical than reinforced concrete & also \prestressed members are decrease in weight reduces the design loads and the cost of construction.

(v) Due to utilization of concrete in the tension zone a savings of 15 to 30% in concrete 60 to 80% savings in steel.

22. Explain partial prestressing.

Under the working load, if the cross section is subjected to no tension after prestressing then it is known as fully prestressed. lll$_{ly}$ under working loads even after the pretsress is apply. If there is some tension. It is known as partial prestressing . Normally the tension portion is reinforced with mild steel reinforcement. This untensioned reinforcement is required so as to resist differential shrinkage temperature effects and handling stresses.



23. Mention the advantages of partial prestressing. **(AUC Nov/Dec 2012 & 2013)**

26. Limited tensile stresses are permitted in concrete under service loads with controls on the maximum width of cracks and depending upon the type of prestressing and environmental condition.
27. Untensioned reinforcement is required in the cross-section of a prestressed member for various reasons, such as to resist the differential shrinkage, temperature effects and handling stresses.
28. Hence this reinforcement can cater for the serviceability requirements, such as

control of cracking, and partially for the ultimate limit state of collapse which can result in considerable reduction in the costlier high tensile steel.

29. Saving in the cost of overall structure.

PART-B QUESTIONS (16 MARKS)

UNIT-1

INTRODUCTION – THEORY AND BEHAVIOUR

1. A rectangular concrete beam 100mm wide & 250mm deep spanning over 8m is prestressed by a straight cable carrying a effective prestressing force of 250kN located at an eccentricity of 40mm. The beam supports a live load of 1.2 kN/m. a) calculate the resultant stress distribution for the centre of the span cross section of the beam assuming the density of concrete as $24kN/m^2$ b) Find the magnitude of prestressing force with an eccentricity of 40mm which can balance the stresses due to dead load & live load at the soffit of the centre span section.

2. A PSC beam of 120mm wide and 300mm deep is used over a span of 6m to support a udl of 4kN/m including its self weight. The beam is prestressed by a straight cable carrying a force of 180kN & located at an eccentricity of 50mm. Determine the location of the thrust line in beam & plot its position at quarter & central span sections.

3. A Prestressed pretensioned beam of 200mm wide and 300mm deep is used over an span of 10m is prestressed with a wires of area $300mm^2$ at an eccentricity of 60mm carrying a prestress of 1200 N/mm^2 Find the percentage of loss of stress, E_c= 35 kN/mm^2 Shrinkage of concrete = 300 x 10^{-6} , creep coefficient =1.6

4. A PSC beam of 120mm wide and 300mm deep is used over an span of 6m is prestressed by a straight cable carrying a force of 200 kN & located at an eccentricity of 50mm. Ec= 38 kN/mm^2 . Find the deflection at centre span a) Under prestress + self weight b) Find the magnitude of live load udl which will nullifythe deflection due to prestress & self weight.

5. A PSC beam of 230mm wide and 450mm deep is used over an span of 4m is prestressed by a cable carrying a force of 650kN & located at an eccentricity of 75mm. The beam supports three concentrated loads of 25kN at each quarter span points. Determine the location of the pressure line in beam at centre, quarter & support sections. Neglect the moment due to self weight of the beam.

6. A PSC beam with rectangular section, 150mm wide 300mm deep is prestressed by three cables each carrying a effective prestress of 200kN. The span of the beam is 12m. The first cable

is parabolic with an eccentricity of 50mm below the centroidal axis at the centre of the span and 50mm above the centroidal axis at the supports.The second cable is parabolic with an eccentricity of 50mm at the centre of the span and zero eccentricity at the supports. The third cable is straight with an eccentricity of 50mm below the centroidal axis.If the beam supports an UDL of 6kN/m and Ec=38kN/mm2 Estimate the instantaneous deflection for the following stages i) Prestress + self weight of the beam ii) Prestress + self weight of the beam + live load

7. (i) Explain why high strength concrete and high strength steel are needed for PSC construction

(ii) State different types of prestressing

8. (i) Explain shrinkage of concrete in PSC members

(ii) Explain durability, fire resistance and cover requirements for PSC members

9. A PSC beam supports an imposed load of 5kN/mm^2 over a simply supported span of 10m.The beam has an I section with an overall depth of 450mm.Thickness of flange and web are 75mm and 1000mm respectively. The flange width is 230mm. the beam is prestressed with an effective prestressing force of 350kN at a suitable eccentricity such that the resultant stress at the soffit of the beam at mid span is zero. Find the eccentricity required for the force.

10. A PSC beam of section 120mm wide and 300mm deep is used over an effective span of 6m to support an udl of 4kN/m including self weight. The beam is prestressed by a straight cable with a force of 180kN and located at an eccentricity of 50mm. Determine the location of thrust line in the beam and plot its position.

UNIT -2

DESIGN FOR FLEXURE AND SHEAR

1. A pretensioned T section has a flange width of 1200mm and 150mm thick. The width and depth of the rib are 300mm and 1500mm respectively. The high tension steel has an area of 4700mm^2 and is located at an effective depth of 1600mm. If the characteristic cube strength of the concrete and the tensile strength of steel are 40 and 1600Mpa respectively; calculate the flexural strength of the section.

2. A PSC beam of effective span 16m is of rectangular section 400mm wide and 1200mm deep. Atendons consist of 3300mm^2 of strands of characteristic strength 1700 N/mm^2 with an effective prestress of 910 N/mm^2. The strands are located 870mm from the top face of the beam. If f_{cu} =60 N/mm^2, estimate the flexural strength of the section as per BS provisions for the following cases: (i) Bonded tendons (ii) Unbonded tendons

3. A post tensioned bridge girder with unbonded tendons is of size 1200mm wide by 1800mm deep is of box section with wall thickness of 150mm. The high tensile steel has an area of

4000mm^2 and is located at an effective depth of 1600mm. The effective prestress in steel after loss is 1000 N/mm^2& effective span is 24m. If f_{ck} = 40 N/mm^2, f_p = 1600 N/mm^2 Estimate the flexural strength.

4. The end block of a PSC beam with rectangular cross section is 100mm wide and 200mm deep. The prestressing force of 100kN is transmitted to the concrete by a distribution plate of 100mm x 50mm, concentrically loaded at the ends. Calculate the position and the magnitude of tensile stress on the horizontal section through the centre and edge of the anchor plate. Compute the bursting tension on the horizontal planes.

5. The end block of a post tensioned concrete beam 300mm X 300mm is subjected to a concentric anchorage force of 800kN by a freyssinet anchorage system of area 1100mm^2. Discuss and detail the anchorage reinforcement for the end block.

6. Discuss the advantages and disadvantages of partial prestressing.

7. A symmetrical I section prestressed beam of 300mm wide and 750mm overall depth with flanges and web 100mm thick. The beam is post tensioned with the cables containing 48 wires of 5mm diameter high strength steel wires at an eccentricity of 250mm. The compressive strength of concrete is 40N/mm^2 and the ultimate tensile strength of wire is 1700N/mm^2. Assuming that the grouting of tendons is 100% effective determine the ultimate moment of section as per IS1343:1980.

8. A PSC beam 250mm wide and 650mm deep is subjected to an effective prestressing force of 1360kN along the centroidal axis. The cable is placed symmetrically over the mild steel anchor plate of area 150mm x 350mm. design the end block.Take f_{ck} = 30N/mm^2. Assume initial prestressing force is 1.2 times the effective prestressing force.

9. (i) Discuss the load deflection behavior of under prestressed, partially prestressed and over prestressed members in detail.

(ii) Explain concept of limit states, partial safety factor.

10. a) What is meant by partial prestressing? Discuss the advantages and disadvantages when partial prestressing is done.

b) Explain about the types of flexure failure occurs in prestressed concrete section

UNIT-3

DEFLECTION AND DESIGN OF ANCHORAGE

1. A cylindrical PSC water tank of internal diameter 30m is required to store water over a depth of 7.5m. The permissible compressive stress in concrete at transfer is 13 N/mm^2 and the

minimum compressive stress under working pressure is 1 N/mm^2. The loss ratio is 0.75. Wires of 5mm diameter with an initial stress of 1000 N/mm^2 are available for circumferential winding and Freyssinet cables made up of 12 wires of 8mm diameter stressed to 1200 N/mm^2 are to be used for vertical prestressing. Design the tank walls assuming the base as fixed. The cube strength of concrete is 40 N/mm^2

2. A prestressed cylindrical pipe is to be designed using a steel cylinder of 1000mm diameter and thickness 1.6mm. The circumferential wire winding consist of a 4mm high tensile wire initially tensioned to a stress of 1000 N/mm^2. The ultimate tensile strength of wire is 1600 N/mm^2. The yield stress of the steel cylinder is 280 N/mm^2. f_{ct}=14 N/mm^2, W_w =0.8 N/mm^2. Determine the thickness of concrete lining required. Fmin = 0; modular ratio = 6

3. Design a free edge water tank of diameter 36m to store water for a depth of 5m. Assume ultimate stress in steel = 1500N/mm^2. Stress in steel at transfer = 70% of ultimate stress. Safe stress in concrete = 0.5f_{ck}. Compressive stress in concrete at service condition= 0.1f_{ck}. Final stress in steel = 0.8 x stress in steel at transfer. Take modular ratio=5.5 f_{ck} = 45N/mm^2

4. Explain the different types of joints between the walls and floor slab of prestressed concrete tanks.

5. Write the design criteria of PSC pipes in detail

6. Explain the step by step design procedure of circular tanks.

7. Explain the types of PSC pipes with neat sketch

8. A non cylindrical PSC pipe of 1000mm diameter and thickness of concrete shell is 75mm is required to convey water at a working pressure of 1.5 N/mm^2. The length of the pipe is 6m. The loss ratio is 0.8. Determine the circumferential wire winding of using 5mm diameter wires stretched to 1000 N/mm^2. The maximum permissible tensile stress is 11.2 N/mm^2

9. Design a non cylindrical PSC pipe of 600mm internal diameter to withstand a working hydrostatic pressure of 1.05 N/mm^2using 2.5mm HYSD stressed to 1000N/mm^2 at transfer. Permissible maximum and minimum stresses in concrete at transfer and service load are 14 N/mm^2 and 0.7 N/mm^2. The loss ratio is 0.75. E_s = 210kN/mm^2 and E_c = 35kN/mm^2

10. Explain any one method of circumferential wire winding adopted in circular prestressing with a neat sketch

UNIT-4

COMPOSITE BEAMS AND CONTINUOUS BEAMS

1. Explain the design procedure of Prestressed composite section.

2. A precast pretensioned beam of rectangular section has a breadth of 100mm and depth of 200mm. The beam with an effective span of 5m is prestressed by the tendons with their centroids coinciding with the bottom kern. The initial force in the tendons is 150kN. The loss of prestress is 15%. The top flange width is 400mm with the thickness of 40mm.If the composite beam supports a live load of 8kN/m^2 calculate the resultant stresses developed if the section is propped and unpropped.

3. A composite T beam is made up of pretensioned rib of 100mm wide and 200mm deep and a cast insitu slab of 400mm wide and 40mm thick. Having the modulus of elasticity as 28kN/m^2, if the differential shrinkage is 100 x 10^{-6} determine the shrinkage stresses developed in precast and cast insitu units.

4. A composite T-girder of span 5 m is made up of a pre-tensioned rib, 100 mm wide by 200 mm depth, with an in situ cast slab, 400 mm wide and 40 mm thick. The rib is prestressed by a straight cable having an eccentricity of 33.33 mm and carrying initial force of, 150 kN. The loss of prestress is 15%. Check the composite T-beam for the limit state of deflection if its supports an imposed load of 3.2 kN/m for (i) unpropped(ii) propped. Assume modulus of Elasticity of 35 kN/mm^2 for both precast & in situ cast elements.

5. i) Explain the types of composite construction with neat sketch.

(ii) Explain the precast prestressed concrete stresses at serviceability limit state.

6. A PSC beam of cross section 150 mm x 300 mm is SS over a 6pan of 8m and is prestressed by means of symmetric parabolic cables @ a distance of 76 mm from the soffit @ mid span and 125 mm @ top @ support section. If the force in the cable i.e 350 KN. Calculate deflection @ mid span the beam is supporting its own weight The point load which must be applied at mid span to restore the beam to the level of its support.

UNIT-5

MISCELLANEOUS STRUCTURES

1. What are the advantages of prestressed concrete bridges?

2. Explain the pre tensioned and post tensioned bridge decks commonly used in construction of bridges.

3. Write the design procedure for post tensioned prestressed concrete slab bridge deck.

4. Write the design procedure for pre tensioned prestressed concrete slab bridge deck.

5. Write the design procedure for post tensioned PSC T beam slab bridge deck.

6. State the significance of adopting the combination of pre tensioned and post tensioned tendons in bridge construction.

7. Write the design procedure for post tensioned bridge girders.

8. Explain the methods involved in utilizing precast pretensioned members in bridge construction.

CE6703 – WATER RESOURCES AND IRRIGATION ENGINEERING

TWO MARKS QUESTION AND ANSWERS

SIXTEEN MARKS QUESTION BANK

UNIT – I

1. Briefly discuss about water resources in India and Tamilnadu.
2. Briefly explain the steps involved in water resources planning.
3. Explain the water requirements for irrigation, hydropower generation, navigation, drinking and disposal of sewage and industrial waste?
4. Discuss about single and multipurpose reservoir with its advantages and disadvantages.
5. Discuss the strategies for reservoir operation.
6. Discuss about levees and flood walls?
7. How will you fix the capacity reservoir?
8. Briefly discuss about the flood control methods.
9. Differentiate between structural and non structural measures for flood control?
10. What are the factors affecting sedimentation and control measures for sedimentation?
11. Briefly discuss about the planning of multipurpose reservoir.

UNIT – II

1. Discuss the salient feature of National Water Policy.
2. What is the importance of water resource survey for the development of the country?
3. Mention the importance of various data required for water resource development and how will you collect them?
4. Briefly discuss about Necessity of National Water Policy.

5. Briefly discuss about economics of water resource planning.

6. Briefly discuss about consumptive use of water and the factors affecting consumptive use of water. How will you measure it?
7. Briefly explain the methods for determination of consumptive use?
8. Explain the water characteristics to be investigated?
9. What is Master Plan in water resources? Explain the scope and aims in detail.
10. Briefly discuss about the contents of Master Plan?
11. Briefly discuss about the concept of basin as a unit for development?
12. What are the different characteristics of water? Briefly discuss about it?

UNIT – III

1. Explain the term duty and delta and derive their relationship?
2. Discuss the need for the irrigation projects in the Indian context?
3. Draw the layout of the canal irrigation system and how duty varies between various location from main canal to field channel. Discuss the methods to improve the duty in that system.
4. List out the various type of irrigation efficiencies and discuss any four?
5. Discuss in detail the planning and the development of irrigation project in the Indian context?
6. What are the methods of improving duty?
7. Define irrigation efficiency, draw irrigation system and define all types of efficiencies using that diagram?
8. Discuss some of the important irrigation projects of our country?
9. Discuss I brief the ill effects of irrigation?

UNIT – IV

1. What is the necessity of river training works?

2. Describe in brief different types of river training works?
3. What is mean by guide banks? What are their functions and effects?
4. State the necessity and location of canal falls?
5. Briefly explain about classification of canals?
6. State the factors to be considered for the choice of a suitable type of cross-drainage work?
7. Explain how canals are classified? Discuss the methods to improve canal irrigation system?
8. Explain canal lining?

UNIT – V

1. Briefly explain about canal irrigation?
2. Briefly explain about lift irrigation?
3. Briefly explain about tank irrigation?
4. Briefly explain about flooding methods?
5. Briefly explain about sprinkler irrigation methods?
6. Briefly explain about drip irrigation?
7. Explain briefly the 'Sprinkler' and 'drip' methods of irrigation systems.
8. State the classification of 'surface irrigation'?
9. Classify the method of irrigation and write the objective of any three irrigation methods.
10. Briefly explain about on-farm-development works?
11. What is the need for water user's association?
12. What is meant by 'participatory irrigation management'? Explain the concept briefly. Highlight the relative merits of 'participatory irrigation management' that of a conventional management approach' adopted in Engineering.

CE 6704 ESTIMATION AND QUANTITY SURVEYING

Two Mark Questions and Answers

Unit-I – Estimate of Buildings

1. Define estimate.

An estimate is a computation or calculation of the quantities required and expenditure likely to be incurred in the construction of a work. The estimate is the probable cost of a work and is determined theoretically by mathematical calculation based on the plans and drawing and current rates.

2. Write the importance of estimate.

- Estimate gives an idea of the cost of the work and hence its feasibility can be determined.
- It gives an idea of time required for the completion of the work.
- It is required to invite tenders and quotations and to arrange contract.
- It is also required to control the expenditure during the execution of the work.

3. What is meant by Quantitysurvey?

- Itisthescheduleofallitemsofworkinabuilding.Thesequantitiesarecalculatedfromthedrawing ofthebuilding.Thusquantitysurveygivesquantitiesofworkdoneincaseofeachitemswhenpriced givesthetotalcost.
- In short, quantity survey means calculations of quantities of materials required to complete the work concerned.

4. What are the methods of estimates?

- Detailed estimate
- Abstract estimate

5. What are the types of estimate?

- Preliminary Estimate or Rough cost estimate
- Plinth area estimate
- Cube Rate Estimate or Cubical Content Estimate
- Approximate Quantity Method Estimate
- Detailed Estimate or Item Rate Estimate
- Revised Estimate
- Supplementary Estimate and Revised Estimate.
- Annual Repair or Maintenances Estimate
- Supplementary Estimate

6.Briefly explain about preliminary Estimate.

The estimate which prepared using any rough method to get theapproximatecostconstructionanticipatedinaprojectiscalledanapproximateorroughestimate.Sincethisestimate is normally prepared in the preliminary estimate.

7. Estimate the quantities of brickwork and plastering required in a wall 4m long, 3mhighand30cmthick.Calculatealsothecostiftherateofbrickworkis Rs.32.00percu.mandof plastering is Rs. 8.50 per sq.m.

Quantities of brickwork	= LxBxH
	= 4m x 3m x0.30m
	= 3.6 cu.m
Quantity of plastering (two faces)	= 2 x 4m x 3m
	= 24 sq.m
Cost of brickwork	= 3.6 x320.00
	= Rs.1152.00
Cost of plastering	= 24x8.50
	= Rs.204.00
Total cost	= 1152.00 +204.00
	= Rs.1356.00

8. Define: Detailed estimate.

The estimate, which provides the item wise quantities of works, item wiseunitratesanditemwiseexpenditureanticipatedintheproject/construction, is called a detailed estimate.

9. What are the methods of taking out estimates or methods of detailed estimate?

- Separate wall method or Long wall- short wall method.
- Centre line method
- Crossing method
- Out to Out and in to in method
- Bay method
- Service unit method

10. What are the methods of detailed estimate?

- Plinth area method
- Cube rate method
- Unit cost method.
- Typical bay method.
- Carpet area method.

11. Define: Abstract estimate.

This is the third and final stage in a detailed estimate. The quantities and rates of each item of work, arrived in the first two stages, are now entered in an abstract form. The total cost of each item of work is now calculated by multiplying the quantities and respective rates.

12. Briefly explain about revised estimate.

The estimate, which is prepared when any major change or alteration is made in the plan/ structural arrangement, with or without affecting the estimate cost, and When the estimated cost is likely to exceed by more than 5% during execution, due to increase in the cost of materials and labour or due to increase in the cost of materials and labour or due to alterations in the items of works to get the revised quantities /rates/amount is called a revised estimate.

13. What is meant by Supplementary estimate?

When the work is in progress, some changes or additional works are found to be necessary in a project, an additional estimate is prepared is called supplementary estimate.

14. Define quantity surveyor

A qualified or experienced person who does the above mentioned works (takingoff, squaring, abstracting and billing) is called a quality surveyor.

15. Write the duties of quantity surveyor.

- Preparing bill of quantities (Taking off, squaring, Abstracting and billing)
- Preparing bills for part payments at intervals during the execution of work.
- Preparingbillofadjustmentinthecaseofvariationsordereddduringtheexecution of work
- Giving legal advice in case of court proceedings.

16. Calculate the quality of brickwork in an arch over a 1.80m span opening. The arch is 40cm. thick and the breath of a wall is 40cm.

Radius of the arch = 1.8m
Thickness of arch = 40cm
The breath of wall = 40cm
Mean diameter = 3.60+0.40 = 4m
Mean length of the arch given =1/6 x (22/7) x 4 = 2.1m
Quantity of brickwork = 2.1 x 0.40 x 0.4 = 0.34 cu.m
No of bricks required = 0.34cu.m @ 550 bricks per cu.m = 187

17. Define Centre line method.

This method is suitable for walls of similar cross sections. Here the total Centre line length is multiplied by breadth and depth of respective item to get the total quantity at a time. When cross walls or partitions or verandah walls join with main wall, the centre line length gets reduced by half of breadth for each junction. Such junction or joints are studied carefully while calculating total centre line length. The estimates prepared by this method are most accurate and quick.

18. Write a note on Long wall – short wall method.

In this method, the wall along the length of room is considered to be long wall while the wall perpendicular to long wall is said to be short wall. To get the length of long wall or short wall, calculate first the centre line lengths of individual walls.

19. Briefly explain about bay method.

This method is useful and is generally followed in case of buildings havingseveralbays. Thecostofthetypicalbayisworkedoutandisthenmultipliedbythenumberofbaysinthatbuilding.Th eextracostfortheendwallsanddifferenceinframing, if there is any, should be made, so as to arrive at the correct cost.

20.Write the recommendation for degree of accuracy in measurements.

- Dimensions of works shall be measured to an accuracy of 0.01 m
- Thickness of R.C works shall be measured to an accuracy of 0.0005 m
- Areas of works shall be calculated to the nearest 0.01 m^2
- Volumes of work shall be calculated to the nearest 0.01 m^3
- Volumes of wood shall be calculated to the nearest 0.001 m^3

21. Define: Floor area.

It defined as covered area i.e plinth areas excluding area of walls (generally 10%-15%) sills of the doors are not included in floor area. The floor area of very storey shall be measured separately.

22. Define: Carpet area.

This means area in a building which is useful one i.e area of drawing room, dining room bedroom etc. Areas of kitchens, staircase, stores, verandahs, entrance hall, bathroom, basement etc. are excluded. It is generally 50% to 60% of the plinth area.

23. What is meant by Lineplan?

Line plan is the plan of a particular construction simply showing main features with the help of the single lines of different portions of the constructions. Details of constructions are not generally shown on this plan. This inside and outside dimensions shown on this plan should necessarily be corresponding to actual dimensions.

24. Write a short note on Centre Line Plan.

Thisisactuallyalayoutplandrawntofacilitatethelayingoutoffoundation lines andotherfeatures.Itisgenerallyfixedontheentranceoratexitinthe centralplaceofthecolonyfortheguidanceoftheinhabitantsandoutsiders.

25. Write the unit of measurement of earthwork, foundation, brickwork, plastering, steel work, and painting.

Earthwork	-	cum
Foundation	-	cum
Brickwork	-	cum
Plastering	-	sq.m
Steel work	-	quintal
Painting	-	sq.m

Unit-II – Estimate of Other Structures

1. Define: Culvert.

It is device used to channel water. It may be used to allow water to passunderneath a road, railway, or embankment.

2. What is meant by Load bearing structures?

It is one in which a wall of a structure bears the weight and force resting upon it,conducting the vertical load from the upper structure to the foundation.

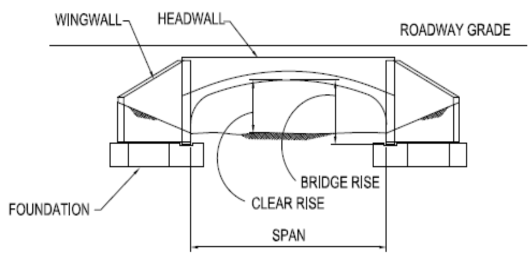
3. What are the main components or parts of culvert?

- Abutments
- Wing walls
- Arch

4. What are the types of culvert?

- Arch culvert
- Slab culvert
- Pipe culvert
- Box culvert

5.Draw differentparts of culvert.



6. What are the methods adopted for the volume calculating?

- From cross-section
- From spot level
- From contours

7. What are factors to be considered in design of septic tank?

- The following factors should be taken into consideration:
- Material should be water proof and corrosion resistant.
- Natural ventilation provided should be adequate
- A manhole should be provided to permit inspection and cleaning.
- Baffles should be limited to one at the inlet and one at the outlet.
- The escape of gas and sludge to effluent pipe should be avoided.

8. Define: Soak pit.

A soak pit, also known as a soak away or leach pit is a covered, porous-walled chamber that allows water to slowly soak into the ground. Pre-settled effluent from a collection and storage/treatment or (semi-) centralized treatment technology is discharged to the underground chamber from which it infiltrates into the surrounding soil.

9. Write the importance of soak pit.

Disposal of effluent from the septic tank may be done by absorption in soil by soak pit; the size and length depend on the number of users and nature of soil.

10. Define lead.

Lead is the crow flying horizontal distance from the centre of borrow pit to the centre of the earth work at site, i.e centre of the area of excavation to the centre of placed earth.

11. Define lift.

Lift is the distance through which the excavated soil is lifted beyond a certain specified depth.

12. How to estimate bituminous and cement concrete road in general.

In both the roads the following steps can be followed.

- Name of the road indicating chainage
- Drawing showing L/S &C/S with all details
- Subgrade soil details
- Design details &intensity of traffic details.
- Life cycle cost that includes cost of construction & maintenance
- Report accompanying the estimate justifying the work to be taken up
- Estimate to be as per the IRC codes.

13. Define: Soling.

Soling is done before laying the foundations and to provide better strength to the foundation as per the specification or requirement at site. It is done by means of stone aggregate of random sizes between 50mm - 150mm.

Types:

Stone flat soling

Brick flat soling

14.Workout the quality of stone metal required for 2Km.Length for wearing coat of a4mwide road. The thickness of the metal road required is 12cmloose.

Quality of metal = 1 X 2 X 1000 X 4 X 0.12 = 960.00cu.m

15. An approach road 2Km.long is to be constructed. Work out the quantity ofmaterials required i.e. stone metal and bricks. Data is given below.

Length = 2Km

Metalled width =3.60m

Soiling of bricks =10cm

Wearing coat of stone metal = 12cmSolution

Solution:

Quantity of bricks = 1 x 2 x 1000 x 3.60 x 0.10 = 720 cu.m

No of bricks = 720.0 x 3.60 x0.12 =3, 60,000

Stone metal = 1 x 2000 x 3.60 x 0.12

= 864cu.m

Bricks = 3, 60,000Nos.

16. A cement concrete road (1:2:3) is to be constructed over the existing waterboundmacadamroad.Thethicknessofslab=10cm.Thelengthoftheroadisonekmandthe width 3.60m.Calculate the quality of cement concrete and the material required,

Solution:

Quality of cement concrete = 1 x 1000 x 3.60 x 0.10 = 360cu.m34.

17. Calculate the quality of earthwork for the construction of an approach road.

Length =1Km

Width of formation = 10 m

Height of embankment = 60cm

Side slope =1:2

Solution

Quantity of earthwork = L $(Bd+Sd^2)$

B=10m; d =0.60m; S =2

Quantity of earthwork = 1000x (10x0.60) +2x0.60x0.60=6720cu.m.

18. List out the various sanitary fittings.

- P trap
- S trap
- Q trap
- Water closet
- Flushing cisterns
- Kitchen sink
- Wash basin, etc

19. Write a short note on Retaining Wall.

It is a structure designed and constructed to resist the lateral pressure of soil when there is a desired change in ground elevation that exceeds the angle of repose of the soil.

20. Write a short note on aqueduct.

It is a water supply or navigable channel (conduit) constructed to convey water. In modern engineering, the term is used for any system of pipes, ditches, canals, tunnels, and other structures used for this purpose

21. Differentiate tube well and open well

Tube well	Open well
A tube well is a type of water well in which a long 100–200 mm (4 to 8 inch) wide stainless steel tube or pipe is bored into an underground aquifer. The lower end is fitted with a strainer, and a pump at the top lifts water for irrigation. The required depth of the well depends on the depth of the water table.	Open wells are almost always water table wells, meaning they are made just deep enough to reach the water table, allowing groundwater to fill the bottom of the well. This type of well commonly has a diameter of at least 3 or 4 feet (0.9-1.2 m), making it big enough for at least one person to stand in the well shaft while it's being constructed, and is usually no deeper than 200 feet (60 m).

22. Write something about Land acquisition.

"Land Acquisition" literally means the acquisition of land for some public purpose by a government agency from individual landowners, as authorised by the law, after paying a government-fixed compensation to cover losses incurred by landowners from surrendering their land to the concerned government agency.

Unit-III – Specification and Tenders

1. Define analysis of rates.

- The determination of rate per unit of a particular item of a work, from the costof quantities of materials, the cost of laborers and other miscellaneous pettyexpenses require for its completion.

2. Define: Schedule of Rates.

In order to determine the rate of a particular item, the factors affecting the rate of thatitem are studied carefully and then finally a rate is decided for that item. This process ofdetermining the rates of an item is termed as analysis of rates or rate analysis.

3. What are the factors affecting Schedule of Rates?

- Specifications of works and material about their quality, proportion and constructional operation method.
- Quantity of materials and their costs.
- Cost of labours and their wages.
- Location of site of work and the distances from source and conveyance charges.
- Overhead and establishment charges
- Profit

4. Define: Owner.

The person of behalf of which work is to be done. He may be an individualorfirm or organization.

5. Define: Drawings.

The section, map, plans etc… which completely define the constructionworkgeometrically is known as drawings

6. Define: Work.

It means the work is to be carried out under thiscontract.

7. Define: Task.

Capacity of doing work by an artisan or skilled lab our in the form of quantity of per day in known as task.

8. What is specification?

Specification is an important document attached with a tender form / contract agreement, which in most cases controls the quality of materials and works.

9.State the different types of specification.

- General or brief specification
- Detailed specification
- Standard specification

10.What are the objects ofspecification?

1. Quality
2. Instruction
3. Aim of the project

11. Describe general specification.

- General specification gives the nature and class of work and materials in general to be used in the various parts of the works, from the foundation to the superstructure.
- General specifications give idea of the whole work or structure and are useful for preparing the estimate.

12. Describe detailed specification

- The detailed specifications form a part of the contract document. The detailed specification of an item of the work specifies the qualities and quantities of materials proportion of mortar workmanship, the method of preparation and execution and method measurement.
- The detailed specifications of different items of work are prepared separately which description what the work should be and how they should execute and constructed.

13. Define:Tender.

Tender is an offer given in writing to execute specified articles or materials at a certain rate, with in a fixed time, under certain conditions of agreement between the contractor and the party, which may be a government department or an individual.

14. What is called Tender Notice?

- The notice inviting tender is called tender notice.
- Tender notice is the publicity of offer to the contractor to quote their ratesforconstructionforconstructionworkosupplied.Sealedtendersareinvited inthemostopen and public manner. It is made public by advisement in leading newspaper, in thegovernment gazette or by notice in English and in the regional languages in public places.

15. Define: Contract and Contractor.

- Contract is merely an agreement being enforceable by law between two persons or parties.
- A person or a firm who undertakes any type of contract is termed as contractor.

16. What are the types of contract?

- Lump-sum contract
- Cost plus percentage of cost contract
- Item rate contract
- Labour contract
- Integrated contracting system

17. Write the essentials requirements of contract.

- The contract language is law full.
- The contract is made by parties competent to contract.
- The contract is made by free consent of the parties.
- The contract is made under valid consideration.
- There shall be a definite proposal and its acceptance.

18. What are the important legal implications of a contract?

- Agreement should not violate the provisions of law.
- It should not have any adverse effect on the morals of the society
- The form of contracts would be in writing and each page of the documents of the contract should be signed by both the parties.
- A contractor who refuses to carry out the work before completion can be used in account of law for breach of contract.

19. What are the types of termination of contract?

- Agreement
- Breach

- Performance
- Impossibility of performance
- Operation of provision of law

20. What are the conditions of contract?

- Conditions relating to documents
- Conditions relating to the execution of work
- Conditions relating to labour and personal

21. What are the types of penalties that are imposed on a contract and why aretheyimposed?

Penalties may be imposed for non-fulfillment of conditions of contract such as not maintaining progress, delay in completion and unsatisfactory work etc. The penalty may be fixed sum per day or a percentage of the estimated cost upto 10%.

22. Why and when the earnest money deposit iscollected?

While submitting a tender, the bidder has to deposit with the departmentanamountequaltoabout2½%oftheestimatedcostoftheworkwhichiscalledearnestmoneydeposit.Thisamountservesasachecktopreventthecontractorfromrefusingtoaccepttheworkorrunway,whenhistenderhasbeenaccepted.Incaseofrefusaltotakeup the work his earnest money is forfeited.

23. Why and when the security deposit is collected?

At the time of execution of the contract agreement, the successful tender has to deposit a further sum of 1% of the contract amount to the department. This amount is known as security deposit. This amount is kept as a check so that the contract or fulfils all terms and conditions of the contract. The security deposit will be refunded to the contract on the satisfactory completion of the whole work, after the observation period of 6months.

24. What is a tendernotice?

Tender notice is the publicity of offer to the contractor to quote there ratesforconstructionforconstructionworkorsupplied.Sealedtendersareinvited in the most open and public manner. It is made public by advisement in leading newspaper, in the government gazette or by notice in English and in the regional languages in public places.

25. What information's should a contract document contain?

Contract Documents			
1	Title page	6	General specifications
2	Index page	7	Detailed specification
3	Tender notice and tender forms	8	Schedule of issue of materials
4	Schedule of quantities	9	Conditions of contract.
5	Drawings		

26. Define: Arbitration.

Arbitration is the settlement of a dispute by the decision not of a court or lawbutof one or more persons chosen by the parties themselves involved in the dispute.

27. Define:Arbitrators.

The persons chosen have the right to take decision are called arbitrators.

28. What are the typesofArbitration?

- Arbitration without intervention of court.
- Arbitration with intervention of court and there is no suit pending
- Arbitration is suits.

29. Define: Bar-bending schedule

- Bar bending schedule is a chart which describe the shape of bars, length of bars and total amount of steel is to be used in the specific design.

30. What is meant by contingencies?

Incidental expenses of miscellaneous character which cannot be classifiedapproximately under any distinct sub-head, but is added in the cost ofconstruction necessarily.

31. How will you analyse the rate of a particular item?

Cost of material and cost of labour is analyzed separately. Contractor's profit of 10% is also checked.

Unit-IV- Valuation

1. Define: Valuation.

Valuation is the process of estimating the cost of a property based on itspresentcondition.Thepropertiesmaybeimmovablepropertieslikeland,buildings,mines, trees, quarries etc., and movable properties such as coal, oil, steel, cement, sand etc.

2. Write the objective of valuation.

It is the technique of estimating and determining the fair price or value of a propertysuch as a building, a factory or other engineering structures of various types, land etc.

3. What is the purpose of valuations?

1. For Buying or Selling Property
2. For assessment of wealth tax, property tax etc
3. For fixation of rent
4. For security of loans or mortgage
5. For insurance, betterment charges etc

6. For compulsory acquisition

4. Write the necessity ofvaluation.

- Rent fixation. It is generally taken as 6% of the valuation of the property
- For buying andselling
- Acquisition of property byGovt.
- To be mortgaged with bank or any other society to raise loan
- For various taxes to be given and fixed, by the Municipal Committee
- Insurance: For taking out on insurancepolicies.

5. What are the methods of valuation?

- Rental Method of Valuation
- Direct Comparisons of the capital value
- Valuation based on the profit
- Valuation based on the cost
- Development method of Valuation
- Depreciation method of Valuation

6. Write a note on Rental Method of Valuation

In this method, the net income by way of rent is found out by deducting all outgoingfrom the gross rent. A suitable rate of interest as prevailing in the market is assumed andYear's purchase is calculated. This net income multiplied by Year's Purchase gives thecapitalized value or valuation of the property. This method is applicable only when the rent is known or probable rent is determined by enquiries.

7. Explain the method of direct comparison with the capital Value.

This method may be adopted when the rental value is not available from the propertyconcerned, but there are evidences of sale price of properties as a whole. In such cases, thecapitalized value of the property is fixed by direct comparison with capitalized value ofsimilar property in the locality.

8. Write a note on Valuation based on profit.

This method of Valuation is suitable for buildings like hotels, cinemas, theatres etc for which the capitalized value depends on the profit. In such cases, the net income is worked outafter deducting gross income; all possible working expense, outgoings, interest on the capitalinvested etc. The net profit is multiplied by Year's Purchase to get the capitalized value. Insuch cases, the valuation may work out to be high in comparison with the cost ofconstruction.

9. Write a note on Valuation based on cost.

In this method, the actual cost incurred in constructing the building or in possessingthe property is taken as basis to determine the value of property. In such cases, necessarydepreciation should be allowed and the points of obsolescence should also be considered.

10. Write a note on Development Method of Valuation.

This method of Valuation is used for the properties which are in the underdevelopedstage or partly developed and partly underdeveloped stage.

11. What is meant by Depreciation Method of Valuation.?

Depreciation is the gradual exhaustion of the usefulness of a property. This may be defined as the decrease or loss in the value of a property due to structural deterioration, life wear and tear, decay and obsolescence.

According to this method of Valuation, the building should be divided into four parts:

- Walls
- Roofs
- Floors
- Doors and Windows

12. What are the methods of depreciation?

1. Straight line Method
2. Constant percentage method or Declining balance method
3. Sinking Fund Method
4. Quantity Survey Method

13. Define: Value.

Value-Present day cost of a engineering structures (saleable value)

14.Define: Scrap Value.

Scrap Value: If a building is to be dismantled after the period its utility is over, some amount can be fetched from the sale of old materials. The amount is known as scrap value of a building. If various from 7% to 10% of the cost of construction according to the availability of the material.

15. Define: Salvage value.

If a property after being discarded at the end of the utility period is sold without being into pieces, the amount thus realized by sale is known as its salvage value.

16.What is meant by capital cost?

Total cost including all the expenditure incurred from beginning tothe completion of a work.

17. Define: Capitalized value.

It is defined as that amount of money whose annual interest at the highest prevailing rate will be equal to the net income received from the property. To calculate the capitalized value, it is necessary to know highest prevailing on such properties and income from the property.

Capitalized Value = Net income x year’s purchase

18. Define: Market value.

Market value: The market value of a property is the amount, which can be obtained at any particular time from the open market if the property is put for sale. The market value will differ from time to time according to demand and supply.

19. Define: Book value.

Book value is the amount shown in the account book after allowing necessary depreciations. The book value of a property at a particularly year is the original cost minus the amount of depreciation up to the previous year.

20. What is meant by rate of cost?

The cost per unit of subhead which is arrived at by dividing the up-todatefinal charges on a sub-head by its up-to-date progress.

21. Define: Gross income.

Total amount of the income received from the property during the year, without deducting outgoings.

22. Define: Net income.

An amount left at the end of the year after deducting all useable outgoings

23. Define: Obsolescence.

The value of property decreases if its style and design are outdated i.e rooms not properly set, thick walls, poor ventilation etc. The reason of this is fast changing techniques of construction, design, ideas leading to more comfort etc.

24. Define: Sinking fund.

A fund which is gradually accumulated and set aside to reconstruct the property after the expiry of the period of utility is known as sinking fund. The sinking funds may be found out by taking a sinking fund policy with any insurance company or deposition some amount in the bank. Generally while calculating the sinking fund, life of the building is considered. 90% of the cost of construction is used for calculations 10% is left out as scrap value.

Sinking fund (I) = Si / (1+i) n-1

Where I = Annual installment required

n = Number of year required to create sinking fund

i = Rate of interest expressed in decimal i.e 5% as 0.05S = Sinking fund

25. Define: Annuity.

Annuity is the net instalment of periodical or annual payment of the capital amount invested in a property for a certain specified period. Annuity is paid either in the beginning or at the end of each project.

26. What are the types of annuity?

1. Annuity certain
2. Annuity due
3. Perpetual certain
4. Deferred annuity.

27. Write a short note on mortgage and lease.

- Advancing money against any form of security is called mortgage.
- The Property given no rent for a definite period under terms and conditions is said to be on lease.

28. What are the types of lease?

- Building lease
- Occupation lease
- Sub-lease
- Life lease
- Perceptual lease

29.What is meant by Standard rent?

Standard rent is the rent fixed under the Rent Control Act or even a provision made for the fixation of rent under the Act.

30. What are the important factors influencing the value of building?

1.Type of the building
2.Location of the building
3.Expected life of the building
4.Size and shape of the building
5.The Present condition of the building
6.Legal control of the building

Unit-V Report Preparation

1. **Define: Report.**

   Report describing the various features of the work, is accompanied with estimate. The report should be written in clear language. It should be written in such a way that while studying it one can able to get an idea about the whole work.

2. **What are the points included in the report preparation?**

   The following points are considered in the report,

   1. Brief history of the work
   2. Object necessity and unity of the project with reasons.
   3. Selection of site

4. Surveying
5. General specification and basis of design and calculation.
6. Availability of materials
7. Labour amenities and temporary accommodation for staff.
8. Total cost and financing
9. Return on revenue if any

3. What are the essential qualifications of a good report?

- A report must meet the needs of the readers and answer the questions in their minds
- A report must be at the right level for the readers. Some readers have an in-depth knowledge of the subject while others may be decision-makers without specialized, technical knowledge
- A report must have a clear, logical structure-with clear signposting to show where the ideas are leading
- A report must give a good first impression.
- A report must not make assumptions about the readers' understanding.
- Reports must be written in good English.
- Reports should have a time reference

4. What are the Steps to follow for prepare a Good and Effective Report?

- Define your aim
- Collect your ideas
- Select the material and decide how to show the significance of your facts
- Structure your ideas
- Start on report writing

5. Write the basic structure of technical report.

Report Structure			
1	Cover Page	**9**	Results
2	Title page	**10**	Discussion
3	Table of Contents	**11**	Conclusion
4	Acknowledgments	**12**	Implications
5	Executive Summary	**13**	Recommendations
6	Introduction	**14**	Further development
7	Objectives	**15**	Extension and Adoption
8	Method	**16**	Project coverage

6. Write down the basic principles of report writing?

1. Need for the project
2. Investigation and design carried out
3. Details of proposals
4. Schedule of rates
5. Cost

7. Write a report on a residential building.

1.Types and location of building
2. Types of soil and foundation
3. Details of Structure element
    - Super structure
    - Lintel cum sunshade
    - Beams and Columns
    - Slab
4. Joinery detail
5. Plumbing details
6. Estimate cost of the project

8.Write a report on the construction of a culvert.

1. Types and location of building
2. Types of soil and foundation
3. Catchment area detail
4. Loading classification
5. Details of structural Element
    - Abutment
    - Slab
    - Parapet wall
6. Estimated cost of the project

9. Write a report on the road construction.

1. Location and class of the road
2. Type of soil, subgrade and sub-base
3. Alignment of road
4. Plane table survey
5. Loading classification
6. HFL details
7. Details of culverts across the road if any
8. Estimate cost of a project.

10. Give the estimate for the sanitary installation.

Sanitary works usually consist of providing flush type latrines and connecting with sewer lines of septic tank. For estimating the numbers of different fittings are found out and rates are taken per number for supply and fixing in position.

11. Give the estimate for the water supply.

The water supply works mainly consist of pipe lines the connection with water mains should be enumerated stating size and length of pipe from the water main up to the boundary of the property together with the charge of water and local authorities.

12. What are the types of Report?

- Information only report
- Research report
- Case-study and analysis report

PART-B Sixteen Mark Questions

<u>Unit-I – Estimate of Buildings</u>

1. Explain the types of estimates and differentiate detailed estimate from revised estimate.
2. What are the difference between preliminary estimates, detailed estimates, supplementary estimates and revised estimates? Under what circumstances each one is prepared and what statements and drawings are to be attached with each one them.
3. Explainthefollowinggeneralitemsofworkinvolvedintheestimationforabuilding along with the process of calculations.
   a. Earthwork in excavation.
   b. Earthwork infilling.
   c. Brick at soling.
   d. Cement concrete in foundation.
   e. Masonry work in foundation.
   f. Damp proof course.
   g. Masonry work in superstructure.
   h. 10 cm thick brickwork.
4. Enumerate different methods for estimating building works along with a suitable example.
5. List and explain any four approximate methods of estimating for buildings.
6. Explain the following area of measurements.
   a. Plinth area
   b. Floor area
   c. Circulation area

d. Carpet area

7. The plan and sectional elevation of a building are given in figure .Estimate he quantities of the following of items of work of the

   (i) Doors and windows

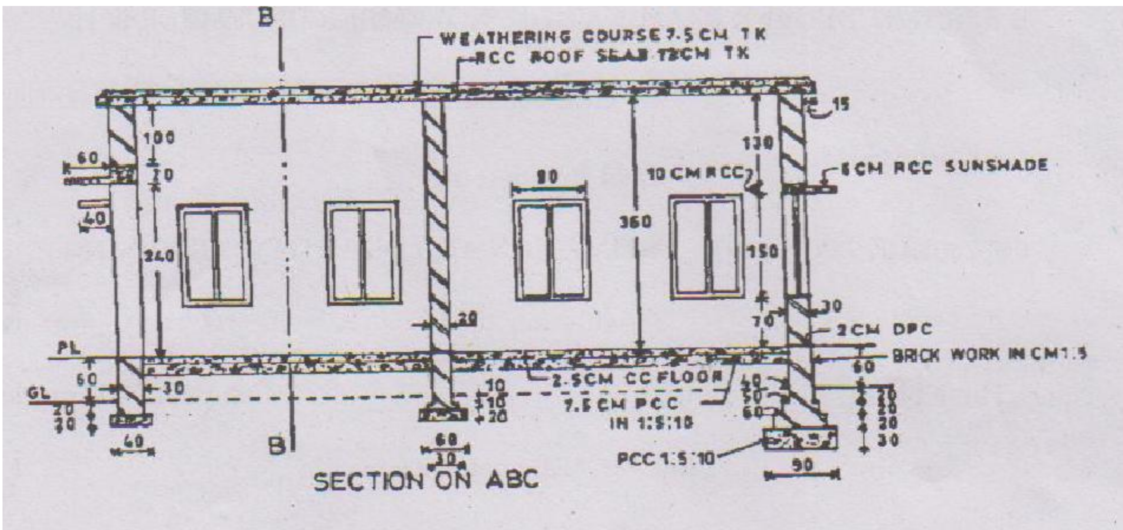
   (ii) R.C.C work in roof slabs, lintels and sunshades

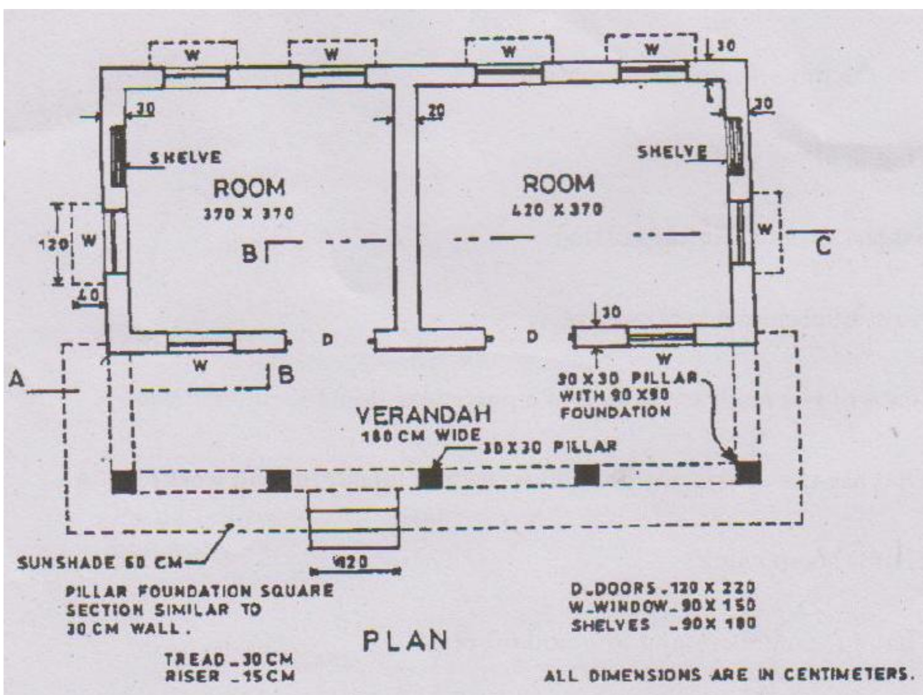
   (iii) Plastering internal

   (iv) First class brick work in C.M. 1:6 in super structure.

   (v) P.C.C. in fou4dation

   (vi) Ceiling plastering.





8. Enumerate different methods for estimating building works along with a suitable example.

9. Distinguish clearly between
    a. Rough cost estimate and detailed estimate
    b. Revised estimate and supplementary estimate
    c. Dismantling and demolition
    d. Administrative approval and technical sanction
    e. Sundries and supervision charges
10. Write down the units of measurement and units of payments in respect of the following
    a. Earthwork
    b. Supply of W.S.pipe
    c. Gusset plate
    d. Painting of doors and windows
    e. Holdfast
    f. L.C. in roof terracing
    g. Tar felting
    h. Pre cost jelly works
    i. Mosaic flooring
    j. Doorframes

Unit-II – Estimate of Other Structures

1. Estimate the quantity of following items of Septic tank fig.

    (i) Earth work excavation

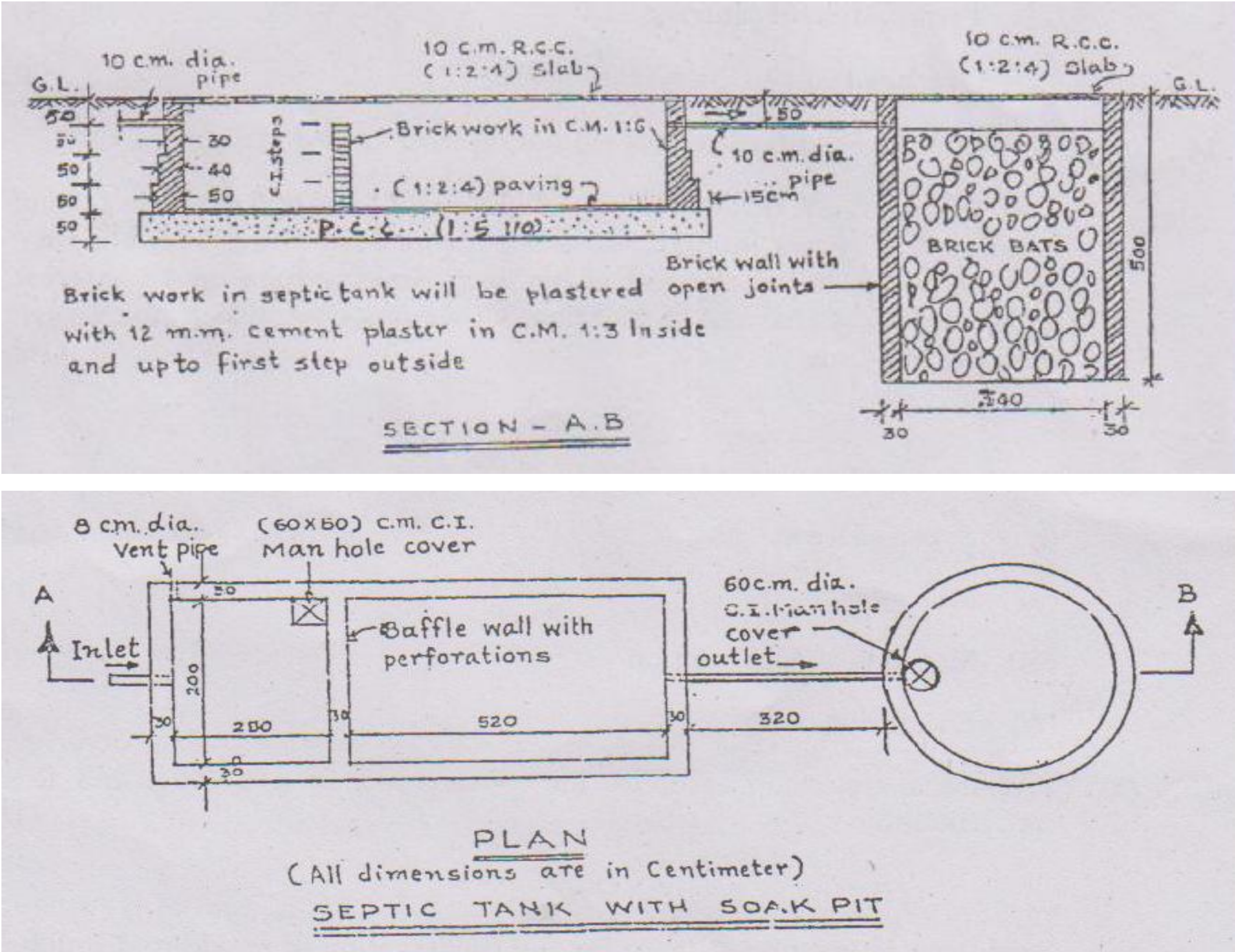
    (ii) P.C.C. 1:5:10

    (iii) Brick Bats in soak pit.

    (iv) Internal Plastering

    (v) Brick work in C.M. 1:6 in septic tank

    (vi) R.C.C cover slab for septic tank and soak pit.



PLAN

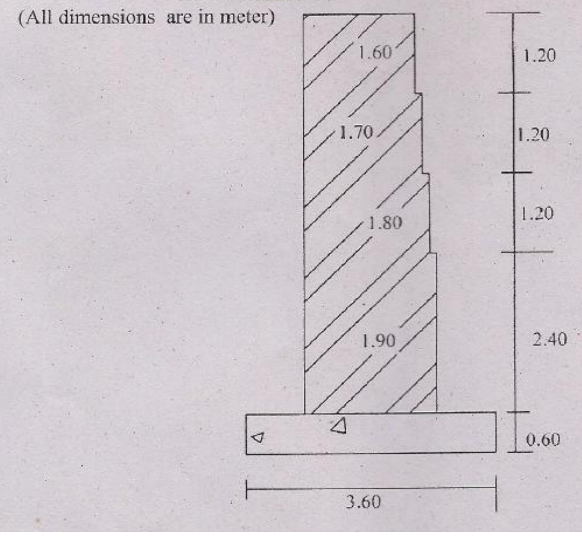
(All dimensions are in Centimeter)

SEPTIC TANK WITH SOAK PIT

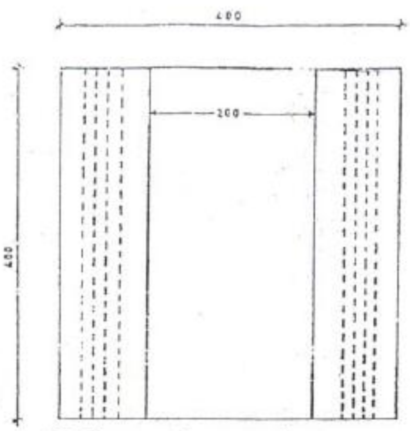
2. Prepare the detailed estimate of the quantities for the following items for a 100 m length of retaining wall shown in fig.
    1. Brick work in CM 1:4
    2. Plain cement concrete 1:4:8
    3. Number of bricks
    4. Number of cement bags.

Fig 2. Retaining wall

(All dimensions are in meter)



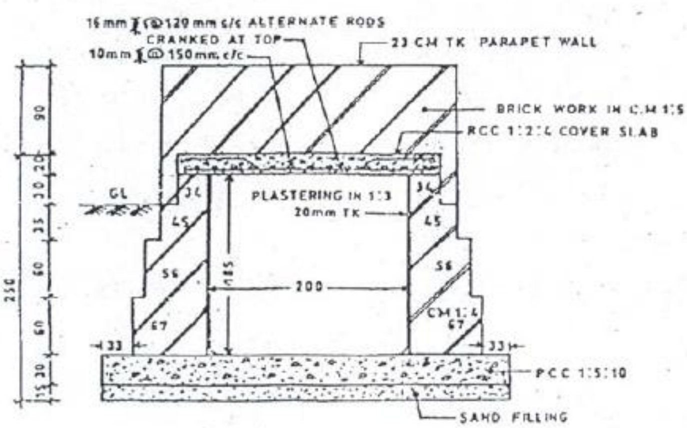
3. Estimate the following quantity for the below figure?
    (i) Earth work excavation
    (ii) PCC
    (iii) Brickwork above & below Ground level
    (iv) RCC work



All dimensions are in cms.

PLAN

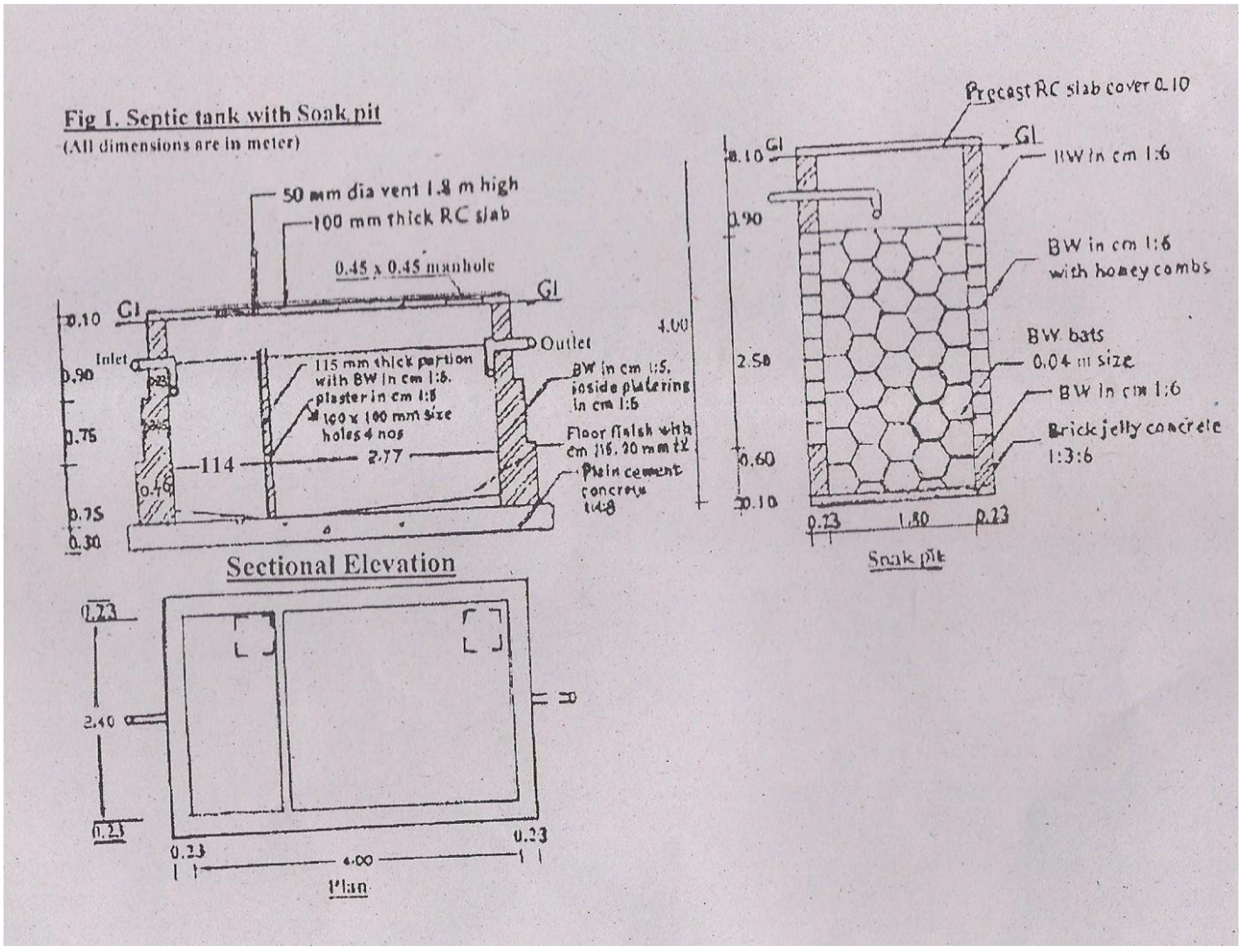
Box Culvert



Cross Section of Box Culvert

4. What are the methods of estimation of roads?
5. Draw the different types of sanitary fittings?
6. Explain in detail about septic tank and dispersion trench?
7. Details of a R.C.C retaining wall 25m long. Prepare a detail estimate for the work.
8. Determine the quantities of following works in the septic tank & soak pit shown in fog.
    1. Earthwork excavation

2. Plain cement concrete
3. Brickwork in CM 1:5 in septic tank.
4. RCC 1:2:4 in cover slab of septic tank.



9. Prepare a detailed estimate of a septic tank with soak pit for 25 users.
10. Prepare a detailed estimate for an open well assuming suitable data.
11. Prepare a detailed estimate for a tube well assuming suitable data.
12. Explain the quantities of different items of work for an abutment with wing wall for a culvert.

Unit-III – Specification and Tenders

1. Explain the contents of a typical tender notice.
2. Write down the general specification of modern road.
3. Explain the contents of a typical contract documents.
4. Discuss the necessity of specification in detail.
5. Using current schedule of rates for materials and labours, prepare data for the following items of work:

(i) Half brick with CM 1:1, for $1m^3$,

(ii) Plain cement concrete 1:5:10 for $1m^3$

6. Briefly explain the following items.

   (i) Tender

   (ii) Contract document

   (iii) Termination of contract

   (iv) Over head cost in analysis of rates.

7. Write down the specifications of the following

   1. Cement concrete in foundation
   2. Plastering in cement mortar 1:6.

8. (i) Explain the sequential procedure of opening of tenders.

   (ii) Discuss about three forms of contractors.

9. Explain various types of contract.
10. Explain in detail about the penalties to contractors.
11. Explain in details about arbitration.
12. What is schedule of rates? What information does it give? Explain in detail.
13. Analyses the rate of Reinforced cement concrete beam and column.
14. Analyses the rate of cement concrete of ratio 1:2:4 and 1:3:6.
15. Explain the procedure of opening the tenders, acceptance of tenders and the execution of agreement for carrying out a work.

Unit-VI – Valuation

1. Explain different methods of valuation.
2. Explain in detail about the four methods of calculation of depreciation.
3. Discuss the importance of valuation.
4. Discuss purpose of valuation in detail.
5. Explain the steps involved in the calculation of standard rent for buildings.
6. Calculate the Standard rent of a building with following data:

   Cost of land: Rs. 7, 00,000.00, cost of building 16, 00,000, Expected life of building = 65 years- Returns expected 5% on land and 8% on building. Annual repair 1% on the cost of building. Sinking fund on 4% interest basis on 90% of the cost of building. Other out going 30% of the return from the building.

7. Explain the following:

   (i) Types of lease

(ii) Mortgage

(iii) Methods of depreciation

(iv) Escalation

8. Explain the following

    1. Scrap value
    2. Salvage value
    3. Market value
    4. Book value.
    5. Capitalised value
    6. Sinking fund

9. Calculate the annual rent of a building with the following data.

    Cost of land = Rs.20000/-

    Cost of building = Rs.80000/-

    Estimate life = 80years

    Return expected = 5% on land 6% on building

13. A newly constructed building stands are a plot costing Rs. 60,000. The construction cost of the building is Rs.20, 000. And the estimated life of the building is 66 years. The investor's desires to have 8% return on the construction cost and 5% return on the land cost. Assuming annual repairs to be at 0.50% of the cost of construction and other outgoing at 30% of the gross rent, Calculate the annual rent that will have to be changed for the building. The annual instalment of the sinking fund for a life of 66 years of the building at 3% may be taken as 0.2 paise per rupee.
14. A machinery was purchased for rs.20,000 in the year 1970. Salvage value of the machine after 7 year is 6000. Calculate depreciation and value for each year.
15. A construction machinery was purchased for Rs.50, 000. The salvage value of machinery after 5 years is Rs.15, 000. Find the book value at the end of the 3 years.
    1. Straight line method
    2. Constant percentage method.

Unit-V – Report Preparation

1. Briefly explain the principle for the preparation of report of water supply scheme?
2. Prepare a report on estimate for construction of a Culvert and Road construction.
3. Prepare a report estimate for the construction of residential building and water supply and sanitary installation.

4. Write a report to accompanying an estimate for a village water supply scheme.
5. Explain in detail how you will work out standard rent of a government building.
6. What are the points to be considered in the report writing and explain the residential building report?
7. Prepare the report on the estimate for the construction of road.
8. Explain the report on estimation for construction of Tube well?
9. Explain the report on estimation for construction of open well?
10. Explain the report on estimation for construction of water supply & sanitary works?

CE 6023 – INDUSTRIAL WASTE MANAGEMNT

TWO MARKS

UNIT – 1 INTRODUCTION

1. Define the term Population Equivalent.

Population Equivalent is the term which indicates the strength of the industrial waste waters for estimating the treatment required at the municipal sewage treatment plant. , it is define as the ratio of standard BOD's of industrial sewage to the standard BOD of domestic sewage per person per day.

Population Equivalent = Standard DOD of industrial sewage/standard BOD of domestic sewage.

2. What is Hazardous waste?

Hazardous waste is the waste which by their nature and quality may be potentially detrimental to human health and or the environmental and which require special treatment and disposal.

3. What is mean by sewage sickness?

When sewage is continuously applied on a piece of land the pores or void in the soil gets clogged and the free circulation of air will be prevented .This is known as sewage sickness.

4. What are the characteristics of industrial waste water?

P^H, BOD, Turbidity, Colour, COD, Suspended solids, Dissolved solids, Settle able solids.

5. What is the Process of treatment of industrial waste?

Preliminary treatment, Primary treatment, Secondary treatment, Tertiary treatment.

6. What is biodegradable organic matter?

The organic matter which is decomposed by bacteria ,under biological action, is called biodegradable organic matter.

7. The method used to dispose radioactive hazardous wastes?

- Diluted and Disperse method
- Delay and decay method
- Concentrate and contain method
- Reclamation method

8. Distinguish between sewage and sewerage?

SEWAGE: The society's wastes are mixed with sufficient quantity of water and carried through closed conduits under the condition of gravity flow. This mixtures of water and waste products, popularly called sewage.

SEWERAGE: The art of collecting, treating and finally disposing of sewage is known as sewerage.

9. What is a Bioassay study?

- Determination of the strength or biological activity of a substance, such as a drug, by comparing its effects with those of a standard preparation on a test organism.
- A test used to determine such strength or activity

10. What is percapita water demand?

It is the annual average amount of daily water required by one person, and includes the domestic use, industrial and commercial use, public use, wastes, thefts etc.

11. What is called Chronic & Acute?

Chronic:

It estimates longer term effects that measures the ability of an organism to reproduce growth or behave normally.

Acute:

It estimates the short term effects it expresses the concentration of the organism.

12. What are the major types of pollutants?

Carbon monoxide, sulfur dioxide, Nitrous oxide, Hydrogen sulphate, Photochemical smog.

13. What are the objectives of waste water treatment?

- To remove the pollutants from the waste water
- Reducing undesirable effect to receiving streams or lands.
- To remove the pathogens and microorganism in the waste water.

14. What are the causes of industrial pollution?

- Lack of control pollution
- Use of out dated technologies

- Unplanned industrial growth
- Limitation of money for treatment process

15. Classification of industries based on its effects

- Red: Sugar industries, Paper and pulp industries, Tanneries industry, Pharmaceuticals industry.
- Orange: Cotton and spinning industry, Hotel, Crushers, Pesticides.
- Green : Mineralized water, Ice cream industries, Furniture industry, Gold, Handloom, Carpet.

UNIT – 2

CLEANER PRODUCTION

1. What are the 3R Concept?

Reduce, Reuse, Recycle

2. Define cleaner production.

Cleaner production is a preventive, company-specific environmental protection initiative. It is intended to minimize waste and emissions and maximize product output.

3. What are the waste minimization techniques?

Inventory management, Equipment modification, Process modification, Byproduct recovery, Reduce, Recycle, reuse.

4. What is meant by waste audit?

A waste audit is an analysis of your facility's waste stream. It can identify what types of recyclable materials and waste your facility generates and how much of each category is recovered for recycling or discarded.

5. What are the steps involved in waste audit?

- Pre – audit process
- Activity at site
- Post audit activates

6. What is the process involved in volume reduction?

Segregation, Conservation, Product modification, Recycle, Reuse and Reduce.

7. How will you reduce the strength of waste?

Process modification, Equipment modification, Equalization waste, Byproduct recovery, proportioning waste.

8. Write the application of byproduct recovery.

Ammonia liquor is recovered from steel plant caustic soda is recovered from the paper mills.

9. What is called segregation of waste?

Segregation of waste is the process of segregating the concentrated waste water from the diluted waste water.

10. What is called process change?

Process change is the method of change the raw materials or manufacturing process in order to reduce the pollution.

11 What is called equipment modification?

Equipment modification is the process of changing or altering the equipment in order to reduce the pollution.

12. What is meant by end of pipe technique?

Methods used to remove already formed contaminants from a stream of air, water, waste, product or similar. These techniques are called 'end-of-pipe' as they are normally implemented as a last stage of a process before the stream is disposed of or delivered.

13. Example for reuse and recycle

- Reuse
- Save and reuse scrap paper for writing notes and always try to write on both sides of paper.
- Use refillable containers (like Tupperware or reusable mugs) and rechargeable batteries.
- Wash and reuse plastic food storage bags and heavy aluminum.
- Donate unwanted clothing to charities or second-hand clothing stores.
- Recycle
- Donate clothing, furniture, appliances, and electronics to not-for-profit organizations.
- Hold a yard sale to recycle unwanted items. One man's garbage is another man's treasure.
- Place all recyclable items in the blue box for weekly collection.

14. What is meant by Hazardous waste water?

A substance is chemically reactive when it is unstable or could react when exposed to another compound. The waste water having the hazardous characteristics and its pH varies from ≤ 2 and ≥ 12.5 is called Hazardous waste water.

Ex: NaoH, Vehicle batteries, Plating water.

UNIT - 3

POLLUTION FROM MAJOR INDUSTRIES

1. What are the pollution characteristics of sugar industrial waste?

- P^H = 4.6 - 7.1
- BOD = 600 - 4300 ppm
- COD/BOD = 1.3 - 2.0

2. What is called waste water reclamation?

The concept of the reclamation of water is for sustainability and water conservation rather than discharging waste in river, land and ocean.

3. What are the effects of tannery waste on receiving waters and sewers?

Streams will get Odour, color change increasing of chemical contents, aquatic organisms will get affect, ground water get pollute,

Sewer will get eroded, corrosion, presence of solid particles will clog the sewer and damaging of sewer.

4. What are the sources of liquid effluent from sugar mill?

Mill House, Waste Water from Boiling House, Waste Water from Boiler Blow-down, Condenser cooling water, Soda and Acid Wastes.

5. What is the major waste water generating unit in textile industry?

Cleaning, Bleaching and Dyeing.

6. State any two uses of studying manufacturing process of an industry in managing its sewage.

To reduce the waste generation from the industry, reduce the environmental pollution from the industry and to increase the production of the product.

7. State the effluent sources of a thermal power plant.

Coal, burning of fossil fuels, waste water from boiler.

8. Define black liquor in distilleries.

A distilled beverage, spirit, liquor, hard liquor or hard alcohol is an alcoholic beverage produced by distillation of a mixture produced from alcoholic fermentation. This process purifies it and removes diluting components like water, for the purpose of increasing its proportion of alcohol content (commonly expressed as alcohol by volume, ABV).

As examples, this term does not include beverages such as beer, wine, and cider, as they are fermented but not distilled. These all have relatively low alcohol content, typically less than 15%.

9. What are the industries under red categories?

Red: Sugar industries, Paper and pulp industries, Tanneries industry, Pharmaceuticals industry.

10. Name the two waste water reclamation techniques.

Disinfection and Reverse osmosis.

UNIT- 4

TREATMENT TECHNOLOGIES

1) What is called equalization or aim of equalization?

Equalization is a process of making the effluent discharge is fairly uniform in its characteristics like pH,color,turbidity,BOD,COD,etc..,

2) Equalization process or equalization types?

Wall baffle, mechanical agitation, Aeration, combination of these.

3) Purpose of injecting air in equalization tank?

Better mixing, chemical oxidation, biological oxidation, to prevent the settling of solids.

4) What is called neutralization?

Neutralization is the process of converting the acidic or alkaline waste water into neutral by treating with suitable acid or base.

5) How do you acidic waste into neutral?

By treating the acidic waste with Passing the acidic waste into beds of the limestone Lime slurry Caustic soda treatment

6) How do you convert alkaline waste water in neutral?

The over alkaline of waste water is treated by Blowing waste boiler flue gas Adding compressed CO_2 to alkaline waste Producing CO_2 in alkaline waste Adding sulphuric acid

7) What are the methods to remove the suspended solids?

Chemical coagulation , filteration bed like contact bed , trickling filter , intermediate sand filter,microscreenig , ultrafilteration.

8) Define coagulation

Coagulation is the process of settling the suspended solids in the waste water by adding the flocc foming chemical.

9) What are the chemicals added in coagulation ?

Aluminium sulphate, ferrous sulphate , lime , ferric chloride, ferric sulphate

10) Advantages of coagulation process.

Chemical coagulation is found to be more economical than plain sedimentation Less space is enough for operation . This process is simple in operation

11) what are the methods for removal of dissolved organic solids ?

- Adsorption method:
- Activated carbon treatment
- Granular carbon treatment
- Powdered activated carbon treatment
- Chemical oxidation method :
- Ozone treatment
- Chlorination
- UV rays treatment

12) Methods of removal of dissolved inorganics?

The operations in the removal of the inorganic dissolved substances are,

- Chemical precipitation
- Ion exchangers
- Ultrafilteration
- RO
- Electrolysis

13) Define dewatering of sludge.

It is a physical unit operation used to reduce the moisture content of the sludge.

14) Write dewatering methods?

Sludge drying methods, sludge lagooning , vaccum filters , mechanical sludge thickener.

15) What is the purpose of dewatering?

- To reduce the cost
- Easy to handle
- To make sludge odourless
- To treat the sludge in Safeway

16) What are the methods of disposal sludge?

Spreading on farm land , dumping , land filling , sludge lagooning , disposing in sea or river.

17) What is meant by combined treatment of industrial and municipal waste?

In this method industrial waste is discharged in municipal waste with suitable proportion and the treatment is carried out in municipal waste water treatment unit.

18) What are the advantages of combined treatment of industrial & municipal waste?

Cost economy, To improve the quality of treated water

19) Define proportioning Proportioning means discharge of the industrial waste in proportion to the flow of municipal sewage in the sewer.

UNIT – 5

HAZARDOUS WASTE MANAGEMENT

1) What is meant by hazardous waste?

It is a waste , which posses major threats to public health or environment

2) What are the characteristics of hazardous waste ?

Ignitability , reactivity , corrosivity , toxicity.

3) What is called hazardous waste management?

Hazardous waste management is the process of collection ,treatment and disposal of waste material which when improperly handled can cause severe harm to health or environment.

4) What is meant by secure landfills?

Landfilling of hazardous solids waste is regulated more strictly than municipal waste landfilling

5) What are the types of landfills?

Dilute and disperse landfill, containment landfill, dedicated landfill.

6) What are the classifications of hazardous waste for landfilling.

Type A : those wastes unsuitable for landfilling

Type B: wastes which could be suitable for dedicated landfills

Type C: those wastes which may be appropriate for co disposal with normal municipal refuse.

7) What is meant by solidification?

It is the process of convert the liquid waste into solid form by adding selected absorbents.

8) What are the aims for solidification ?

- The liquids waste is convenient to dispose.
- Eco friendly

- Economically

9) Define the term incineration ?

It is the process of burnt the hazardous waste and solid waste at a high temperature which are not suitable for treatment process

10) What are all the hazardous waste incinerators?

- Rotary kilns
- Fluidized bed units
- Liquidized injection tanks

11) What are the method involved in incinerator?

- Rotary kilns
- Fluidized bed units
- Liquidized injection tanks
- Fixed hearth units

12) What is called landfill?

Landfill is the process of disposing the waste by alternate layers of waste and soil in under the soil.

PART B (16 MARKS)

UNIT I INTRODUCTION

1. What is the effect of stream in industrial waste?
2. Explain process modification?
3. Write brief note on environmental legislation?
4. Explain by product recovery with example?
5. Explain waste strength reduction methodology?

UNIT II CLEANER PRODUCTION

1. Lit it method of reducing waste strength explain them with example?
2. Discus the waste volume reduction technique?
3. What is meant by ate audit? Explain with methodology?
4. How change and housekeeping proceed can lead to waste minimization?
5. What are the different waste management approaches?

UNIT III POLLUTION FROM MAJOR INDUSTRIES

1. What is the necessary of dewatering?
2. Explain the various methods of land treatment?
3. What do you mean by auditing?

4. How could strength reduction calculated?

5. List the advantages of common effluent treatment plant?

UNIT IV TREATMENT TECHNOLOGIES

1. With a neat flow diagram explain the treatment methodology for a pulp and paper mill?
2. What are the various sources a distillery unit?
3. Discuss the problems associated with refinery effluent?
4. Discuss the pollution aspects of thermal power?
5. Illustrate the steps involved in textile?

UNIT V HAZARDOUS WASTE MANAGEMENT

1. Define characteristics of hazardous ate?
2. Define physical chemical and biological treatment of hw?
3. Describe the method of land filling?
4. What are the wastes incinerated?
5. What is meant by solidification?

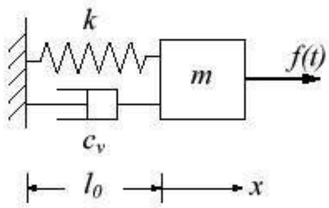
CE6007- HOUSING PLANNING AND MANAGEMENT

CE6701 – STRUCTURAL DYNAMICS AND EARTHUAKE ENGINEERING

SINGLE DEGREE OF FREEDOM

SINGLE DEGREE OF FREEDOM

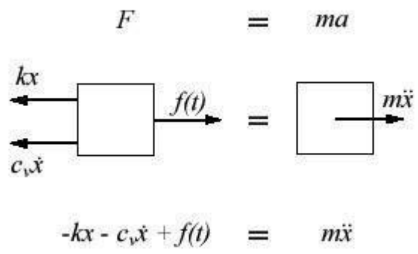
The simplest vibratory system can be described by a single mass connected to a spring (and possibly a dashpot). The mass is allowed to travel only along the spring elongation direction. Such systems are called *Single Degree-of-Freedom* (SDOF) systems and are shown in the following figure,



Single degree of freedom with damper

EQUATION OF MOTION

SDOF vibration can be analyzed by Newton's second law of motion, $F = m*a$. The analysis can be easily visualized with the aid of a free body diagram,



Free body Diagram

The resulting equation of motion is a second order, non-homogeneous, ordinary differential equation:

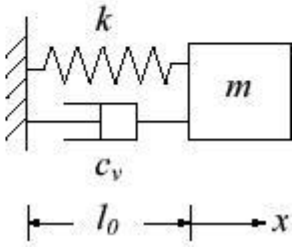
$$m\ddot{x} + c_v\dot{x} + kx = f(t)$$

with the initial conditions,

$$\begin{cases} x(t=0) = x_0 \\ \dot{x}(t=0) = v_0 \end{cases}$$

The solution to the general SDOF equation of motion is shown in the damped SDOF discussion.

Free vibration (no external force) of a single degree-of-freedom system with viscous damping can be illustrated as,



Free Vibration

Damping that produces a damping force proportional to the mass's velocity is commonly referred to as "viscous damping", and is denoted graphically by a dashpot.

For an unforced damped SDOF system, the general equation of motion becomes,

$$m\ddot{x} + c_v\dot{x} + kx = 0$$

with the initial conditions,

$$\begin{cases} x(t=0) = x_0 \\ \dot{x}(t=0) = v_0 \end{cases}$$

This equation of motion is a second order, homogeneous, ordinary differential equation (ODE). If all parameters (mass, spring stiffness and viscous damping) are constants, the ODE becomes a linear ODE with constant coefficients and can be solved by the Characteristic Equation method. The characteristic equation for this problem is,

$$ms^2 + c_v s + k = 0$$

which determines the 2 independent roots for the damped vibration problem? The roots to the characteristic equation fall into one of the following 3 cases:

1.	If < 0, the system is termed underdamped. The roots of the characteristic equation are complex conjugates, corresponding to *oscillatory motion* with an *exponential decay* in amplitude.
2.	If = 0, the system is termed critically-damped. The roots of the characteristic equation are repeated, corresponding to *simple decaying motion* with at most *one*

$c_v^2 - 4mk$

	overshoot of the system's resting position.
3.	If $c_v^2 - 4mk > 0$, the system is termed overdamped. The roots of the characteristic equation are purely real and distinct, corresponding to simple *exponentially decaying* motion.

To simplify the solutions coming up, we define the critical damping c_c, the damping ratio □, and the damped vibration frequency $□_d$ as,

$$c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n$$

$$\zeta = \frac{c_v}{c_c}$$

where the natural frequency of the system $□_n$ is given by,

$$\omega_n = \sqrt{\frac{k}{m}}$$

Note that $□_d$ will equal $□_n$ when the damping of the system is zero (i.e. undamped). The time solutions for the free SDOF system are presented below for each of the three case scenarios.

Underdamped Systems

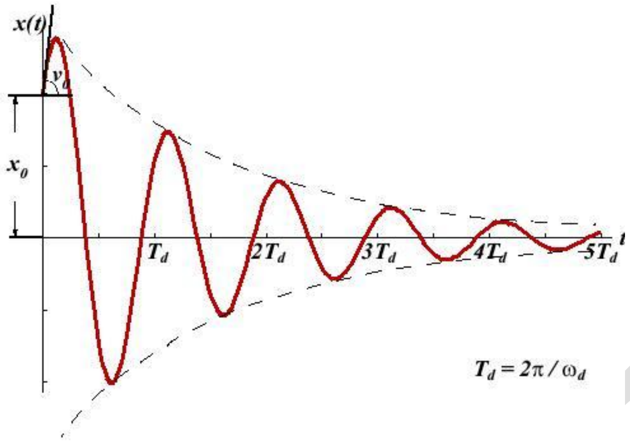
When $c_v^2 - 4mk < 0$ (equivalent to $\zeta < 1$ or $c_v < c_c$), the characteristic equation has a pair of complex conjugate roots. The displacement solution for this kind of system is,

$$\begin{aligned} x(t) &= c_1 e^{\left(-\zeta + i\sqrt{1-\zeta^2}\right)\omega_n t} + c_2 e^{\left(-\zeta - i\sqrt{1-\zeta^2}\right)\omega_n t} \\ &= e^{-\zeta\omega_n t}\left[d_1 \cos\left(\omega_d t\right) + d_2 \sin\left(\omega_d t\right)\right] \\ \Rightarrow x(t) &= \underbrace{e^{-\zeta\omega_n t}}_{\text{Exponentially decay}} \underbrace{\left[x_0 \cos\left(\omega_d t\right) + \frac{v_0 + \zeta\omega_n x_0}{\omega_d} \sin\left(\omega_d t\right)\right]}_{\text{Periodic motion}} \end{aligned}$$

An alternate but equivalent solution is given by,

$$x(t) = A_0 \underbrace{e^{-\zeta\omega_n t}}_{\text{Exponentially decay}} \underbrace{\cos\left(\omega_d t - \varphi_0\right)}_{\text{Periodic}}$$

The displacement plot of an underdamped system would appear as,



Time Period Acceleration

Note that the displacement amplitude decays exponentially (i.e. the natural logarithm of the amplitude ratio for any two displacements separated in time by a constant ratio is a constant; long-winded!),

$$\frac{A_k}{A_{k+1}} = \frac{A_0 e^{-\zeta\omega_n(kT_d)}\cos(\varphi_0)}{A_0 e^{-\zeta\omega_n[(k+1)T_d]}\cos(\varphi_0)} = \frac{e^{-\zeta\omega_n(kT_d)}}{e^{-\zeta\omega_n[(k+1)T_d]}} = e^{\zeta\omega_n T_d}$$

$$\ln\left(\frac{A_k}{A_{k+1}}\right) = \zeta\omega_n T_d = \zeta\omega_n \frac{2\pi}{\omega_d} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

Where $T_d = \frac{1}{f_d} = \frac{2\pi}{\omega_d}$ is the period of the damped vibration?

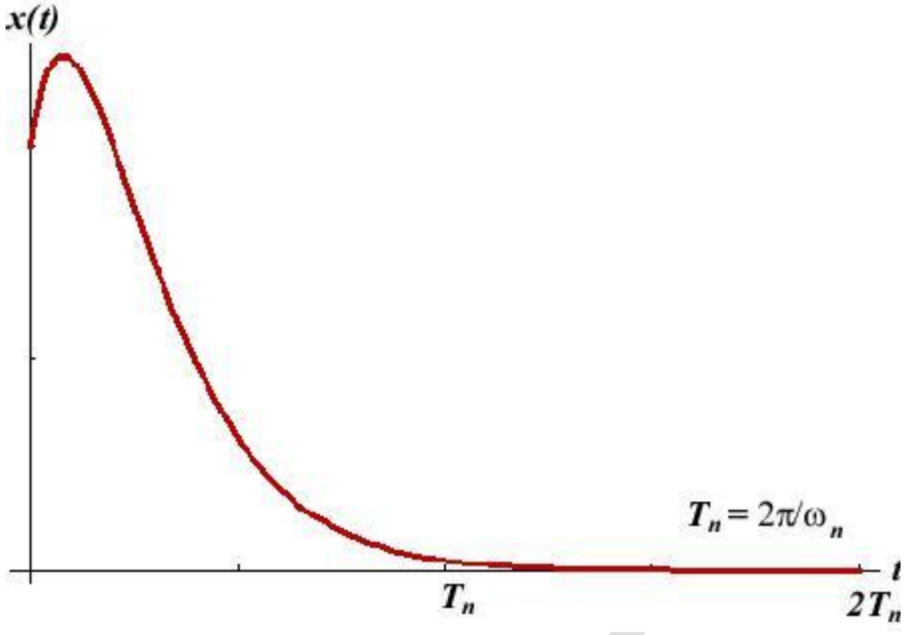
Critically-Damped Systems

When $c_v^2 - 4mk$ = 0 (equivalent to ζ = 1 or $c_v = c_c$), the characteristic equation has repeated real roots. The displacement solution for this kind of system is,

$$x(t) = (c_1 + c_2 t)e^{-\omega_n t}$$
$$\Rightarrow x(t) = e^{-\omega_n t}\left[x_0 + (v_0 + \omega_n x_0)t\right]$$

The critical damping factor c_c can be interpreted as the *minimum damping* those results in non-periodic motion (i.e. simple decay).

The displacement plot of a critically-damped system with positive initial displacement and velocity would appear as,



Time Period acceleration of critically damped system

The displacement decays to a negligible level after one natural period, T_n. Note that if the initial velocity v_0 is negative while the initial displacement x_0 is positive, there will exist one overshoot of the resting position in the displacement plot.

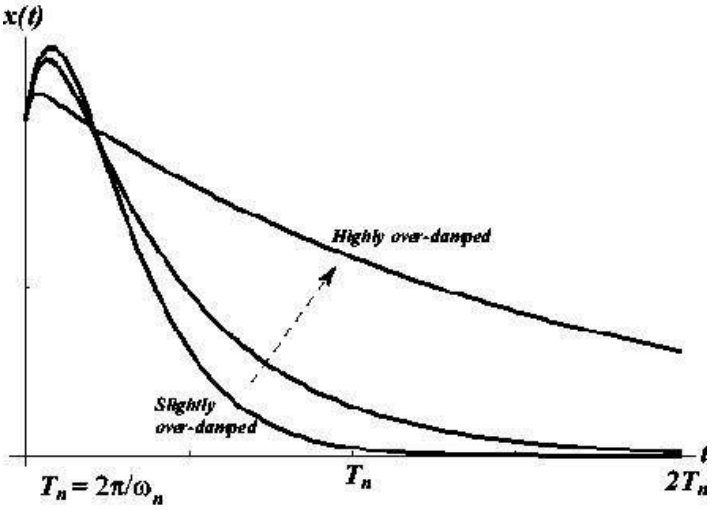
Overdamped Systems

When $c_v^2 - 4mk > 0$ (equivalent to $\zeta > 1$ or $c_v > c_c$), the characteristic equation has two distinct real roots. The displacement solution for this kind of system is,

$$x(t) = c_1 e^{\left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n t} + c_2 e^{\left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t}$$

$$\Rightarrow x(t) = \frac{x_0\omega_n\left(\zeta + \sqrt{\zeta^2 - 1}\right) + v_0}{2\omega_n\sqrt{\zeta^2 - 1}} e^{\left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n t} + \frac{-x_0\omega_n\left(\zeta - \sqrt{\zeta^2 - 1}\right) - v_0}{2\omega_n\sqrt{\zeta^2 - 1}} e^{\left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t}$$

The displacement plot of an overdamped system would appear as,



Displacement plot for Overdamped system

The motion of an overdamped system is non-periodic, regardless of the initial conditions. The larger the damping, the longer the time to decay from an initial disturbance
If the system is heavily damped, $\zeta \gg 1$, the displacement solution takes the approximate form,

$$x(t) \approx x_0 + \frac{v_0}{2\zeta\omega_n}\left(1 - e^{-2\zeta\omega_n t}\right)$$

2.3.1Simple Harmonic Motion

In the diagram a simple harmonic oscillator, comprising a mass attached to one end of a spring, is shown. The other end of the spring is connected to a rigid support such as a wall. If the system is left at rest at the equilibrium position then there is no net force acting on the mass. However, if the mass is displaced from the equilibrium position, a restoring elastic force which obeys Hooke's law is exerted by the spring.

Mathematically, the restoring force **F** is given by

$$\mathbf{F} = -k\mathbf{x},$$

where **F** is the restoring elastic force exerted by the spring (in SI units: N), *k* is the spring constant ($N \cdot m^{-1}$), and **x** is the displacement from the equilibrium position (in m).

For any simple harmonic oscillator:

- When the system is displaced from its equilibrium position, a restoring force which resembles Hooke's law tends to restore the system to equilibrium.

Once the mass is displaced from its equilibrium position, it experiences a net restoring force. As a result, it accelerates and starts going back to the equilibrium position. When the mass moves closer to the equilibrium position, the restoring force decreases. At the equilibrium position, the net restoring force vanishes. However, at $x = 0$, the mass has momentum because of the impulse that the restoring force has imparted. Therefore, the mass continues past the equilibrium position, compressing the spring. A net restoring force then tends to slow it down, until its velocity vanishes, whereby it will attempt to reach equilibrium position again.

As long as the system has no energy loss, the mass will continue to oscillate. Thus, simple Harmonic motion is a type of periodic motion.

Dynamics of simple harmonic motion

For one-dimensional simple harmonic motion, the equation of motion, which is a second-order linear ordinary differential equation with constant coefficients, could be obtained by means of Newton's second law and Hooke's law.

$$F_{net} = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -kx,$$

where m is the inertial mass of the oscillating body, x is its displacement from the equilibrium (or mean) position, and k is the spring constant.

Therefore,

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\left(\frac{k}{m}\right)x,$$

Solving the differential equation above, a solution which is a sinusoidal function is obtained.

$$x(t) = c_1\cos(\omega t) + c_2\sin(\omega t) = A\cos(\omega t - \varphi),$$

Where

$$\omega = \sqrt{\frac{k}{m}},$$

$$A = \sqrt{{c_1}^2 + {c_2}^2},$$

$$\tan\varphi = \left(\frac{c_2}{c_1}\right),$$

In the solution, c_1 and c_2 are two constants determined by the initial conditions, and the origin is set to be the equilibrium position.[A] Each of these constants carries a physical meaning of the motion: A is the amplitude (maximum displacement from the equilibrium position), $\omega = 2\pi f$ is the angular frequency, and φ is the phase.[B]

Using the techniques of differential calculus, the velocity and acceleration as a function of time can be found:

$$v(t) = \frac{\mathrm{d}x}{\mathrm{d}t} = -A\omega \sin(\omega t - \varphi),$$
$$a(t) = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -A\omega^2 \cos(\omega t - \varphi).$$

Acceleration can also be expressed as a function of displacement:

$$a(x) = -\omega^2 x.$$

Then since $\omega = 2\pi f$,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}},$$

and since $T = 1/f$ where T is the time period,

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

These equations demonstrate that the simple harmonic motion is isochronous (the period and frequency are independent of the amplitude and the initial phase of the motion).

Energy of simple harmonic motion

The kinetic energy K of the system at time t is

$$K(t) = \frac{1}{2}mv^2(t) = \frac{1}{2}m\omega^2A^2\sin^2(\omega t - \varphi) = \frac{1}{2}kA^2\sin^2(\omega t - \varphi),$$

and the potential energy is

$$U(t) = \frac{1}{2}kx^2(t) = \frac{1}{2}kA^2\cos^2(\omega t - \varphi).$$

The total mechanical energy of the system therefore has the constant value

$$E = K + U = \frac{1}{2}kA^2.$$

Derivation of Time Period through Energy Method Total Energy of SHM = constant

NATURAL FREQUENCY

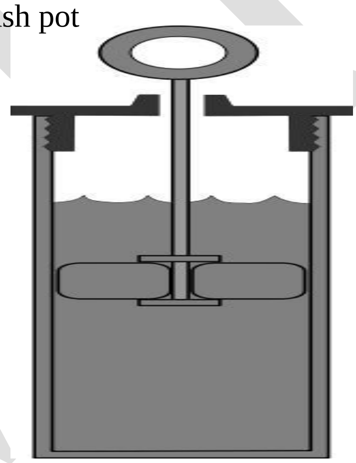
The natural frequency of vibrati**on is ω =**√ (k/m) in radians per second

The time period is the time taken for one cycle is T = 2**Π/ω**

The frequency of vibration f = 1/ T = **ω/2Π**

VISCOUS DAMPING

When the system is made to vibrate in a surrounding viscous medium that is under the control of highly viscous fluid, the damping is called viscous damping. This type of damping is Achieved by means of hydraulic dash pot



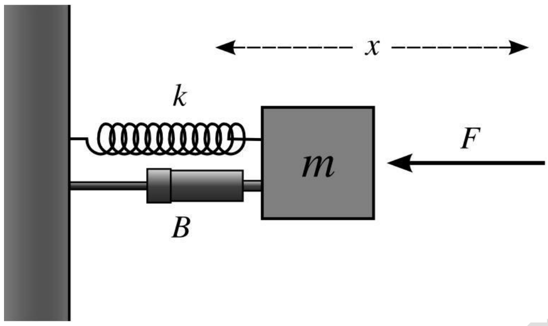
Viscous Dampers

If such force is also proportional to the velocity, as for a simple mechanical viscous damper (dashpot), the force F may be related to the velocity v by

$$F = -cv\,,$$

where *c* is the *damping coefficient*, given in units of newton-seconds per meter.

Equation of motion for Viscous Damping



Forced Vibrations with Dampers

An ideal mass–spring–damper system with mass m, spring constant k and viscous damper of damping coefficient c is subject to an oscillatory force

$$F_s = -kx$$

and a damping force

$$F_d = -cv = -c\frac{dx}{dt} = -c\dot{x}.$$

The values can be in any consistent system of units; for example, in SI units, m in kilograms, k in newton's per meter, and c in newton-seconds per meter or kilograms per second.

Treating the mass as a free body and applying Newton's second law, the total force F_{tot} on the body is

$$F_{tot} = ma = m\frac{d^2x}{dt^2} = m\ddot{x}.$$

Where a is the acceleration of the mass and x is the displacement of the mass relative to a fixed point of reference.

Since $F_{tot} = F_s + F_d$,

$$m\ddot{x} = -kx + -c\dot{x}.$$

This differential equation may be rearranged into

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0.$$

The following parameters are then defined:

$$\omega_0 = \sqrt{\frac{k}{m}}$$

The first parameter, ω_0, is called the (undamped) natural frequency of the system. The second parameter, ζ, is called the *damping ratio*. The natural frequency represents an angular frequency, expressed in radians per second. The damping ratio is a dimensionless quantity.

The differential equation now becomes

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x = 0.$$

Continuing, we can solve the equation by assuming a solution *x* such that:

$$x = e^{\gamma t}$$

Where the parameter γ (gamma) is, in general, a complex number.

Substituting this assumed solution back into the differential equation gives

$$\gamma^2 + 2\zeta\omega_0\gamma + \omega_0^2 = 0\,,$$

which is the characteristic equation.

Solving the characteristic equation will give two roots, γ_+ and γ_-. The solution to the differential equation is thus [1]

$$x(t) = Ae^{\gamma_+ t} + Be^{\gamma_- t}\,,$$

Where *A* and *B* is determined by the initial conditions of the system:

$$A = x(0) + \frac{\gamma_+ x(0) - \dot{x}(0)}{\gamma_- - \gamma_+}$$

RESPONSE TO UNDAMPED AND DAMPED FREE AND FORCED VIBRATION

Free Vibration of Un damped structures

We will examine the case when there is no damping on the SDOF system

$$\ddot{u}(t) + \omega^2 u(t) = 0$$

$$\lambda^2 + \omega^2 = 0$$

$$\lambda_{1,2} = \pm i\omega$$

Respectively, where i = _1. Using these roots and by using Euler's equation we get the general solution:

$$u(t) = A\cos\omega t + B\sin\omega t$$

Where A and B are constants to be obtained from the initial conditions of the system and so:

$$u(t) = u_0 \cos\omega t + \left(\frac{\dot{u}_0}{\omega}\right)\sin\omega t$$

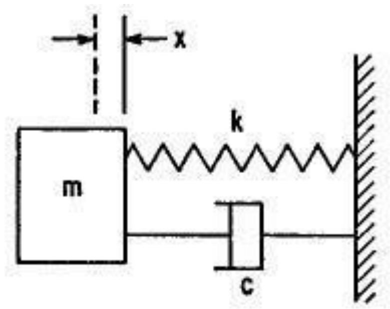
Where U0 and U0 are the initial displacement and velocity of the system respectively.

$$T = \frac{2\pi}{\omega}$$

The natural frequency of the system is got from

Free Vibration of damped Structures

Figure shows a single degree-of-freedom system with a viscous damper. The differential equation of motion of mass m, corresponding to Equation for the undamped system, is



Free Vibration with Dampers

$m\ddot{x} + c\dot{x} + kx = 0$

The form of the solution of this equation depends upon whether the damping coefficient is equal to, greater than, or less than the *critical damping coefficient Cc*:

$C_c = 2\sqrt{km} = 2m\omega_n$

The ratio $\zeta = c/c_c$ is defined as the *fraction of critical damping.*

If the damping of the system is less than critical, $\zeta < 1$; then the solution

$$x = e^{-ct/2m}(A \sin \omega_d t + B \cos \omega_d t)$$
$$= Ce^{-ct/2m} \sin(\omega_d t + \theta)$$

When $\zeta > 1$, the solution

$$x = e^{-ct/2m}(Ae^{\omega_n\sqrt{\zeta^2-1}\,t} + Be^{-\omega_n\sqrt{\zeta^2-1}\,t})$$

TWO DEGREES OF FREEDOM- EQUATIONS OF MOTION AND EIGEN VALUE PROBLEM

The system which requires two independent coordinates to describe the motion completely is called as two degree of freedom system.

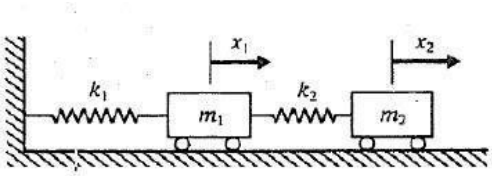


Fig 2.11 Free Vibration on TDOF

UNDAMPED FREE AND FORCED VIBRATION OF TDOF

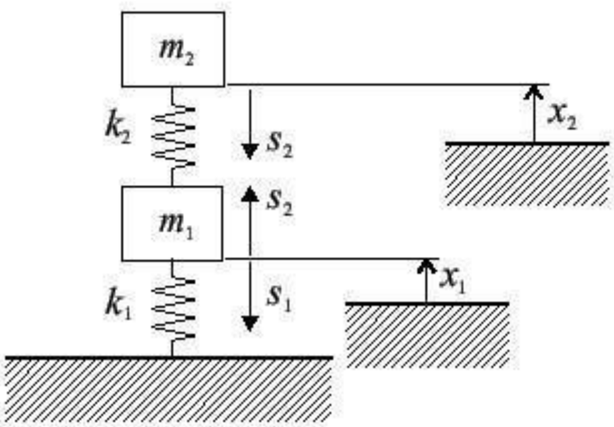


Fig 2.12 Free Vibration on TDOF

The equations of motions are

$$m_1\ddot{x}_1 = -S_1 + S_2,$$
$$m_2\ddot{x}_2 = -S_2.$$

The spring forces are proportional to their extensions

Substituting, rearranging

$$m_1\ddot{x}_1 = -k_1x_1 + k_2(x_2 - x_1),$$
$$m_2\ddot{x}_2 = -k_2(x_2 - x_1).$$

$$m_1\ddot{x}_1 + x_1(k_1 + k_2) + x_2(-k_2) = 0,$$
$$m_2\ddot{x}_2 + x_1(-k_2) + x_2(k_2) = 0.$$

and rewriting into the matrix form

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

we finally get

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\}$$

where [M],[k]are mass and stiffness matrices respectively. These equations are linear and homogeneous of the second order and their solutions can be assumed in the form

$$\begin{aligned} x_1 &= C_1 \sin(\Omega t + \gamma) \quad \Rightarrow \quad \ddot{x}_1 = -\Omega^2 C_1 \sin(\Omega t + \gamma), \\ x_2 &= C_2 \sin(\Omega t + \gamma) \quad \Rightarrow \quad \ddot{x}_2 = -\Omega^2 C_2 \sin(\Omega t + \gamma). \end{aligned}$$

Substituting into the equations of motion and 'dividing out by the sine function' we get

$$\begin{aligned} -m_1 C_1 \Omega^2 &= -k_1 C_1 + k_2 (C_2 - C_1), \\ -m_2 C_2 \Omega^2 &= -k_2 (C_2 - C_1). \end{aligned}$$

This is a set of homogeneous linear algebraic equations in $\mathbf{C_1}$ and $\mathbf{C_2}$, which have a nontrivial solution only if the determinant of the coefficient vanishes, that is

$$\begin{vmatrix} -m_1\Omega^2 + k_1 + k_2 & -k_2 \\ -k_2 & -m_2\Omega^2 + k_2 \end{vmatrix} = 0.$$

This is called the characteristic - or the - frequency equation of the system from which the values of natural frequencies can be determined. Solving the frequency equation we get

$$\Omega_{1,2} = \sqrt{\frac{1}{2}\left(\frac{k_1+k_2}{m_1} + \frac{k_2}{m_2}\right) \pm \sqrt{\frac{1}{4}\left(\frac{k_1+k_2}{m_1} + \frac{k_2}{m_2}\right)^2 - \frac{k_1 k_2}{m_1 m_2}}}.$$

The natural frequencies, also called Eigen frequencies, can be found by Mat lab both analytically and numerically.

Generally, the Eigen frequencies of a mechanical system with **n** degrees of freedom described by could be found assuming

$$\{x\} = \{x_0\} e^{i\Omega t} \quad \Rightarrow \quad \{\ddot{x}\} = -\Omega^2 \{x_0\} e^{i\Omega t}.$$

Substituting the assumed vibrations into the equation of motion we get a so-called generalized eigenvalue problem

$$([K] - \Omega[M])\{x_0\} = \{0\},$$

that could be transformed into a standard eigenvalue problem

$$\left(\underbrace{[M]^{-1}[K]}_{[C]} - \lambda[I]\right)\{x_0\} = \{0\} \quad \text{where} \quad \lambda = \Omega^2.$$

The particular integrals, describing the steady-state response of the system, could be assumed in the form

$$\begin{aligned} x_1 &= A_1 \sin(\omega t) \qquad \Rightarrow \qquad \ddot{x}_1 = -\omega^2 A_1 \sin(\omega t), \\ x_2 &= A_2 \sin(\omega t) \qquad \Rightarrow \qquad \ddot{x}_2 = -\omega^2 A_2 \sin(\omega t). \end{aligned}$$

Substituting into the equations of motion we get

$$\begin{aligned} -m_1 A_1 \omega^2 + k_1 A_1 - k_2 (A_2 - A_1) &= P_1, \\ -m_2 A_2 \omega^2 + k_2 (A_2 - A_1) &= P_2, \end{aligned}$$

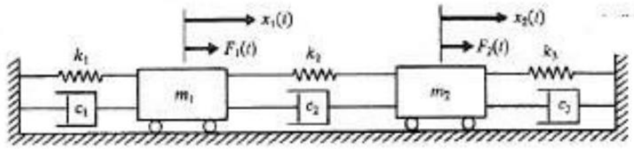
$$\begin{bmatrix} -m_1\omega^2 + k_1 + k_2 & -k_2 \\ -k_2 & -m_2\omega^2 + k_2 \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}.$$

The amplitudes are

$$A_1 = \frac{P_1(k_2 - m_2\omega^2) + P_2 k_2}{(k_1 + k_2 - m_1\omega^2)(k_2 - m_2\omega^2) - k_2^2},$$

$$A_2 = \frac{P_2(k_1 + k_2 - m_2\omega^2) + P_1 k_2}{(k_1 + k_2 - m_1\omega^2)(k_2 - m_2\omega^2) - k_2^2}.$$

2.11 FREE VIBRATION RESPONSE, FORCED VIBRATION RESPONSE TO HARMONIC EXCITATION



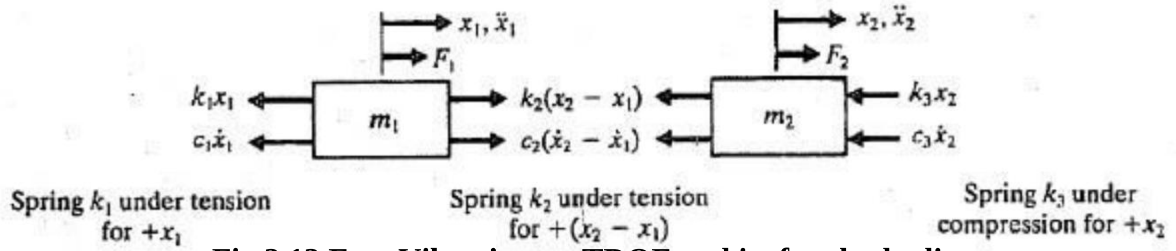


Fig 2.13 Free Vibration on TDOF and its free body diagram

The equations can be written in matrix form as:

$$[m]\,\ddot{\vec{x}}(t) + [c]\,\dot{\vec{x}}(t) + [k]\,\vec{x}(t) = \vec{F}(t)$$

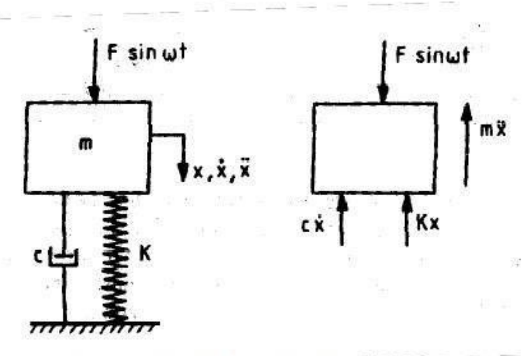
Where [m], [c] and [k] are mass, damping and stiffness matrices, respectively and x (t) and F (t) are called the displacement and force vectors, respectively which are given by:

$$[m] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \qquad [c] = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{bmatrix} \qquad [k] = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix}$$

It can be seen that the matrices [m], [c] and [k] are all 2x2 matrices whose elements are the known masses, damping coefficient, and stiffness of the system, respectively. Further, these matrices can be seen to be symmetric, so that:

Forced Vibration due to Harmonic excitation

$$[m]^T = [m], \qquad [c]^T = [c], \qquad [k]^T = [k]$$



Model and free body diagram

Free body Diagram

The equation of motion is

As soon as the harmonic force is applied there will be a transient response coupled with the forced response. The transient part is the one which dies out after some time.

Steady state excitation frequency **ω.**

Hence neglecting the transient response, we have

$$x = A \sin \omega t + B \cos \omega t$$

Substituting this in the equation of motion we get A and B as

$$A = \frac{(K - m\omega^2)\, F}{(K - m\omega^2)^2 + (c\omega)^2}$$

$$B = \frac{-c\omega F}{(K - m\omega^2)^2 + (c\omega)^2}$$

The response equation can also be written as

$$x = X \sin(\omega t - \varphi)$$

Where $X = \frac{F}{\sqrt{(k - m\omega)^2 + (c\omega)^2}}$

$$\varphi = \tan^{-1} \frac{c\omega}{k - m\omega^2}$$

MODAL ANALYSIS

Modal analysis is the study of the dynamic properties of structures under vibrational excitation.

Modal analysis is the field of measuring and analysing the dynamic response of structures and or fluids when excited by an input. Examples would include measuring the vibration of a car's body

when it is attached to an electromagnetic shaker, or the noise pattern in a room when excited by a loudspeaker.

Modern day modal testing systems are composed of transducers (typically accelerometers and load cells), or non-contact via a Laser vibrometer, an analog-to-digital converter frontend (to digitize analog instrumentation signals) and a host PC (personal computer) to view the data and analyze it.

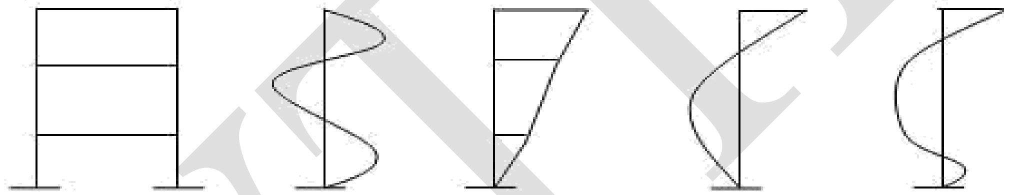
Classically this was done with a SIMO (single-input, multiple-output) approach, that is, one excitation point, and then the response is measured at many other points. In the past a hammer survey, using a fixed accelerometer and a roving hammer as excitation, gave a MISO (multiple-input, single-output) analysis, which is mathematically identical to SIMO, due to the principle of reciprocity. In recent years MIMO (multi-input, multiple-output) has become more practical, where partial coherence analysis identifies which part of the response comes from which excitation source.

Typical excitation signals can be classed as impulse, broadband, swept sine, chirp, and possibly others. Each has its own advantages and disadvantages.

The analysis of the signals typically relies on Fourier analysis. The resulting transfer function will show one or more resonances, whose characteristic mass, frequency and damping can be estimated from the measurements.
The animated display of the mode shape is very useful to NVH (noise, vibration, and harshness) engineers.
The results can also be used to correlate with finite element analysis normal mode solutions.

The free vibration response of an MDOF system consists of coupled differential equations with respect to mass, stiffness and damping. Solution of the coupled equations is a complicated problem because it has dependent variables. To simply the response of system

The coupled differential equations must be transformed to independent or uncoupled equations which contain only one dependent variable. This process of uncoupling the coupled differential equations is called decoupling of equation. Thus the modal superposition is used to uncouple the coupled differential equations by means of transformation of coordinates which incorporates the orthogonally principles of mode shape



Deflected shape Mode 1 Mode 2 Mode 3

Mode shapes

EXAMPLE 7.1 Determine the natural frequencies and mode shape of the given system.

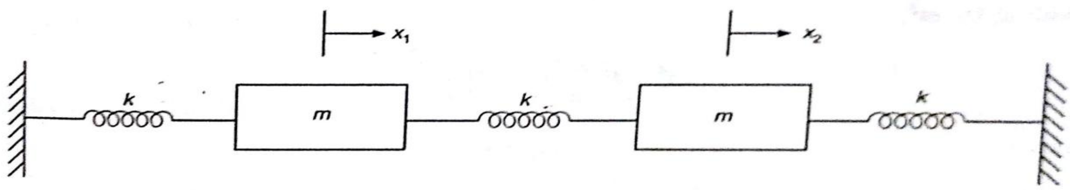


Figure 7.7

Solution:

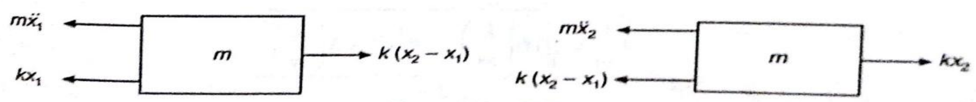


Figure 7.8 FBD.

The governing differential equation of motion can be written as

$$m\ddot{x}_1 + kx_1 - kx_2 + kx_1 = 0 \quad \text{or} \quad m\ddot{x}_1 + 2kx_1 - kx_2 = 0$$
$$m\ddot{x}_2 + kx_2 - kx_1 + kx_2 = 0 \quad \text{or} \quad m\ddot{x}_2 - kx_1 + 2kx_2 = 0$$

Writing the equations on previous page into matrix form

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \{0\}$$

The characteristics equation given by

$$\left|[k] - \omega_n^2[m]\right| = 0$$

$\Rightarrow$

$$\left|\begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix} - \omega_n^2 \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\right| = 0$$

$\Rightarrow$

$$\begin{vmatrix} 2k - m\omega_n^2 & -k \\ -k & 2k - m\omega_n^2 \end{vmatrix} = 0$$

$\Rightarrow$

$$(2k - m\omega_n^2)^2 - k^2 = 0$$

$\Rightarrow$

$$4k^2 - 2km\omega_n^2 - 2km\omega_n^2 + m^2\omega_n^4 - k^2 = 0$$

$\Rightarrow$

$$4k^2 - 4km\omega_n^2 + m^2\omega_n^4 - k^2 = 0$$

Rearranging

$$m^2\omega_n^4 - 4km\omega_n^2 + 3k^2 = 0$$

Dividing by m^2,

$$\omega_n^4 - 4\left(\frac{k}{m}\right)\omega_n^2 + 3\left(\frac{k}{m}\right)^2 = 0$$

Let

$$\omega_n^2 = s$$

$$s^2 - 4s\frac{k}{m} + 3\left(\frac{k}{m}\right)^2 = 0$$

$\Rightarrow$

$$s_{1,2} = \frac{4\frac{k}{m} \pm \sqrt{16\left(\frac{k}{m}\right)^2 - 4\times 1\times 3\left(\frac{k}{m}\right)^2}}{2}$$

$\Rightarrow$

$$= \frac{4\frac{k}{m} \pm 2\frac{k}{m}}{2}$$

$$s_{1,2} = \frac{3k}{m} \text{ or } \frac{k}{m}$$

Thus, $\omega_1^2 = k/m$ (Take lesser value for fundamental mode)

$$\omega_2^2 = \frac{3k}{m}$$

Hence the two natural frequencies are,

$$\omega_1 = \sqrt{\frac{k}{m}}$$

$$\omega_2 = \sqrt{\frac{3k}{m}}$$

First mode

To find the first mode shape, substitute the value of ω_1 in Eq. (2)

$$\left[\begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix} - \omega_n^2 \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\right] \begin{Bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{Bmatrix} = 0 \qquad (2)$$

$$\Rightarrow \qquad \left| k\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - m\omega_n^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| \begin{Bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{Bmatrix} = 0$$

Dividing by k

$$\left[\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \frac{m}{k}\omega_n^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{Bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{Bmatrix} = 0$$

Substituting the value of $\omega_n^2 = \frac{k}{m}$, we get

$$\left[\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \frac{m}{k}\frac{k}{m} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{Bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{Bmatrix} = 0$$

$$\Rightarrow \qquad \left[\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{Bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{Bmatrix} = 0$$

$$\Rightarrow \qquad \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{Bmatrix} = 0$$

$$x_1^{(1)} - x_2^{(1)} = 0$$

$$-x_1^{(1)} + x_2^{(1)} = 0$$

$$\frac{x_1^{(1)}}{x_1^{(1)}} = 1$$

$$\frac{x_2^{(1)}}{x_1^{(1)}} = 1$$

Thus, the mode shape corresponding to the fundamental frequency is given as

$$= \begin{Bmatrix} \dfrac{x_1^{(1)}}{x_1^{(1)}} \\ \dfrac{x_2^{(1)}}{x_1^{(1)}} \end{Bmatrix}$$

$$\{\phi_1\} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

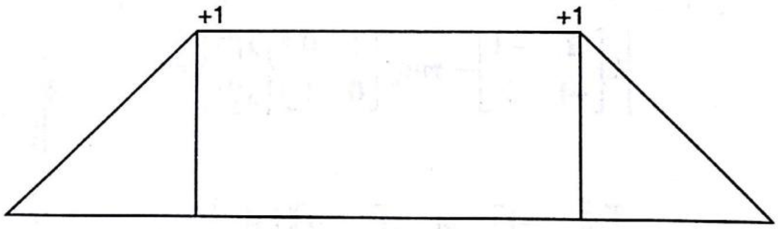


Figure 7.9 First mode shape.

Second mode

Substitute the value of ω_2^2 in Eq. (2) to obtain the second mode shape of vibration.

$$\Rightarrow \quad \left[\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - 3\frac{k}{m}.\omega_n^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{Bmatrix} x_1^{(2)} \\ x_2^{(2)} \end{Bmatrix} = 0$$

$$\Rightarrow \quad \left[\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right] \begin{Bmatrix} x_1^{(2)} \\ x_2^{(2)} \end{Bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} x_1^{(2)} \\ x_2^{(2)} \end{Bmatrix} = 0$$

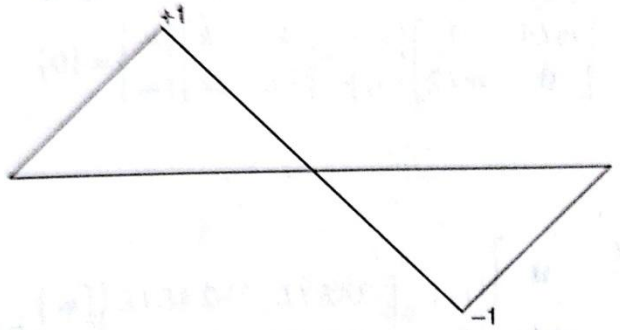
$$\Rightarrow \quad -x_1^{(2)} - x_2^{(2)} = 0$$

$$\Rightarrow \quad \frac{x_2^{(2)}}{x_1^{(2)}} = -1$$

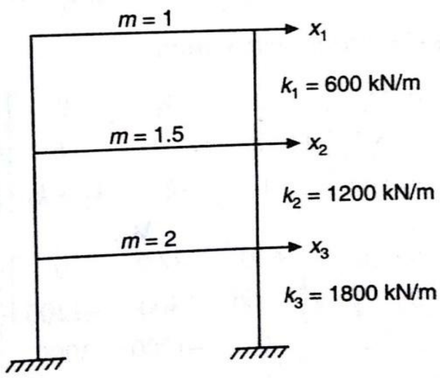
$$\Rightarrow \qquad \frac{x_1^{(2)}}{x_1^{(2)}} = 1$$

Thus the mode shape corresponding to $\omega_2 = \begin{Bmatrix} \dfrac{x_1^{(2)}}{x_1^{(2)}} \\ \dfrac{x_2^{(2)}}{x_1^{(2)}} \end{Bmatrix}$

$$\{\phi_2\} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$



EXAMPLE 8.1 Determine the natural frequencies and the mode shapes for the shear building as shown in Figure 8.4.



This is equivalent to

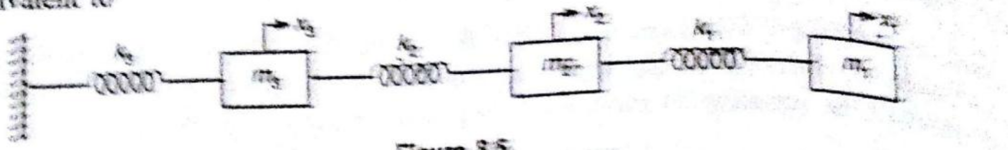


Figure 8.5.

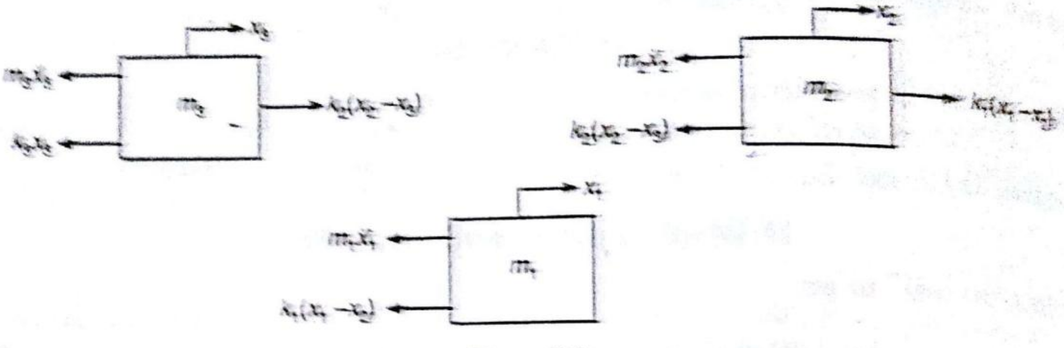


Figure 8.6

Let $m_3 = 2$
$m_2 = 1.5$
$m_1 = 1$

Free body diagrams for the equivalent system are.

$$m_3\ddot{x}_3 + k_3x_3 - k_2x_2 + k_2x_3 = 0$$

$\Rightarrow$
$$m_3\ddot{x}_3 + (k_2 + k_3)x_3 - k_2x_2 = 0$$

$$m_2\ddot{x}_2 + k_2x_2 - k_2x_3 - k_1x_1 + k_1x_2 = 0$$

$\Rightarrow$
$$m_2\ddot{x}_2 - k_1x_1 + (k_1 + k_2)x_2 - k_2x_3 = 0$$

and
$$m_1\ddot{x}_1 + k_1x_1 - k_1x_2 = 0$$

Writing the Eqs. (1), (2) and (3) into a matrix form.

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ 0 & -k_2 & k_2 + k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \{0\}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} 600 & -600 & 0 \\ -600 & 1800 & -1200 \\ 0 & -1200 & 3000 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = 0$$

The characteristic equation is $|[k] - \omega_n^2[m]| = 0$

Substitute these values into characteristic equation, we get

$$\left| 600\begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 5 \end{bmatrix} - \omega_n^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \right| = 0$$

Dividing by 600

$$\left| \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 5 \end{bmatrix} - \frac{\omega_n^2}{600} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \right| = 0$$

Let $$\frac{\omega_n^2}{600} = \lambda$$

$$\left| \begin{bmatrix} (1-\lambda) & -1 & 0 \\ -1 & (3-1.5\lambda) & -2 \\ 0 & -2 & (5-2\lambda) \end{bmatrix} \right| = 0$$

Expanding the above determinant, we get

$$\lambda^3 - 5.5\lambda^2 + 7.5\lambda - 2 = 0$$

The roots are,

$$\lambda_1 = 0.351$$

$$\lambda_2 = 1.61$$

$$\lambda_3 = 3.54$$

We know that $$\lambda = \frac{\omega_n^2}{600}$$

$$\lambda_1 = \frac{\omega_1^2}{600} = 0.351$$

$$\omega_1 = 14.5 \text{ rad/s}$$

$$\lambda_2 = \frac{\omega_2^2}{600} = 1.61$$

$$\omega_2 = 31.1 \text{ rad/s}$$

$$\lambda_3 = \frac{\omega_3^2}{600} = 3.54 \Rightarrow \omega_3 = 46.1 \text{ rad/s}$$

Mode shapes

By using static condensation technique,

$$\begin{bmatrix} (1-\lambda) & \vdots & -1 & 0 \\ \cdots & & \cdots & \cdots \\ -1 & \vdots & (3-1.5\lambda) & -2 \\ 0 & \vdots & -2 & (5-2\lambda) \end{bmatrix} \begin{Bmatrix} x_1 \\ \overline{x_2} \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \overline{0} \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} -1 \\ 0 \end{Bmatrix} x_1 + \begin{bmatrix} (3-1.5\lambda) & -2 \\ -2 & (5-2\lambda) \end{bmatrix} \begin{Bmatrix} x_2 \\ x_3 \end{Bmatrix} = 0$$

$$\begin{Bmatrix} x_2 \\ x_3 \end{Bmatrix} = -\begin{bmatrix} (3-1.5\lambda) & -2 \\ -2 & (5-2\lambda) \end{bmatrix}^{-1} \begin{Bmatrix} -1 \\ 0 \end{Bmatrix} x_1$$

First mode

Assume $x_1^{(1)} = 1$

Substituting $\lambda = \lambda_1 = 0.351$,

$$\begin{Bmatrix} x_2{}^{(1)} \\ x_3{}^{(1)} \end{Bmatrix} = -\begin{bmatrix} (3-1.5\times 0.351) & -2 \\ -2 & (5-2(0.351)) \end{bmatrix} \begin{Bmatrix} -1 \\ 0 \end{Bmatrix} (1)$$

$$= \begin{Bmatrix} 0.644 \\ 0.30 \end{Bmatrix}$$

The eigen vector (or) mode shape corresponding to $\omega_1 = 14.5$ rad/s is

$$\{\phi_1\} = \begin{Bmatrix} 1 \\ 0.644 \\ 0.30 \end{Bmatrix}$$

Second mode

Assume $x^{(2)} = 1$

Substituting $\lambda = \lambda_2 = 1.61$

$$\begin{Bmatrix} x_2^{(2)} \\ x_3^{(2)} \end{Bmatrix} = -\begin{bmatrix} (3-1.5\times 1.61) & -2 \\ -2 & 5-2(1.61) \end{bmatrix} \begin{Bmatrix} -1 \\ 0 \end{Bmatrix} (1)$$

$$= \begin{Bmatrix} -0.601 \\ -0.676 \end{Bmatrix}$$

Thus, the eigen vector (or) mode shape corresponding to $\omega_2 = 31.1$ rad/s is

$$\{\phi_2\} = \begin{Bmatrix} 1 \\ -0.601 \\ -0.676 \end{Bmatrix}$$

Third mode

Assume $x_1^{(3)} = 1$

Substituting $\lambda = \lambda_3 = 3.54$

$$\begin{Bmatrix} x_2^{(3)} \\ x_3^{(3)} \end{Bmatrix} = -\begin{bmatrix} (3-1.5\times 3.54) & -2 \\ -2 & 5-2(3.54) \end{bmatrix} \begin{Bmatrix} -1 \\ 0 \end{Bmatrix} (1)$$

$$= \begin{Bmatrix} -2.57 \\ 2.47 \end{Bmatrix}$$

The eigen vector (or) mode shape corresponding to $\omega_3 = 46.1$ rad/s is,

$$\{\phi_3\} = \begin{Bmatrix} 1 \\ -2.57 \\ 2.47 \end{Bmatrix}$$

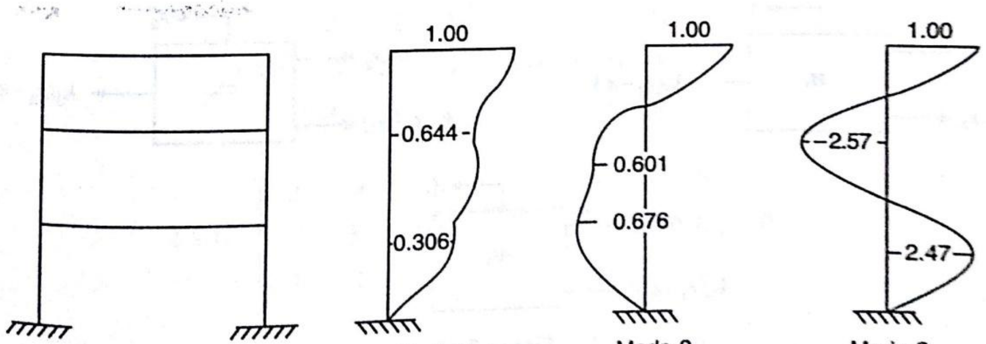
The natural frequencies are,

$\omega_1 = 14.5$ rad/s (fundamental frequency)

$\omega_2 = 31.1$ rad/s

$\omega_3 = 46.1$ rad/s

The shapes of the modes are,



Shear building

Mode-1 $\omega_1 = 14.5$ rad/s First mode shape

Mode-2 $\omega_2 = 31.1$ rad/s Second mode shape

Mode-3 $\omega_3 = 46.1$ rad/s Third mode shape

UNIT-III
ELEMENTS OF SEISMOLOGY

1. Define Seismology And Earthquake

- Seismology is the study of the generation, propagation generation and recording of elastic waves in the earth and the sources that produce them.
- An Earthquake is a sudden tremor or movement of the earth's crust, which originates naturally at or below the surface. About 90% of all earthquakes results from tectonic events, primarily movements on the faults.

2. What are the causes of Earthquake?

Earthquake originates due to various reasons, which may be classified into three categories. Decking waves of seashores, running water descending down waterfalls and movement of heavy vehicles and locomotives, causes feeble tremors these earthquakes are feeble tremors, which don't have disastrous effects.

Contrary to the volcanic earthquake and those due to superficial causes, which can be severe, only locally, the more disastrous earthquakes affecting extensive region are associated with movements of layers or masses of rocks forming the crust of the earth. Such seismic shocks, which originate due to crustal movements, are termed as tectonic earthquakes

3. What is mean by Epicenter and focus?

The point at which the rupture begins and the first seismic wave originates is called focus or hypocenter. The point on the ground directly above the focus is called epicenter.

4. Write a short note on Plate Tectonic Theory

Tectonic is the study of deformations of earth materials that result from deformation. Plate tectonics refers to deformation on a global scale. The basic hypothesis of plate tectonics is that the surface of the earth consists of a number of large plates. These plates move relative to one another. The present six important plates are namely

1. African plate
2. American plate
3. Antarctic plate
4. Australian – Indian plate
5. Eurasian plate
6. Pacific plate

5. Write a short note on Seismic waves.

Large strain energy released during an earthquake travel as seismic waves in all directions through the earth's layers, reflecting at each interface. These waves area of two types, body waves and surface waves

6. Write a short note on Magnitude.

The magnitude of an earthquake is a measure of the amount of energy released. The earthquake scale is devised by Charles F. Richter, an American seismologist be based on the total amount o f energy released during earthquake be called magnitude.

7. What is mean by seismogram?

A seismogram is the graph output by a seismograph. It is a record of ground motion at a measuring station. The energy measured in a seismogram may result from earthquake or from some other source

8. Write a note on Intensity.

Intensity indicates the intensity of shaking or extent of damage at a given location due to particular earthquake. Thus the intensity of some earthquake will be different at different places. Intensity is a measure earthquake in qualitative way by judging what actually happens on the ground, the damage to the buildings and other structures caused by earthquake waves.

9. What is Elastic rebound theory?

The concept of possible mode of origin of tectonic earthquakes is known as Elastic Rebound theory

10. Name the types of fault.

(i) Dip-Slip fault
(ii) Strike-Slip fault
(iii)Oblique-Slip fault

11. Name the types of fault.

(i) Dip-Slip fault
(ii) Strike-Slip fault
(iii)Oblique-Slip fault

12. What are the types of Body waves and surface waves?

Body waves are mainly of two types, they are

(i) Primary waves (or) P-waves
(ii) Secondary waves (or) S-waves

Surface waves also having two types, they are

(i) Love waves
(ii) Rayleigh waves

13.Compare: Magnitude and Intensity of an earthquake.

Sl.No	Magnitude	Intensity
1.	Magnitude measures the energy release at the source of the earthquake. It is determined from measurements on seismographs.	Intensity measures the strength of shaking produced by the earthquake at a certain location. It is determined from the effects on people, structure and natural environment.
2.	Magnitude of an earthquake is a quantitative measure of its size. Thus the magnitude of the earthquake is a single number which does not vary from	Intensity is a qualitative measure of an earthquake, based on the damage caused by them.

14. How the earthquakes are classified?

Earthquake can be classified into the following types.

(a) According to plate boundaries

(b) According to its depth of focus

(c) According to its origin of the earthquakes

(d) Based on magnitude (M).

15. What is the difference between Inter plate earthquakes and Intra plate earthquakes?

(i) **Inter plate earthquakes:** The earthquake occurring along the boundaries of the tectonic plates are called as inter plate earthquakes.

Example: 1987, Assam Earthquake

(ii) **Intra plate earthquakes:** The earthquakes occurring within a plate are called as intra plate earthquakes.

Example: 1993, Latur Earthquake

16. What are the factors influences the ground motion? The

factors which influence the ground motion are:

(i) Magnitude of earthquake

(ii) Epicentral distance

17. What is the difference between shallow, intermediate and deep focus earthquake?

(i) **Shallow-focus earthquake:** In this case, the seismic shocks originate at a depth of about less than 70 km. Nearly 80% of the world's earthquakes are shallow-focus earthquakes.

(ii) **Intermediate-focus earthquake:** In this case, the seismic waves originate at a depth between 70 km to 300 km.

Deep-focus earthquake: Here, the point of origin of the seismic wave is at a depth of greater than 300 km.

18. What is Seismograph?

Seismograph is an instrument used to recording motions of the earth's surface caused by seismic waves, as a function of time. A modern seismograph includes five basic parts: a clock, a sensor called a seismometer that measures intensity of shaking at the instruments location, a recorder that traces a chart or seismogram, of the seismic arrivals, an electronic amplifier and a data recorder that stores the information for later analysis.

19. Explain volcanic Earthquake?

Earthquakes associated with volcanoes are more localized both in the extent of damage and in the intensity of the waves produced in comparison to those which are associated with faulting motions. Deep below the centre of volcano, hot magma moves slowly through underground passages under pressure, as its makes it ways towards the earth's surface. As this happens, the surrounding rock is put under pressure as the magma pushes against it. This causes the rock to fracture and small earthquakes to occur.

20. What are the basic difference between Focus and Epicentre?

Focus is the location within the earth where fault rupture actually occurs whereas the epicentre is the location on the surface above the focus.

21. What is hypocenter?

Focus is an exact location within the earth where seismic waves are generated by sudden release of stored elastic building. It is called as hypocenter.

22. What is accelerogram?

A graph plotted between acceleration of ground and time is called accelerogram. The nature of accelerogram's may vary depending on energy released at focus, type of faults, geology along the fault plane and local soil.

23. Explain Uttarkashi earthquake of 1991?

An earthquake of magnitude 6.6 struck the districts of Uttarkashi, Tehri and Chamoli in the state of Uttar Pradesh on October 20, 1991. About 768 persons lost their lives, with about 5,066 injured. Maximum peak ground acceleration of about 0.31g was record at Uttarkashi. Many four-storey buildings in Uttarkashi with RC frame and infill walls sustained the earthquake. Howe ever, some of the ordinary RC buildings collapsed.

25. What is Focal depth and Epicentral distance?

(i) **Focal depth:** The distance between the epicentre and the focus is called focal depth.

(ii) **Epicentral distance:** The distance from epicentre to any of interest is called epicentral distance.

PART-B

CAUSES OF EARTH QUAKE:

Earth quake can be caused due to both manmade and natural sources, they are follows

Natural sources

- Caused due to movement of Tectonic plates - Tectonic earthquake
- Caused due to volcanic eruption - volcanic earthquake
- Caused due to movements of fault

Manmade sources:

- Controlled sources – manmade explosion
- Reservoir induced earthquake
- Mining induced earthquake

TECTONIC EARTHQUAKE:

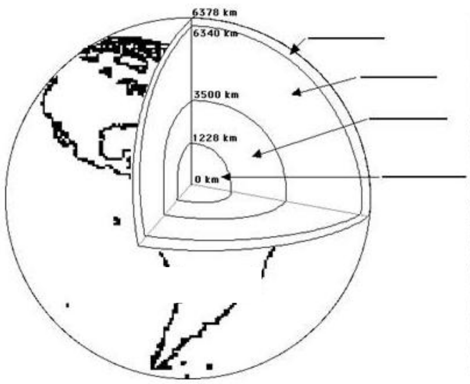
It refers to earthquake caused due to the movement of the tectonic plate, almost all the major earth are caused due to this type only.

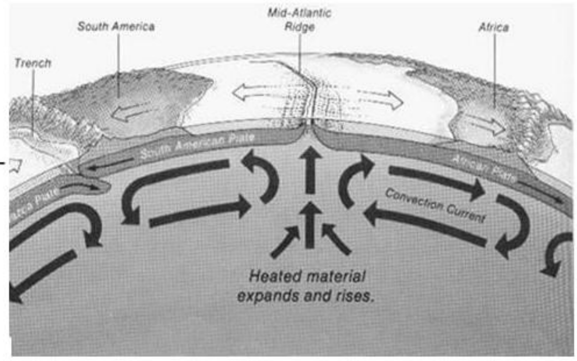
PLATE TECTONICS THEORY:

The plate tectonic theory is the starting for understanding the forces within the earth that causes. Tectonics is the study of deformation of the earth material and the structures that result from deformation

THE Earth Consist Of

- Inner Core - 1290km thick - solid and consists of heavy metals
- Outer Core - 2200 km thick - Outer Core is liquid in form
- Mantle - 2900 km thick - is in semi solid state
- Crust - 5 to 40 km thick - Crust consists of light materials





- ❖ Convection currents is develop in the viscous Mantle, because of prevailing high temperature and pressure gradients between the Crust and the Core, like the convective flow of water when heated in a beaker

- ❖ The energy for the above circulations is derived from the heat produced from the incessant decay of radioactive elements in the rocks throughout the Earth's interior. These convection currents result in a **circulation** of the earth's mass

- ❖ The convective flows of Mantle material cause the Crust and some portion of the Mantle, to slide on the hot molten outer core. This sliding of Earth's mass takes place in pieces called **Tectonic Plates.**

The surface of the Earth consists of six major tectonic plates and many smaller ones, They are as follows

- ❖ African plate
- ❖ American plate
- ❖ Antarctic plate
- ❖ Australian-indian plate
- ❖ Eurasian plate
- ❖ Pacific plate

These plates move in different directions and at different speeds from those of the neighboring ones. Sometimes, the plate in the front is slower then the plate behind it comes and collides (and **mountains** are formed). On the other hand, sometimes two plates move away from one another (and **rifts** are created). In another case, two plates move side-by-side, along the same direction or in opposite directions.

These three types of inter-plate interactions.they are

- ❖ **Convergent** boundaries
- ❖ **divergent** boundaries
- ❖ **Transform** boundaries

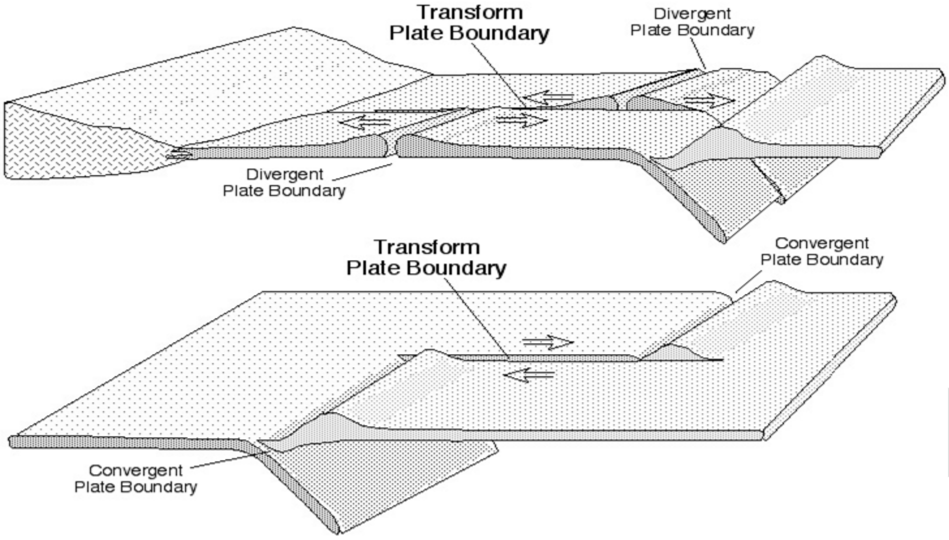
CONVERGENT BOUNDARIES:

- ❖ In the convergent boundary the plate move towards each other and collide
- ❖ When the dense continental plate collides with a less dense plate the more denser plate moves downward causing the uplift of the less denser plate
- ❖ A collision between two continental plates in general uplift to form a mountain

- The movement of one plate over another creates a considerable strain and creates earthquake when the boundary slip and cause the release of stored energy due to the strain
- The earth quake caused due to convergent boundary is severe

DIVERGENT BOUNDARIES:

- In the divergent boundary the plate away from each other and thereby form a gap
- The hot molten material from the mantel moves upsward through the graph due to this there is a possibility of volcanic eruption
- At the divergent boundary the earth quakes are shallow



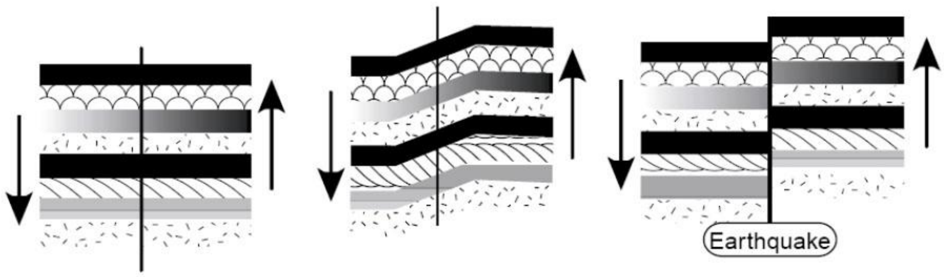
TRANSFORM BOUNDARY:

- If two plate slide by one another in opposite direction or same direction

ELASTIC REBOUND THEORY:

The concept of possible mod of origin of tectonic earth quakes is known as elastic rebound theory

- Rocks are made of elastic material, and so elastic strain energy is stored in them during the movement of the fault (a crack in the rocks where movement has taken place)
- If the stress is continued for a long period of time, or if it is increased in magnitude the rock will first take a permanent deformation and eventually rupture.
- When the rupture occurs, rocks on either side of the fault tend to return to their original shape because of their elasticity and an elastic rebound occurs.
- This rebound sets up the seismic waves
- Thus the energy accumulated or stored in the system through decades, is released instantaneously causing underground dislocation of the rocks and setting up vibration which causes earthquake



Rocks on each side of the fault are forced to move, but rocks at the fault are "locked" together.

Because rocks are locked together, they bend, storing energy like a coiled spring.

Rocks suddenly "let go", springing back, causing an earthquake.

FAULTS:

Fault is a fracture having appreciable movement parallel to the plane of fracture. Due to this large amount of strain energy is stored in the along the edges of the fault and when this energy is released and it produces shock wave (vibration) and causes earthquake.

- If the earthquake is caused near the active plane boundaries is called **inter-plate** earth quake
- If the earthquake is caused far from the active plane boundaries is called **intra-plate** earth quake

TYPES OF FAULT:

Faults are classified based on the direction of movement of the blocks as follows

- Dip-slip fault
- Strike-slip fault
- Oblique-slip fault

DIP-SLIP FAULT:

if the blocks of earth moves in vertical direction to one another it is called dip-slip fault. Dip-slip fault is further classified into two types.

(a) **Normal fault** –the block above the fault moves down relative to the block below. The fault motion is caused by tensional forces and results in extension. It is also called as tension fault.

(b) **Reverse fault -** the block above the fault moves upward relative to the block below. The fault motion is caused by compressional forces and results in shortening. It is also called as thrust fault.

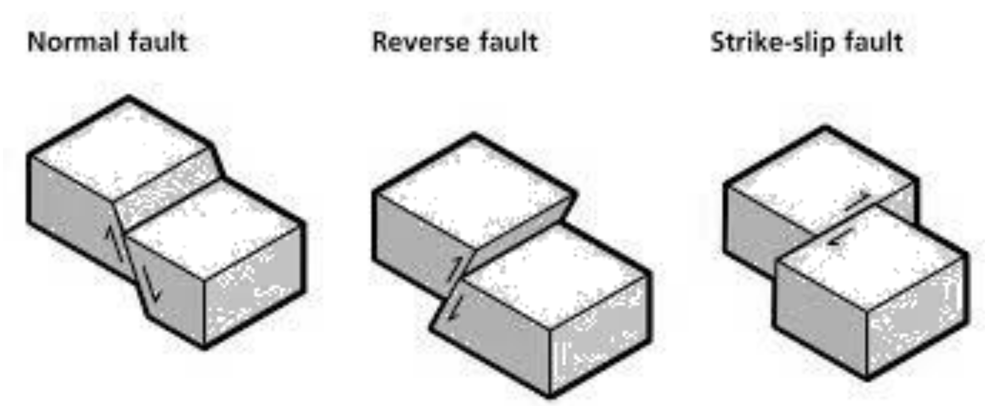
STRIKE-SLIP FAULT:

If the movement of the block along the fault are horizontal. it is called strike slip fault

- If the block on the far side to the left of the fault moves to the left it is called left-lateral
- If the block on the far side to the right of the fault moves to the left it is called right-lateral

OBLIQUE-SLIP FAULT:

Oblique-slip faulting suggest both dip-slip faulting and strike slip faulting. It is caused by a combination of shearing and tensional or compressional forces



ESTIMATION OF EARTHQUAKE PARAMETER :

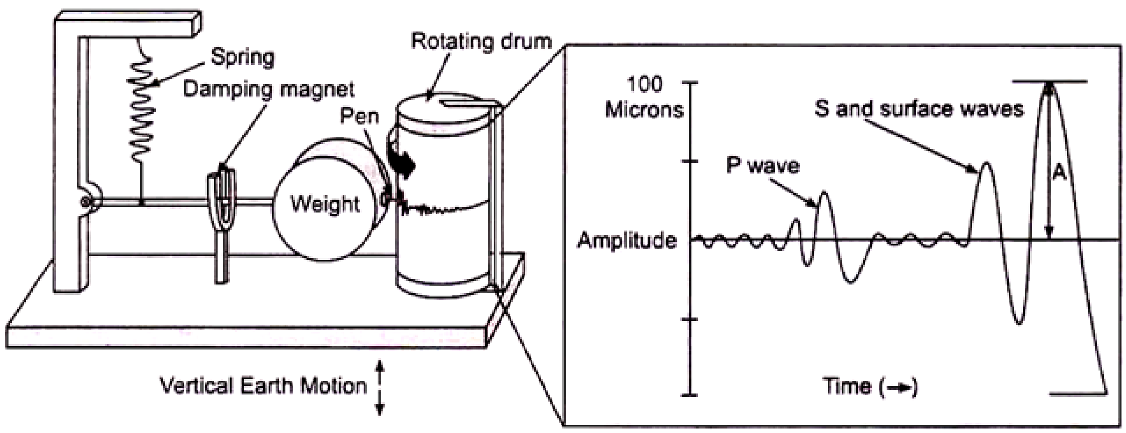
MEASUREMENT OF EARTHQUAKE:

A seismograph is an instrument used to record motions of the earth's surface caused by seismic waves, as a function of time.

PRINCIPLE OF SEISMOGRAPH: Seismographs operate on the principle of inertia of stationary objects

PARTS OF SEISMOGRAPH:

- **Clock**
- **Seismometer** – a sensor that measures intensity of shaking at the instrument location
- **Recorde**r – a device used to trace a chart or seismograph
- **Spring And Mass Arrangement** –a spring attached to a pendulum that moves in accordance to the ground motion
- **Paper**



RESPONSE OF STRUCTURES TO EARTHQUAKE

STEP B Y STEP PROCEDURE FOR SEISMIC ANALYSIS OF RC BUILDING

STEP1: Determination of Natural Period of Vibration

For moment resisting frame without infill panel

$$T_a = 0.075\, h^{0.75} \quad \text{for RC frame building}$$
$$= 0.085\, h^{0.75} \quad \text{for steel frame building}$$

For a moment resisting frame with infill panels

$$T_a = \frac{0.09}{\sqrt{d}}$$

Where

h-height of the building in m

d-base dimension of the building at plinth level in m

STEP2: Determination of average response co-efficient factor

Average response acceleration coefficient Sa/g is calculated from the fundamental natural frequency.

For rocky, or hard soil sites

$$\frac{S_a}{g} = \begin{cases} 1 + 15T; & 0.00 \le T \le 0.10 \\ 2.50 & 0.10 \le T \le 0.40 \\ 1.00/T & 0.40 \le T \le 4.00 \end{cases}$$

For medium soil sites

$$\frac{S_a}{g} = \begin{cases} 1 + 15T; & 0.00 \le T \le 0.10 \\ 2.50 & 0.10 \le T \le 0.55 \\ 1.36/T & 0.55 \le T \le 4.00 \end{cases}$$

For soft soil sites

$$\frac{S_a}{g} = \begin{cases} 1 + 15T; & 0.00 \le T \le 0.10 \\ 2.50 & 0.10 \le T \le 0.67 \\ 1.67/T & 0.67 \le T \le 4.00 \end{cases}$$

where

T-natural period of vibration

STEP3: Determination of zone factor from table 2 of IS 1893 Page 16

STEP4: Determination of impact factor from table 6 of IS 1893 Page 18

STEP4: Determination of reduction factor from table7 of IS 1893 Page 23

STEP5: determination of horizontal seismic coefficient

$$A_h = \frac{Z\, I\, S_a}{2\, R\, g}$$

Where

z- zone factor

I- impact factor

Sa/g - average response co-efficient factor R- reduction factor

STEP6 : determination of design vertical seismic coefficient

design vertical seismic coefficient = (2/3) X horizontal seismic coefficient

STEP7: determination of base shear

$$V_B = A_h W$$

Where

Ah - horizontal seismic coefficient
W – seismic weight of building

STEP8: determination of equivalent lateral load

$$Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2}$$

Where

Qi – design lateral force at floor i
Wi – seismic weight of floor I
hi – height of floor I measured from base
n – number of storeys in building

EXAMPLE 15.3 A three storeyed symmetrical RC school building situated at Bhuj with the following data:

Plan dimensions	:	7 m
Storey height	:	3.5 m
Total weight of beams in a storey	:	130 kN
Total weight of slab in a storey	:	250 kN
Total weight of column in a storey	:	50 kN
Total weight of walls in a storey	:	530 kN
Live load	:	130 kN
Weight of terrace floor	:	655 kN

The structure is resting on hard rock. Determine the total base shear and lateral loads at each floor levels for 5% of damping using seismic coefficient method.

Solution:

Step 1 Determination of natural period

The fundamental natural period is calculated by assuming the infill panels are provided.

$$T = \frac{0.09h}{\sqrt{d}} = \frac{0.09 \times 10.5}{\sqrt{7}}$$

$$= 0.357 \text{ s}$$

Step 2 Determination of other important factors

For T = 0.357 s, damping of 5% and for hard rock, $s_a / g = 2.5$ [from Figure 14.2]

Bhuj is situated in Zone V, Zone factor z = 0.36 (Table 14.1)

Since the building is used as a school building, the Importance Factor I = 1.5 (Table 14.3)

For a special moment resisting frame, the Response Reduction Factor, R = 5.0 (Table14.4)

Step 3 Determination of design horizontal seismic coefficient

The design horizontal seismic coefficient $A_h = \dfrac{ZIS_a}{2Rg}$

$$= \frac{0.36 \times 1.5 \times 2.5}{2 \times 5}$$

$$= 0.135$$

Step 4 Determination of seismic weight

Weight of one storey = Total weight of beams + Slab + Columns + Walls + Live load

= 130 + 250 + 50 + 530 + 130 kN

= 1090 kN

Thus weight of I and II floor = 1090 kN

Weight of terrance floor = 655 kN

Total weight of building W = 2 × 1090 + 655

= 2835 kN

Step 5 Determination of base shear

Design base shear $V_B = A_h W$

$$= 0.135 \times 2835$$

$$= 382.725 \text{ kN}$$

Step 6 Distribution of equivalent lateral load

The design base shear computed shall be distributed along the height of the building as per the following formula

$$Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2}$$

where, h_i is calculated from base.

$$Q_1 = V_B \left[\frac{W_1 h_1^2}{W_1 h_1^2 + W_2 h_2^2 + W_3 h_3^2} \right]$$

$$= 382.725 \left[\frac{655 \times 3.5^2}{655 \times 3.5^2 + 1090 \times 7^2 + 1090 \times 10.5^2} \right]$$

$$= 16.91 \text{ kN}$$

$$Q_2 = 382.725 \left[\frac{1090 \times 7^2}{655 \times 3.5^2 + 1090 \times 7^2 + 1090 \times 10.5^2} \right]$$

$$= 112.56 \text{ kN}$$

$$Q_3 = 382.725\left[\frac{1090 \times 10.5^2}{655 \times 3.5^2 + 1090 \times 7^2 + 1090 \times 10.5^2}\right]$$

$$= 253.21 \text{ kN}$$

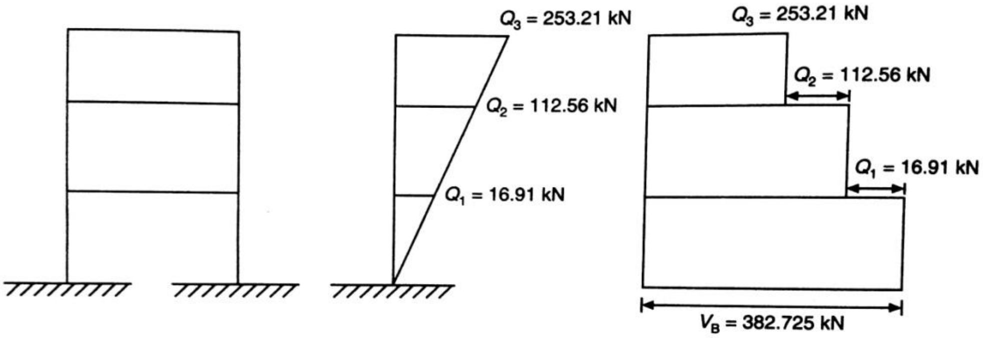


Figure 15.1 Force and shear distribution for three-storey building.

Check :

$$V_B = Q_1 + Q_2 + Q_3$$
$$382.725 \text{ kN} = 382.7 \text{ kN}$$

EFFECTS OF EARTHQUAKE ON REINFORCED CONCRETE STRUCTURE

- Bond failure between concrete and reinforcement
- Direct shear failure of short element such as short column
- Shear cracking in the beam-column joint
- Diagonal cracking of shear walls
- Tearing of slab at discontinuities and junction with stiff vertical elements

PROVISIONS TO PROTECT RC BUILDING DURING EARTHQUAKE

- All frame elements must be detailed with adequate ductility so that is remain elastic at extreme loads
- Non ductility modes such as shear and bond failure must be avoided therefore no anchorage and splicing of bars should be avoided
- Rigid elements should be attached to the structure with ductile or flexible fixings
- Joint should provided at discontinuities with adequate provision for movement so that pounding of the two faces against each other is avoided.

EFFECTS OF EARTHQUAKE ON REINFORCED STEEL STRUCTURE

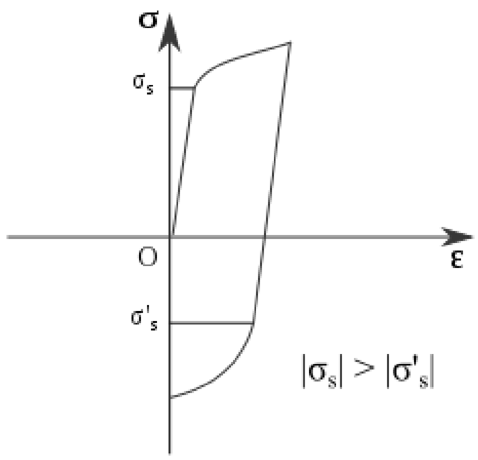
- Local Buckling Of Plate Elements
- Flexural buckling of long columns and braces
- Lateral-tensional buckling of beams
- Uplift of braced frame
- Failure of connection due to shearing and bearing of bolt

PROVISIONS TO PROTECT RC BUILDING DURING EARTHQUAKE

- Brittle failure can be avoided by using mechanical fastener rather than welding
- The use of sufficiently ductile steel
- The ductile design and fabrication of framed members and connection
- Failure mechanism should provide maximum redundancy
- The possibility of failure by local collapse should be avoided

BAUSCHINGER EFFECT:

- The Bauschinger effect refers to a property of materials where the material's stress/strain characteristics change as a result of the microscopic stress distribution of the material. For example, an increase in tensile yield strength occurs at the expense of compressive yield strength..
- While more tensile cold working increases the tensile yield strength, the local initial compressive yield strength after tensile cold working is actually reduced. The greater the tensile cold working, the lower the compressive yield strength.
- The Bauschinger effect is normally associated with conditions where the yield strength of a metal decreases when the direction of strain is changed. It is a general phenomenon found in most polycrystalline metals.
- The basic mechanism for the Bauschinger effect is related to the dislocation structure in the cold worked metal. As deformation occurs, the dislocations will accumulate at barriers and produce dislocation pile-ups and tangles.



10.4.1 Design Spectra

Response spectra developed for displacement, pseudo-velocity and pseudo acceleration in a combined manner for Elcentro earthquake (1940) for various damping ratios as shown in Figure 10.5. This type of spectrum is called as '*tripartite response spectrum*'. Generally a real spectrum has irregular shape with local peaks and valleys. For design purpose, local peaks and valleys should be ignored, since natural period cannot be calculated with that much accuracy. Hence a smooth curve is plotted by considering the average of a number of elastic response spectrums corresponding to various possible earthquakes at a particular site. It is known as the smoothed elastic design response spectrum (SEDRS) as shown in Figure 10.6.

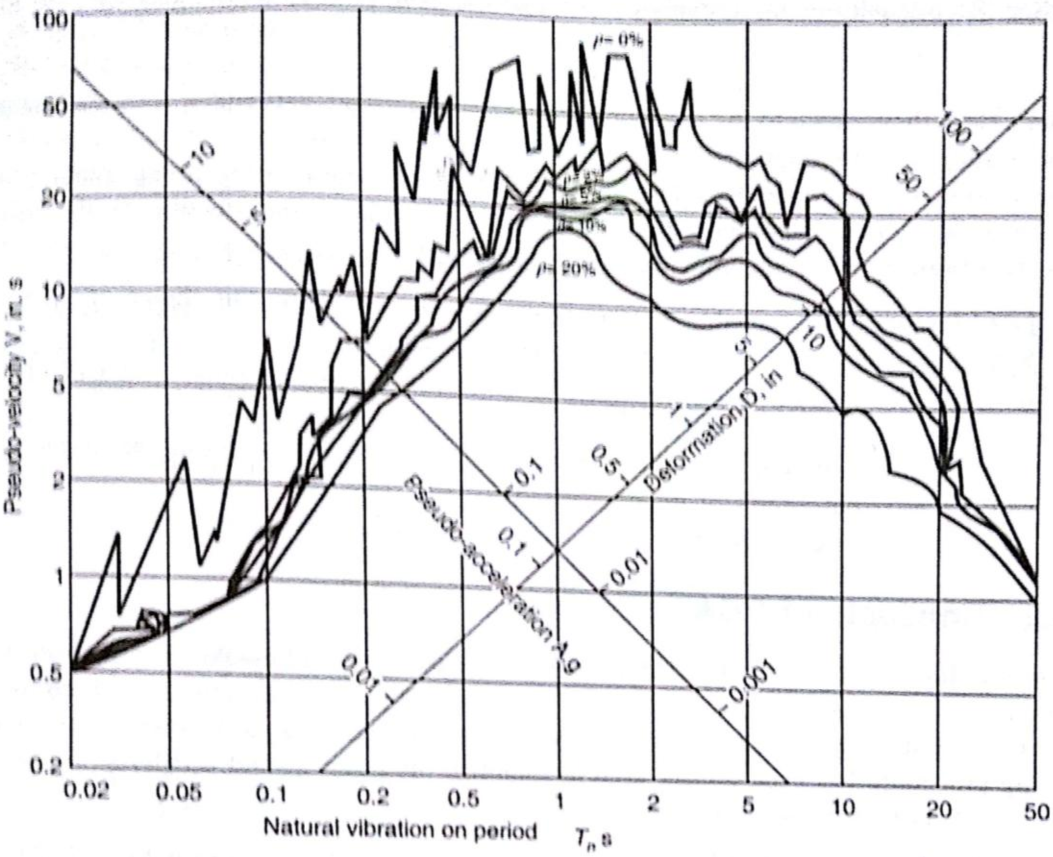


Figure 10.5 Combined displacement, pseudo-velocity and pseudo-acceleration spectrum for El centro ground motion for ρ = 0, 2, 5, 10 and 20%.

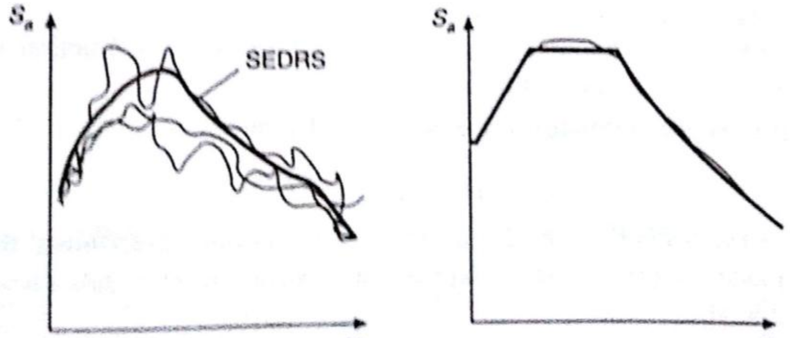


Figure 10.6 Developing the design response spectrum.

Therefore design response spectrum incorporates the spectra for several earthquakes and presents the 'average' response spectrum for design. Generally, it is assumed that the shapes the design spectra are the same for both the design and maximum probable earthquakes but at they differ in intensity as measured by peak ground acceleration. Hence, it is necessary to normalize the intensity of these design spectra to the peak ground acceleration of 1.0g so that,

$$\lim_{\omega \to \alpha} s_a(\rho, \omega) = 1.0g$$

Later scale them down to the appropriate peak acceleration levels which represent the design and maximum probable earthquakes.

Since some damage is expected and accepted in the structure during strong shaking, design spectrum is developed considering the overstrength, redundancy and ductility in the structure. Design spectrum must be accompanied by the following:

1. Load factors or permissible stresses that must be used (different choice of load factors will lead to different seismic, safeties to the structure.)
2. Damping to be used in design (variation in the value of damping used will affect the design force).
3. Method of calculating of natural period (depending on the modelling assumptions, one can get different values of natural period).
4. Type of detailing for ductility.

10.4.2 Concepts of PGA

PGA stands for Peak Ground Acceleration. It is a measure of earthquake acceleration. Unlike Richter scale, it is not a measure of the total size of the earthquake, but rather how hard the earth shakes in a given geographic area. PGA is what is experienced by a particle on the ground. The following procedure has been mostly used to establish the peak ground acceleration values for the maximum probable and design earthquakes:

1. Establish the locations of known active faults in the nearby region of the site. If the active fault is not known, it is required to carry out extensive investigation at the site.
2. To establish the maximum Richter magnitude possible for future earthquakes occurring along each fault, all historical seismic data available from past earthquakes occurring on this active faults are carefully studied.
3. By using empirical relations, mean PGA in expressed as a function of source-to-site distance R and maximum Richter magnitude.

   Campbell gives the following expression to calculate mean PGA

   $$a = b_1[R + b_4 e^{b_5 m}]^{-b_3} e^{b_2 m}$$

   Where, a is mean PGA, b_1, b_2, b_3, b_4 and b_5 are constants determined through nonlinear regression analysis using to the extent possible, strong motion data recorded in the local region of the site.

   R source-to-site distance

   M max Richter magnitude
4. Using the largest value of mean PGA 'a' obtained for all active fault under consideration, the design PGA is then used to scale down the normalized (1g) response spectra to the appropriate level representing the maximum probable earthquake.

UNIT -4
2 MARKS

1. What is peak ground acceleration (PGA)? [M/J-14,N/D-16]

PGA is a measure of earthquake acceleration. Unlike Richter scale, it is not a measure of the total size of the earthquake, but rather how hard the earth shakes in a given geographical area. PGA is experienced by a particle on the ground

2.What do you mean by ductility?[A/M-14,N/D-16]

Ductility as the ability of a structure to undergo larger deformations without collapsing. As per special provisions recommended in codes, the detailing of the structure that lets the structure to gain a larger ductility other than the contributions of material ductility are called as ductility detailing or ductile detailing

3. Write down the steps to improve Global level Ductility?[M/J-13,N/D-15]

(a) Increasing the redundancy of the structure
(b) Weak bam and strong column approach.
(c) Avoiding soft first storey effects

Avoiding Non – ductile failure modes like shear, bond and axial compression at the element level

4. What are the properties of soil on which its spring constant depends?[N/D-15]

The soil spring constant Kz depends on the dynamic soil shear modulus G and poisson's ratio μ.

5. What is zero period acceleration?[N/D-12,M/J-14]

Zero period acceleration implies maximum acceleration experienced by a structure having zero natural period (T =0). An infinitely rigid structure has zero natural period (T = 0). It doesn"t deform. Thus relative motion between its mass and its base, Mass has same acceleration as of the ground. Hence ZPA is the same as peak ground acceleration.

6. Write the short note on soil properties on dynamic analysis.

The soil spring constant Kz depends on the dynamic soil shear modulus G and poisson's ratio μ
The unit weight is needed to calculate the soil density. If the base is resting on loose soil,then it would be either densified or stiffened with admixtures or stone columns

7. Explain site specific response spectra. [N/D-11, M/J-14]

A site specific response spectrum is plotted by taking the average of each record of site specific ground motions. This results in smooth means spectrum. The recorded earthquake motions clearly show that response spectrum shape differs for different types of soil profile at the site.The average shape of response of spectrum

8. What are the different effects of liquefaction (four)? [N/D-11]

- Loss of bearing strength
- Lateral spreading
- Sand boils
- Flow failures
- Ground oscillation
- Flotation
- Settlemen

CE6071-STRUCTURAL DYNAMICS AND EARTHQUAKE ENGINEERING
UNIT-V
DESIGN METHODOLOGY

1. What is “strong column –weak beam” design concept (Nov/Dec-2016)

The design philosophy for frames is to avoid failure of column from both axial load and bending moment considerations. Lack of adequate stiffness or strength in the column will lead to formation of plastic hinges in them. Under such conditions, formation of plastic moment hinges in the beam is preferred to that in the columns. Hence the design is done as beam a weaker member than the column. This concept of designing reinforced concrete frame is called the strong column-weak beam design

2. Write down seismic design philosophy of IS 4326(Nov/Dec-16)

The general principles to be observed in the construction of such earthquake resistant buildings as specified in this standard are Lightness, Continuity of Construction, avoiding/reinforcing Projecting and suspended parts, Building configuration, strength in various directions, stable foundations, Ductility of structure, Connection to non-structural parts and fire safety of structures.

3. What is the formula for finding out the base shear using seismic coefficient method? (Nov/Dec-2015)

The total design lateral force or design seismic base shear (Vb) along any principal direction shall be determined by hte following expression

$$V_b = A_h W$$

Ah – Design horizontal acceleration spectrum value using natural method W – Seismic weight of the building

4. Define Viscous damping (Nov/Dec-15)

Viscous damping is caused by such energy losses as occur in liquid lubrication between moving parts or in a fluid forced through a small opening by a piston, as in automobile shock absorbers. The viscous-damping force is directly proportional to the relative velocity between the two ends of the damping device.

5. Write the formula to find the load factors for plastic design of steel structures(Apr/May-2016)

In plastic design of steel structures, the following load combinations shall be accounted for

b 1.7(DL+IL)
c 1.7(DL+EL)
d 1.3(DL+IL+EL)

When earthquake forces are considered on a structure, these shall be combined as per load combination for plastic design of steel structures and partial safety factor for limit state design of reinforced concrete structures and prestressed concrete structures

6. What are the techniques involved in improving element level ductility?(Apr/May-2016)

Ductility in element level is generally with reference to the displacement and moment curvature relationship of a section. This an be generally improved by

a. Decreasing the tension steel area, yield stress and strain of the tension steel increasing the ultimate compressive strain of concrete
b. Increasing the area of compressive steel
c. Reduction in the axial compression on the section
d. Provision of effective confinement stirrups, hoops or ties such that compressive steel does not buckle and concrete is led in to three dimensional state of stress such that its ultimate compressive strain increases

7. Write the IS13920 provisions for flexural members(Apr/May-2015)

The provisions apply to frame members resisting earthquake induced forces and designed to resist flexure. These members shall satisfy the following provisions

❖ The factored axial stress on the member under earthquake loading shall not exceed 0.1fck
❖ The member shall preferable have a width to depth ratio more than 0.3
❖ Width of the member shall not be less than 200mm

8. Write the methods of dynamic analysis of multi-storeyed structure as per IS 1893 code (Apr/May-2015)

This code states that structures should withstand without structural damage, moderate earthquakes and withstand without total collapse under heavy earthquakes. This code specifies two methods of analysis

❖ Seismic co-efficient methods
❖ Modal analysis or Response Spectrum method

9. Define mass irregularity(Nov/Dec-2014)

Mass irregularity is induced by the presence of a heavy mass on a floor, say a swimming pool. In IS1893 the mass irregularity has been defined as a situation when weight of a floor exceeds twice the weight of the adjacent floor.

10. What are the damages due to seismic effects?(Nov/Dec-2014)

Damages due to

a. Liquefaction
b. Surface faulting
c. Ground shaking
d. Sliding of superstructure on its foundation
e. Structural vibrations

11. Define lateral load analysis of building system(Nov/Dec-12)

Earthquake force is an inertia force which is equal to mass times acceleration. Mass of the building is mainly located at the floors. Transferring the horizontal component of seismic force safely to the ground is the major task in seismic design. The floors should transfer the horizontal force to vertical seismic elements viz. columns, frames, walls and subsequently to the foundation finally to the soil.

12. Define the term symmetry in buildings. Why symmetrical forms are preferred than unsymmetrical forms (Nov/Dec-12)

Symmetry denotes a geometrical property of the plan configuration, where as the structural symmetry means that the center of mass and the center of resistance are located at the same point. In symmetrical configuration/structural system the eccentricity between the center of mass and resistance will produce torsion and stress concentration and therefore the symmetrical forms are preferred to the symmetrical ones

13. What are the factors that govern the architectural configurations?(Apr/May-13)

a. Architectural design
b. Functional requirements
c. Urban design parameters
d. Planning considerations
e. Aesthetic appearance
f. Identity (distinctiveness)

14. What are the objectives of earthquake resistant designs?(Apr/May-13)

a. Resist minor earthquake shaking without damage
b. Resist moderate earthquake shaking without structural damage but possibly with some damage to non-structural members
c. Resist major levels of earthquakes shaking with both structural and non-structural damage, but the building should not collapse thus endangerment of the lives of occupants is avoided

16. Explain two cases of design horizontal earthquake load.

(a) When the lateral resisting elements are oriented along orthogonal horizontal direction, the structure shall be designed for the effects due to full design earthquake load in one horizontal direction at time.

(b) When the lateral load resisting elements are not oriented along the orthogonal horizontal directions, the structure shall be designed for the effect due to full design earthquake load in one horizontal direction plus 30% of the design earthquake load in the other direction.

17. What is additive shear?

Additive shear will be super-imposed for a statically applied eccentricity of ±0.05 with respect to centre of rigidity.

18. Name types of damper's.

(i) Metallic dampers or yielding dampers

(ii) Friction dampers

(iii) Viscous dampers.

19. Write the formula for modal mass (M_k).

The modal mass $\mathbf{M_k}$ of mode k is given by:

$$M_k = \frac{[\sum_{i=1}^{n} W_i \phi_{ik}]^2}{g \sum_{i=1}^{n} W_i (\phi_{ik})^2}$$

20. Explain design eccentricity.

The design eccentricity, to be used at floor I shall be taken as:

$$e_{di} = \begin{Bmatrix} 1.5\, e_{si} + 0.05 b_i \\ or\ e_{si} - 0.05 b_i \end{Bmatrix}$$

Whichever of these gives the more severe effect in the shear of any frame

Where

e_{di} = Static eccentricity
e_{di} = defined as the distance between centre of mass and centre of rigidity
b_i = floor plan dimension of floor

1.) PLANNING & ARCHITECTURAL CONCEPTS:

IS 4326-1993 clause (4.1 to 4.9) & IS 1893(part-1):2002 tables – 4,5&6 page 18

The behavior of a building during earthquakes depends critically on its overall shape, size and geometry, in addition to how the earthquake forces are carried to the ground. Hence, at the planning stage itself, architects and structural features unfavorable features should be avoided and a good building configuration must be chosen. Types of irregularity that effect building during earth quake are as follows

- Vertical irregularity
- Torsional irregularity
- Mass irregularity

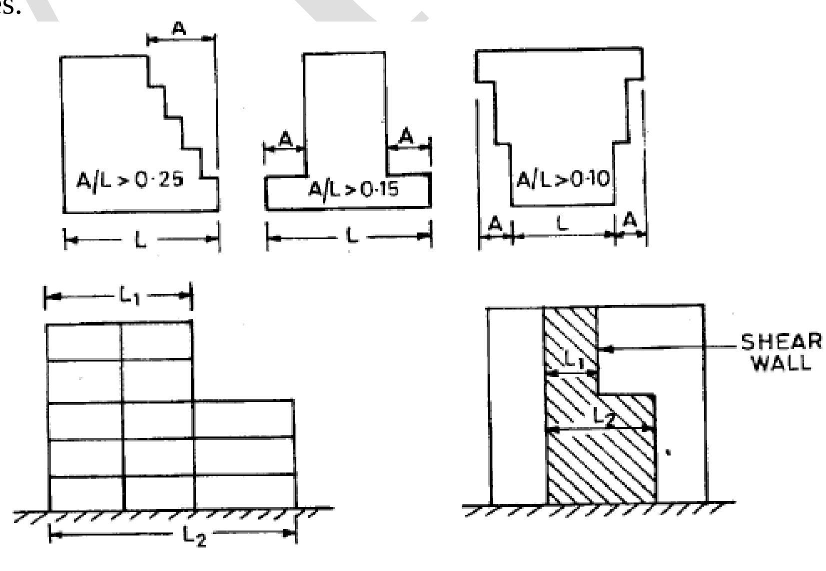
Vertical Irregularity: (IS 1893, tables 5, pg.18)

Vertical irregularity refers to the variation in size, shape, mass and stiffness of one storey with respect to the adjacent storey (ie) the specific storey with respect to the top and bottom storey. Vertical irregularity are as follows

- **Soft Storey** – if the variation in stiffness of one floor is 60% less than the above storey it is considered as a vertical irregularity

- **Vertical Geometric Irregularity-** if the horizontal dimension of the lateral force resisting system in any storey is more than 150 percent of that in its adjacent storey it is considered as a vertical irregularity

- **Mass Irregularity-** Mass irregularity shall be considered to exist where the seismic weight of any storey is more than 200 percent of that of its adjacent stories.



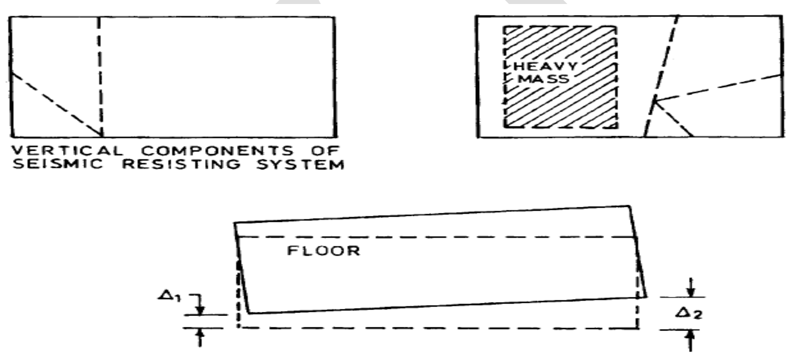
4 C Vertical Geometric Irregularity when $L_2 > 1.5\ L_1$

Torsional Irregularity: (IS 1893, tables 4, pg.18) & (IS 4326,Clause 4,PG.2)

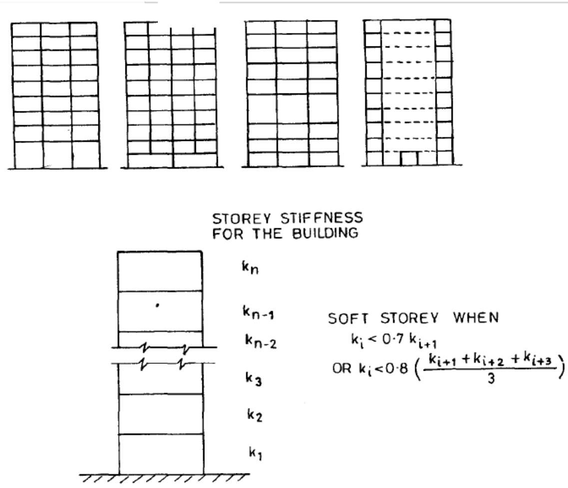
- **structural irregularity:** it is to be considered when floor diaphragms are rigid in their own plan in relation to the vertical structural elements that resist the lateral forces. Torsional irregularity to be considered to exist when the maximum storey drift, computed with design eccentricity, at one end of the structures transverse to an axis is more than 1.2 times the average of the storey drifts at the two ends of the structure

- **Torsional geometric irregularity-** If the building is unsymmetrical with respect to both mass and rigidity such that the centres of mass and rigidity of the building do not coincide with each other torsional irregularity occurs

- if a member consist of unsymmetrical vertical member it causes twisting of the floor memeber



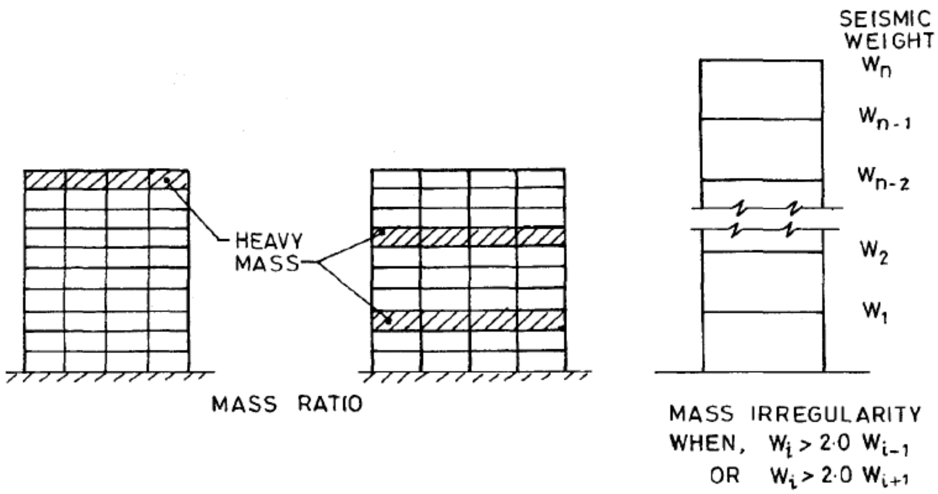
3 A Torsional Irregularity



4 A Stiffness Irregularity

Mass Irregularity: (IS 1893, tables 4&5, pg.18)

- variation of mass in between adjacent floor such as set back building in which the dimension of one floor is either smaller are larger than the another causes irregularity
- lack of infill walls in certain floor causes a variation in the mass of the structure as mass in reduced due to absence of infall wall
- mass irregularity may also occur in the horizontal plane as well this type of mass irregularity cause twisting in structure



4 B Mass Irregularity

VERTICAL IRREGULARITIES — *Continued*

2.) **Design and detailing as per IS 13920-1993(nov/dec2016)**

FLEXURAL MEMBER: (IS 13920,Clause6.1 ,Pg.3)

- The factored axial stress on the member under earthquake loading shall not exceed 0.1 f_{ck}.
- The member shall preferably have a width-to-depth ratio of more than 0.3
- The width of the member shall not be less than 200mm
- The depth D of the member shall preferably be not more than ¼ of the clear span

LONGITUDINAL REINFORCEMENT: (IS 13920,Clause6.2 ,Pg.3)

- The top and bottom reinforcement shall atleast consist of two bars
- The min tension steel ratio on any face shall not be less than $0.24\sqrt{\frac{fck}{fy}}$

- The max tension steel ratio on any face shall not be more than 0.025
- The positive steel at the joint face must be atleast half of the negative steel at the face
- The longitudinal bars shall be spliced, only if the hoop are provided over the entire splice length, at a spacing not exceeding 150mm
- The lap length shall not be less than the bar development length in tension

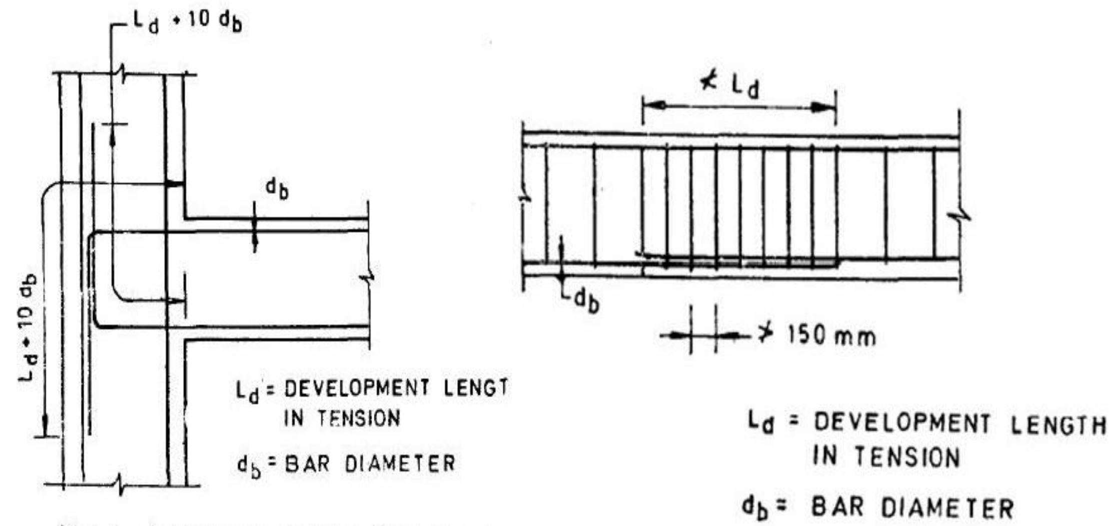


FIG. 1 ANCHORAGE OF BEAM BARS IN AN

Web reinforcement(IS 13920,Clause6.3 ,Pg.4)

- Web reinforcement shall consist of vertical hoops.
- A vertical hoop is a closed stirrup having a 13.5" hook with a 10 diameter extension (but not < 75 mm) at each end that is embedded in the confined core as shown in the diagram below
- it may also be made up of two pieces of reinforcement; a U-stirrup with a 135" hook and a 10 diameter extension (but not c 75 mm) at each end, embedded in the confined core and a crosstie in diagram A crosstie is a bar having a 135" hook with a 10 diameter extension (but not < 75 mm) at each end. The hooks shall engage peripheral longitudinal bars.
- The minimum diameter of the bar forming a hoop shall be 6 mm. However, in beams with clear span exceeding 5 m, the minimum bar diameter shall be 8 mm.

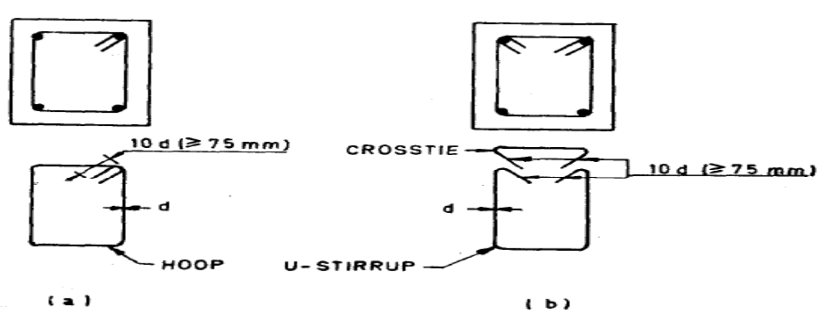


FIG. 3 BEAM WEB REINFORCEMENT

The shear force to be resisted by the vertical hoops shall be the maximum of :
a) calculated factored shear force as per analysis, and
b) shear force due to formation of plastic hinges at both ends of the beam plus the factored gravity load on the span.

i) for sway to right:

$$V_{u,a} = V_a^{D+L} - 1.4 \left[\frac{M_{u,lim}^{As} + M_{u,lim}^{Bh}}{L_{AB}} \right]$$

$$\text{and } V_{u,b} = V_b^{D+L} + 1.4 \left[\frac{M_{u,lim}^{As} + M_{u,lim}^{Bh}}{L_{AB}} \right], \text{ and}$$

ii) for sway to left:

$$V_{u,a} = V_a^{D+L} + 1.4 \left[\frac{M_{u,lim}^{Ah} + M_{u,lim}^{Bs}}{L_{AB}} \right]$$

$$\text{and } V_{u,b} = V_b^{D+L} - 1.4 \left[\frac{M_{u,lim}^{Ah} + M_{u,lim}^{Bs}}{L_{AB}} \right],$$

WHERE:

Mt_u^{As}lim, Mt_u^{Ah}lim, and Mt_u^{Bs}lim, Mt_u^{Bh}lim are the sagging and hogging moments of resistance of the beam section at ends A and B respectively. These are to be calculated as per IS 456 -1978. L_{AB} is clear span of beam. V_a^{D+L} and V_b^{D+L} are the shears at ends A and B, respectively, due to vertical loads with a partial safety factor of 1.2 on loads. the larger of the two values of $V_{u,r}$, computed above. The design shear at end A shall be Similarly, the design shear at end B shall be the larger of the two values of $V_{u,b}$ computed above.

DESIGN OF COLUMNS AND FRAME MEMBERS:(nov/dec2017)
GENERAL: (IS 13920,Clause7.1,Pg.4)

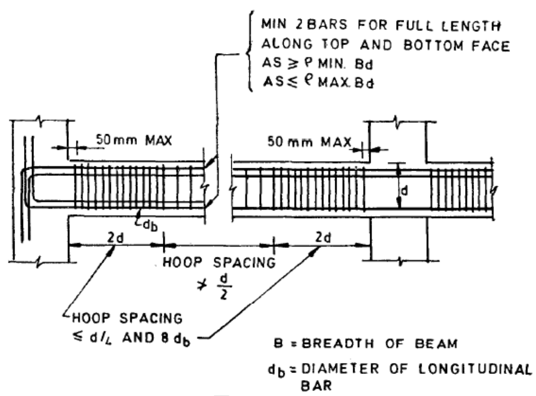
- ❖ These requirements apply to frame members which have a factored axial stress in excess of 0.1 f_{ck} under the effect of earthquake forces.

- ❖ The minimum dimension of the member shall not be less than 200 mm. However, in frames which have beams with centre to centre span exceeding 5 m or columns of unsupported length exceeding 4 m, the shortest dimension of the column shall not be less than 300 mm.

- ❖ The ratio of the shortest cross sectional dimension to the perpendicular dimension shall preferably not be less than 0.4.

LONGITUDINAL REINFORCEMENT: (IS 13920,Clause7.2,Pg.5&6)

- ❖ Lap splices shall be provided only in the central half of the member length. It should be proportioned as a tension splice. Hoops shall be provided over the entire splice length at spacing not exceeding 150 mm centre to centre. Not more than 50 percent of the bars shall be spliced at one section.

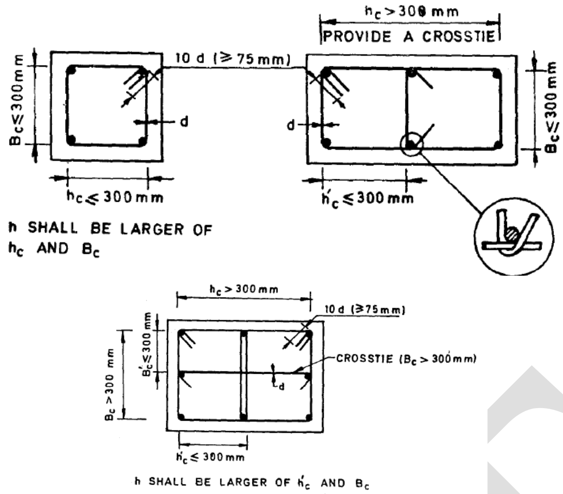


Transverse Reinforcement: (IS 13920,Clause7.3 ,Pg.6)

- ❖ Transverse reinforcement for circular columns shall consist of spiral or circular hoops. In rectangular columns, rectangular hoops may be used. A rectangular hoop is a closed stirrup, having a 135” hook with a 10 diameter extension but not less then 75mm at each that IS embedded in the confined core

- ❖ The parallel legs of rectangular hoops shall be spaced not more than 300 mm centre to centre.

- ❖ If the length of any side of the hoop exceeds 300 mm, a crosstie shall be provided Alternatively, a pair of overlapping hoops may be provided within the columm The hooks shall engage peripheral longitudinal bars.



7C OVERLAPPING HOOPS WITH A CROSSTIE

Special Confining Reinforcement:

This requirement shall be met with, unless a larger amount of transverse reinforcement is required from shear strength considerations.

Mechanical Vibration and Structural Dynamics

Unit 1: Introduction - Single degree-of-freedom system

Contents

Lecture No.	Date	UNIT	TOPIC	Reference	Pages
		I	**Introduction to Single-Defree-of-Freedom-System**		
1		1.1	Simple Harmonic motion (SHM), terminology		
		1.2	Degrees of freedom		
2		1.3	Free vibration and forced vibration		
			Examples of single-degree-of-freedom mechanical vibrations		
			Equation of motion		
		1.4	Spring, inertia and damping elements		
3		1.5	Undamped natural frequency		
			Damped natural frequency		
			Damping ratio		
4		1.6	Mechanism of damping		
			Equivalent viscous damping		
5		1.7	Forced vibrations		
			Examples		
			Resonance		
			Amplitude and phase response diagram		
6		1.8	Vibration measuring instrunent		
7			D'Alembert Principles		

1.0 Some historical background

- Historically studies on **vibration** (acoustics) started long ago (around 4000BC)
- Musicians and philosophers have sought out the rules and laws of sound production, used them in improving musical instruments, and passed them on from generation to generation
- Music had become highly developed and was much appreciated by Chinese, Hindus, Japanese, and, perhaps, the Egyptians.
- These early peoples observed certain definite rules in connection with the art of music, although their knowledge did not reach the level of a science.
- Early applications (by Egyptian) to single or multiple string instruments known as Harps
- Our present system of music is based on ancient Greek civilization.
- The Greek philosopher and mathematician Pythagoras (582-507 B.C.) is considered to be the first person to investigate musical sounds on a **scientific basis** [later on we will be talking about **Mathematical Basis** as well]

1.1 Introductory Remarks

- Most human activities involve vibration in one form or other. For example, we hear because our eardrums vibrate and see because light waves undergo vibration

- Any motion that repeats itself after an interval of time is called *vibration or oscillation.*

- The general terminology of "Vibration" is used to describe oscillatory motion of mechanical and structural systems

- The Vibration of a system involves the transfer of its potential energy to kinetic energy and kinetic energy to potential energy, alternately

-

1.1 Introductory Remarks

- Any object in this world having mass and elasticity is capable of vibration

- We are mainly interested in vibration of mechanical system

- When subjected to an oscillating load, this system undergoes a vibratory behavior

- Vibrations are an engineering concern in these applications because they may cause a catastrophic failure (complete collapse) of the machine or structure because of excessive stresses and amplitudes (resulting mainly from resonance) or because of material fatigue over a period of time

Example: - Failure of Tacoma Narrows Bridge in 1940 due to 42-mile-per-hour wind undergoing a torsional mode resonance
- Vibration of machine components generate annoying noise
- Vibration of string generate pleasing music (already discussed before)

- Vibrations in mechanical system (or more preciously flight vehicles) is dissipated by inherent damping of the material

- Vibration of mechanical system is model as a combination of spring-mass-damper

1.1 Introductory Remarks

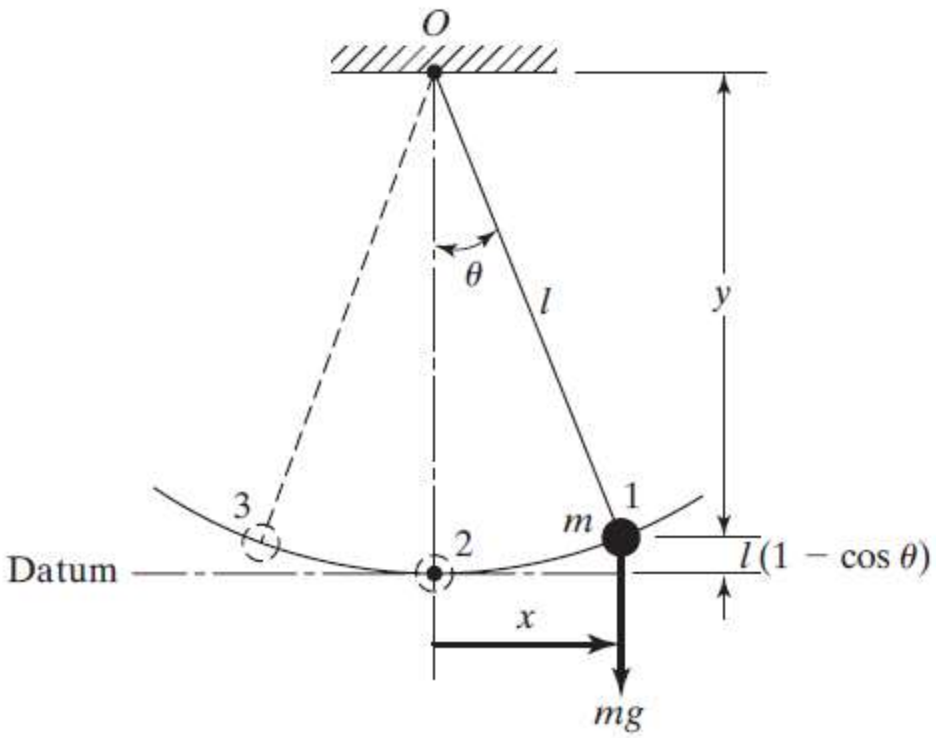
- In some system it may be clearly visible – for example vibration of automobiles

  - The body mass represented by concentrated mass m
  - The Stiffness of suspension system is represented by linear/nonlinear spring k
  - The shock absorber is represented by damper c

- In most of the cases (like in continuous system) it may not be possible clearly identify spring-mass-damper system

  - Vibration of flight vehicle
  - Vibration of machine component etc

1.2 Degrees of freedom



“Period of vibration” is the time that it takes to complete one cycle. It is measured in seconds.
“Frequency” is the number of cycles per second. It is measured in Hz (1 cycle/second). It could be also measured in radians/second.

Period of vibration: T
Frequency of vibration: f = (1/T) Hz or ω = (2π/T) radians/s T=(2 π/ω) = (1/T)

Types of Vibratory Motion

Oscillatory motion may repeat itself regularly, as in the case of a simple pendulum, or it may display considerable irregularity, as in the case of ground motion during an earthquake.

If the motion is repeated after equal intervals of time, it is called *periodic motion. The simplest type of periodic motion is harmonic motion.*

Harmonic motion

It is described by sine or cosine functions.

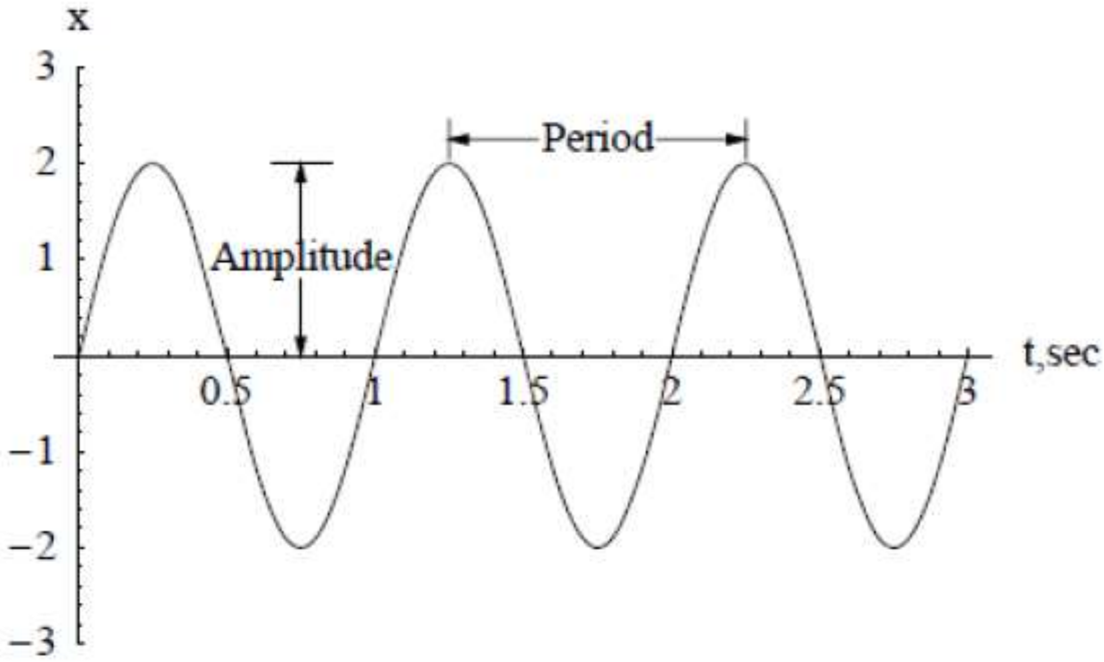
$x(t) = A\sin(\omega\, t)$

A is the amplitude while ω is the frequency (radians/sec)

$$\dot{x}(t) = \omega\ A\cos(\omega\ t)$$

$$\ddot{x}(t) = -\omega^2 A\sin(\omega\ t) = -\omega^2 x(t)$$

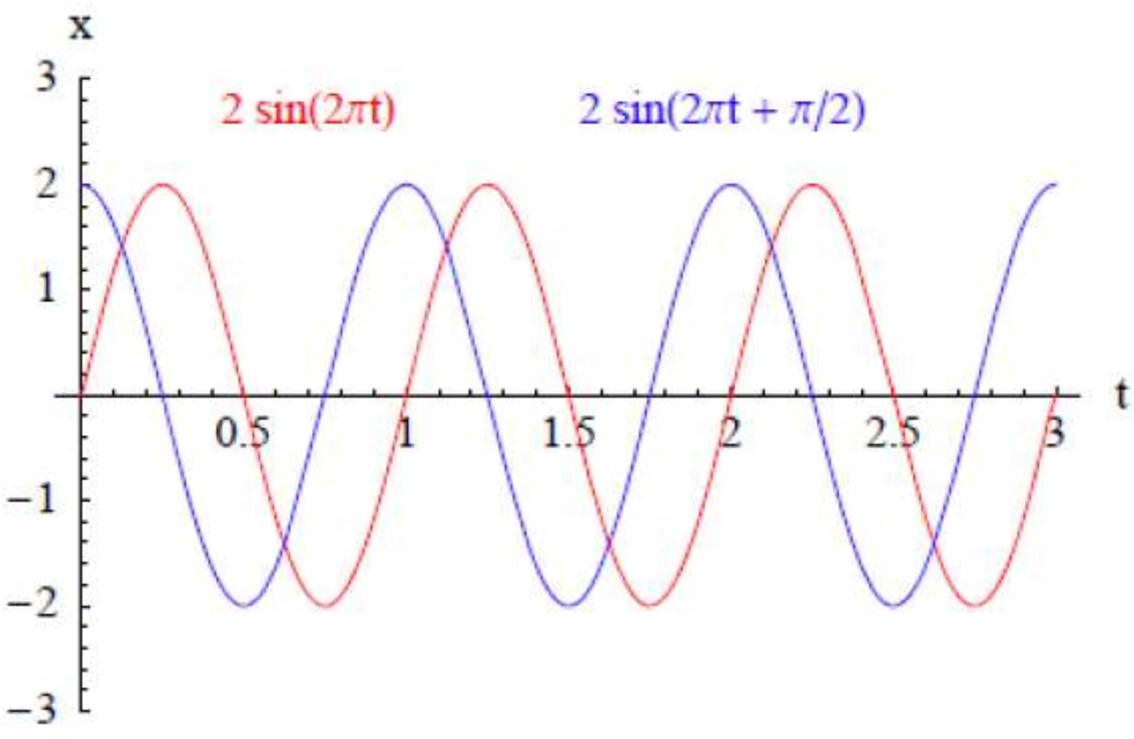
Types of Vibratory Motion



Plot of $x(t) = 2\sin(2\pi t)$

Types of Vibratory Motion

Two harmonic motions having the same period and/or amplitude could have different phase angle



Plot of two harmonic functions $2\sin(2\pi t)$ and $2\sin(2\pi t + \pi/2)$

Types of Vibratory Motion

- A harmonic motion can be written in terms of exponential functions.

$$\sin \omega t = \frac{e^{i\omega t} - e^{-i\omega t}}{2i}; \quad \cos \omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

so that

$$e^{i\omega t} = \cos \omega t + i \sin \omega t$$

A harmonic motion could be written as

$$x(t) = a\, e^{i\omega t}$$

- Alternative forms for harmonic motion

Generally, a harmonic motion can be expressed as a combination of sine and cosine waves.

$$y(t) = A \cos \omega t + B \sin \omega t \quad \Longleftrightarrow \quad y(t) = Y \sin(\omega t + \theta)$$

Types of Vibratory Motion

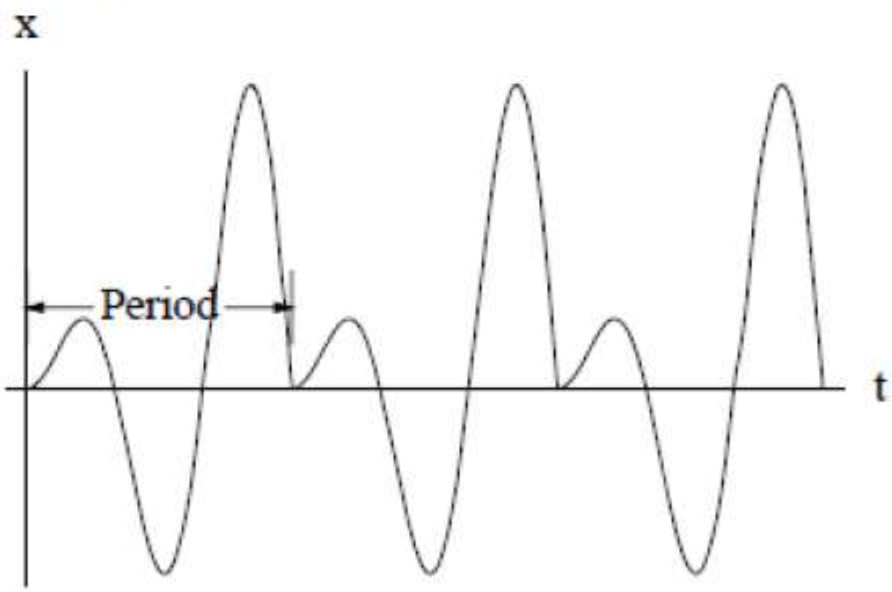
$$Y = \sqrt{A^2 + B^2} \qquad \theta = \tan^{-1}(A/B)$$

or

$$y(t) = A\cos\omega t - B\sin\omega t \iff y(t) = -Y\sin(\omega t - \theta) = Y\cos(\omega t - \theta)$$

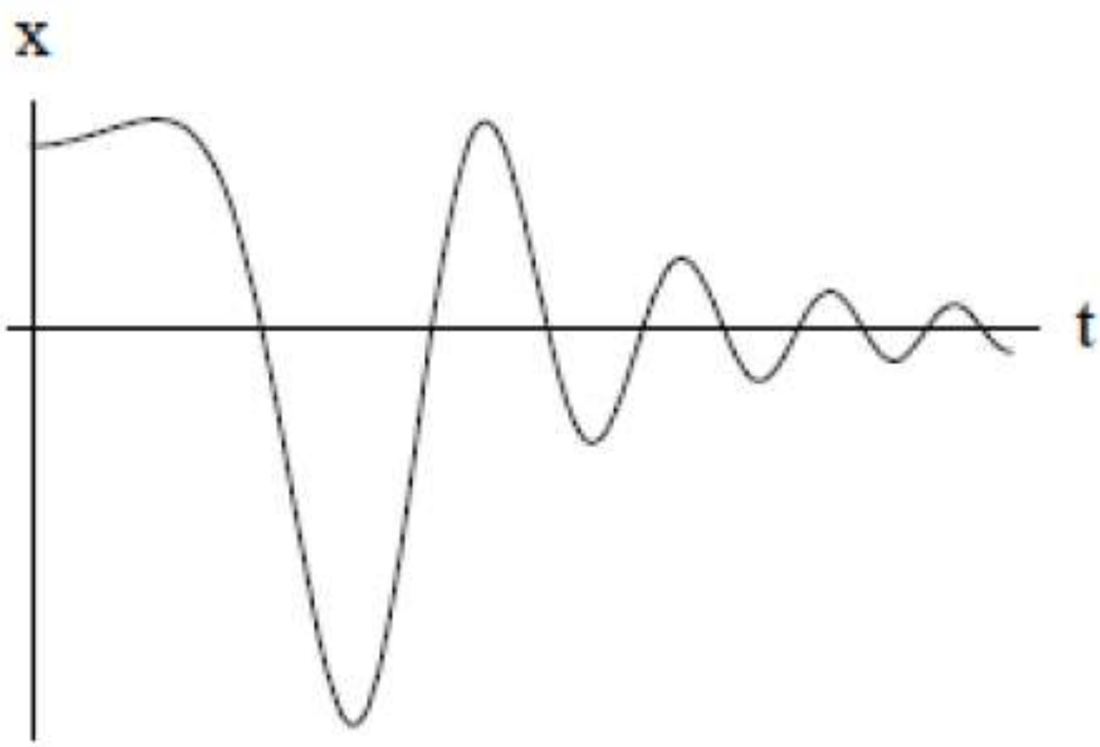
- Periodic motion

The motion repeats itself exactly.

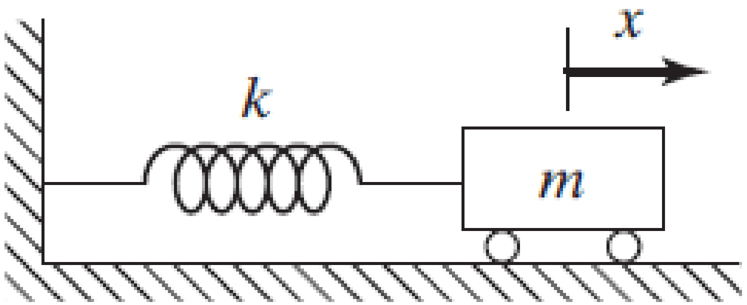


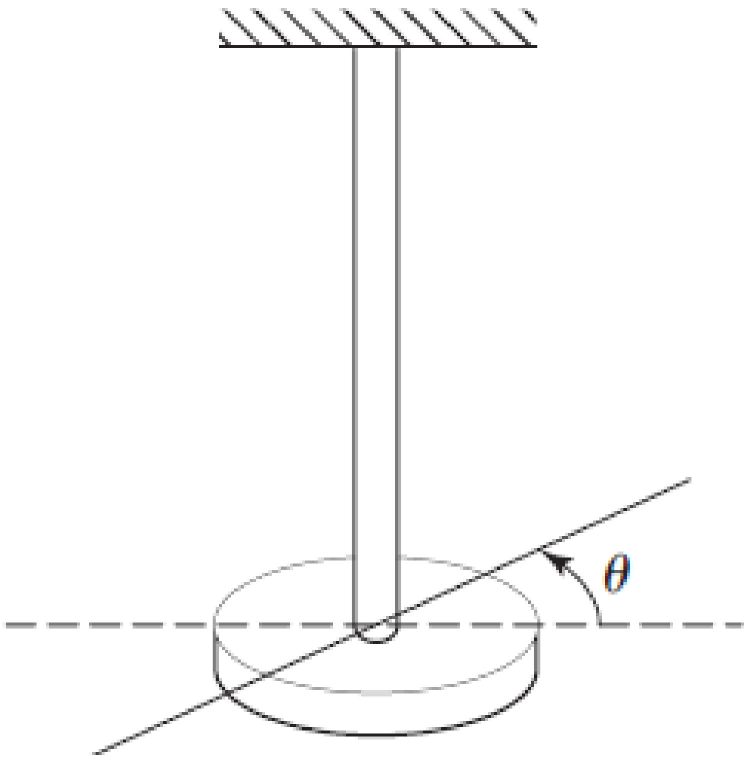
Types of Vibratory Motion

A general vibratory motion doesn't have a repeating pattern.

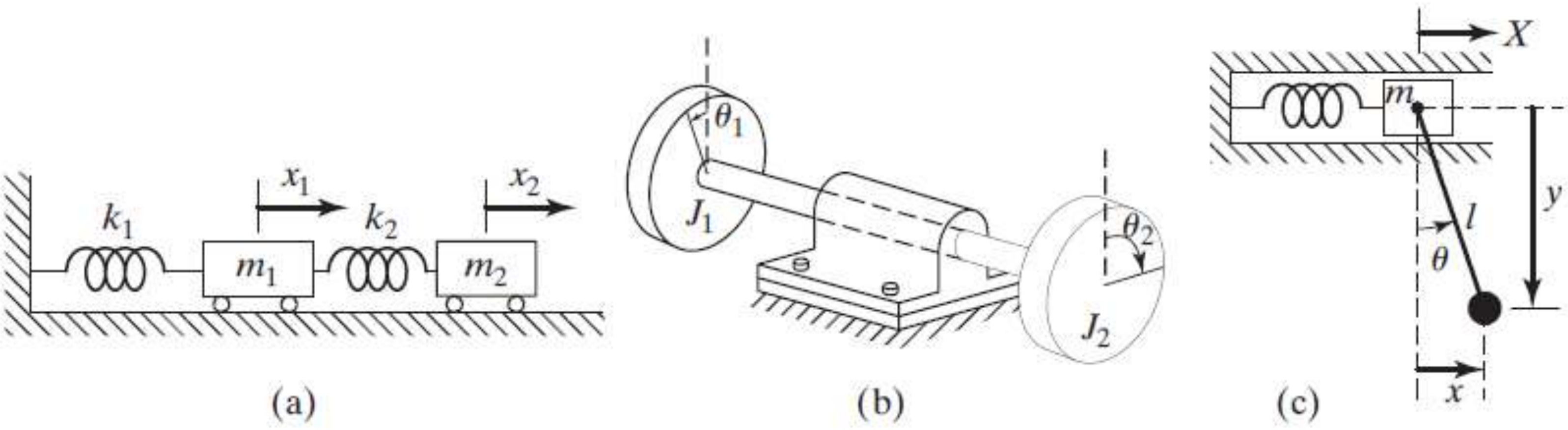


1.2 Degrees of freedom (cont…)

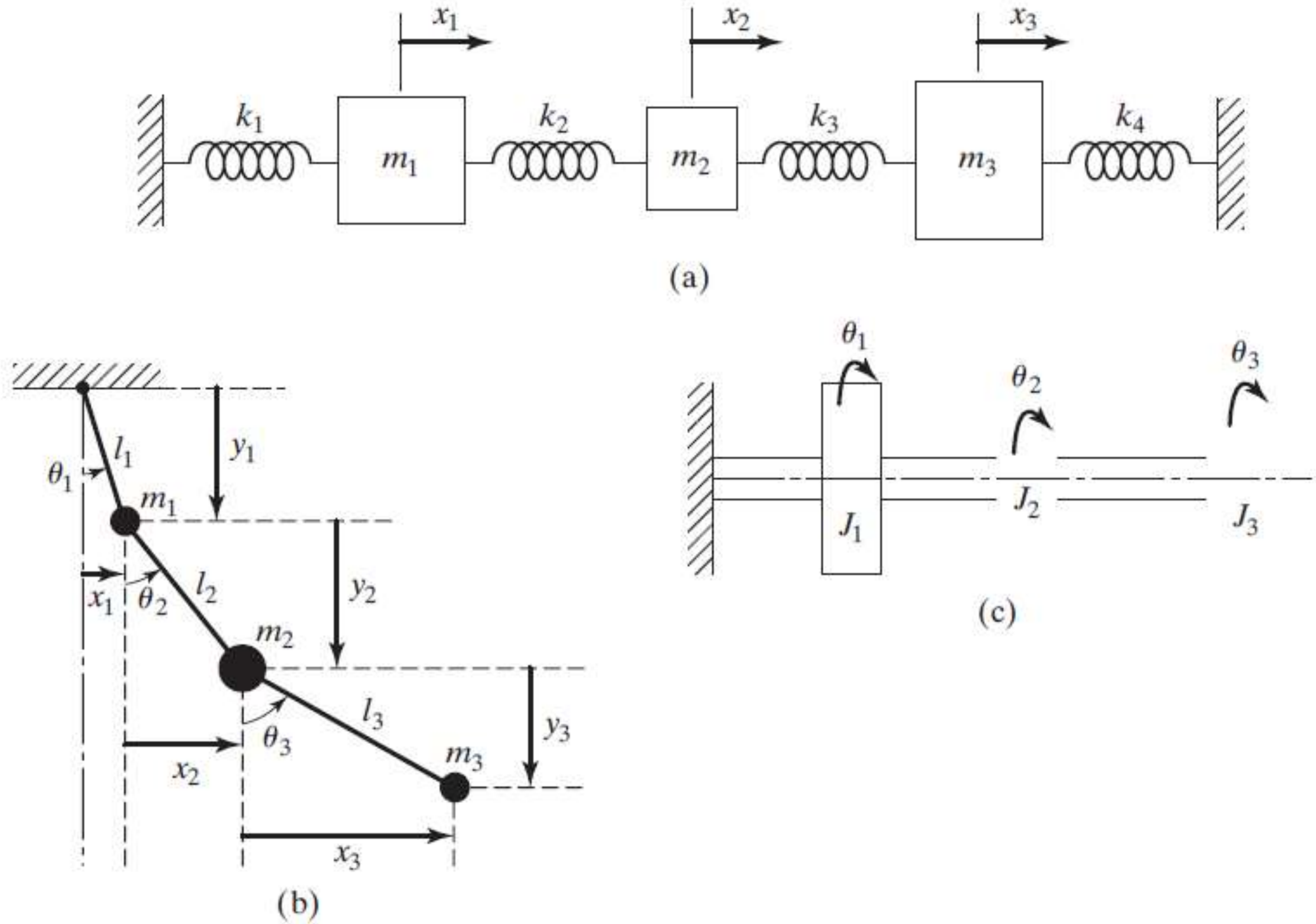




1.2 Degrees of freedom (cont…)



1.2 Degrees of freedom (cont…)



1.2 Degrees of freedom (cont…)

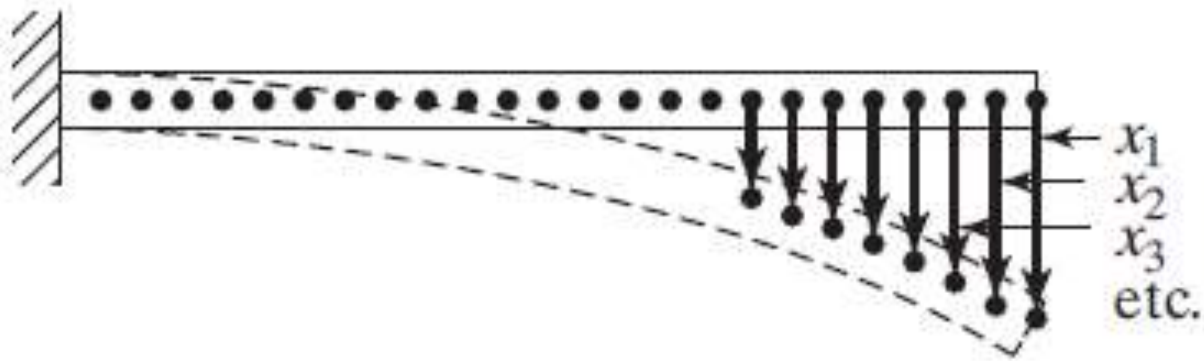


FIGURE 1.14 A cantilever beam (an infinite-number-of-degrees-of-freedom system).

1.3 Classification Vibration

Vibration can be classified in several ways. Some of the important classifications are as
follows.

a) Free and forced vibration
b) Undamped and damped vibration
c) Linear and nonlinear vibrations
d) Deterministic and random vibration

The terminology of **"Free Vibration"** is used for the study of natural vibration modes in the absence external loading.

The terminology of **"Forced Vibration"** is used for the study of motion as a result of loads that vary rapidly with time. Loads that vary rapidly with time are called dynamic loads.

1.3 Classification Vibration

If no energy is lost or dissipated in friction or other resistance during oscillation, the vibration is known as "**undamped vibration**".

If any energy is lost in this way, however, is called "**damped vibration".**

If the system is damped, some energy is dissipated in each cycle of vibration and must be replaced by an external source if a state of steady vibration is to be maintained.

Importance of Dynamic Analysis

Load magnification and Fatigue effects

A static load is constant and is applied to the structure for a considerable part of its life. For example, the self weight of building. Loads that are repeatedly exerted, but are applied and removed very slowly, are also considered static loads.

Fatigue phenomenon can be caused by repeated application of the load. The number of cycles is usually low, and hence this type of loading may cause what is known as low-cycle fatigue.

Quasi-static loads are actually due to dynamic phenomena but remain constant for relatively long periods.

Most mechanical and structural systems are subjected to loads that actually vary over time. Each system has a characteristic time to determine whether the load can be considered static, quasi-static, or dynamic. This characteristic time is *the fundamental period of free vibration of the system.*

Importance of Dynamic Analysis

Dynamic Load Magnification factor (DLF) is the ratio of the maximum dynamic force experienced by the system and the maximum applied load.

The small period of vibration results in a small DLF.

Fatigue phenomenon can be caused by repeated application of the load. The number of cycles and the stress range are important factors in determining the fatigue life.

1.3 Classification Vibration

1.4 Spring, inertia and damping elements

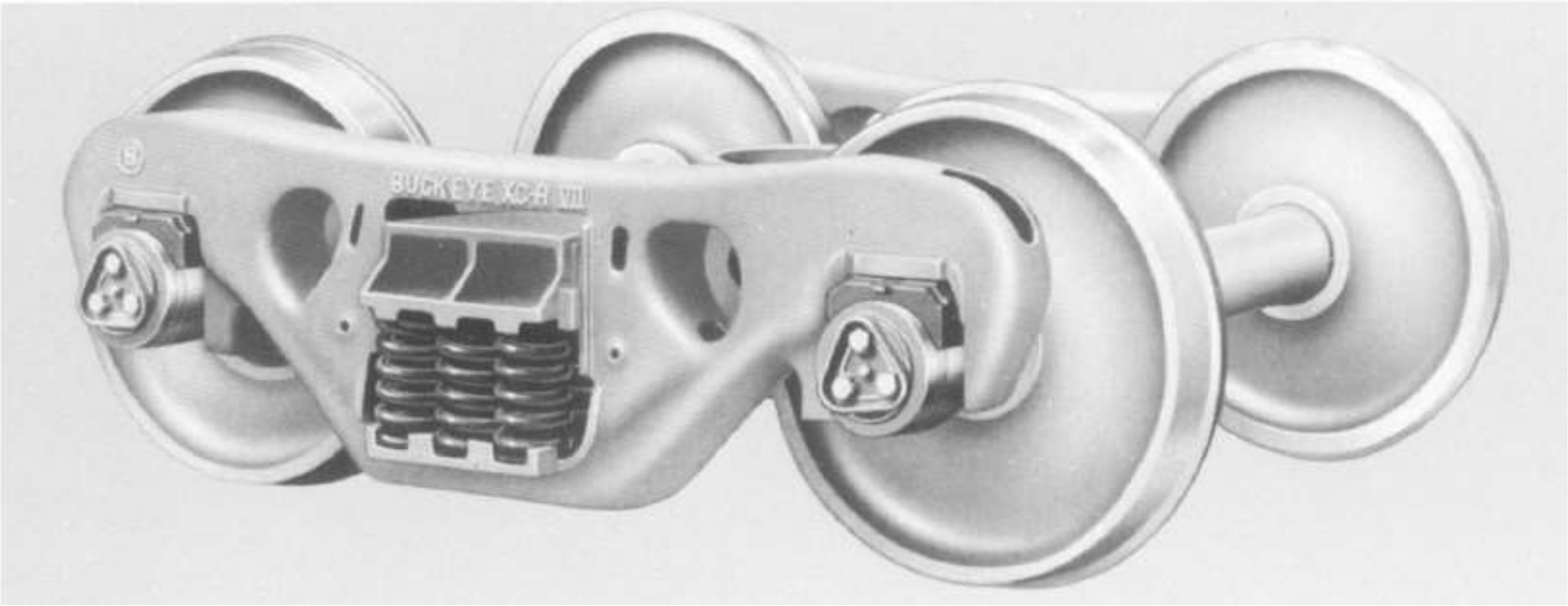
A vibratory system, in general, includes a means for storing potential energy (spring or elasticity), a means for storing kinetic energy (mass or inertia), and a means by which energy is gradually lost (damper).

The minimum number of independent coordinates required to determine completely the positions of all parts of a system at any instant of time defines the degree of freedom (DOF) of the system.

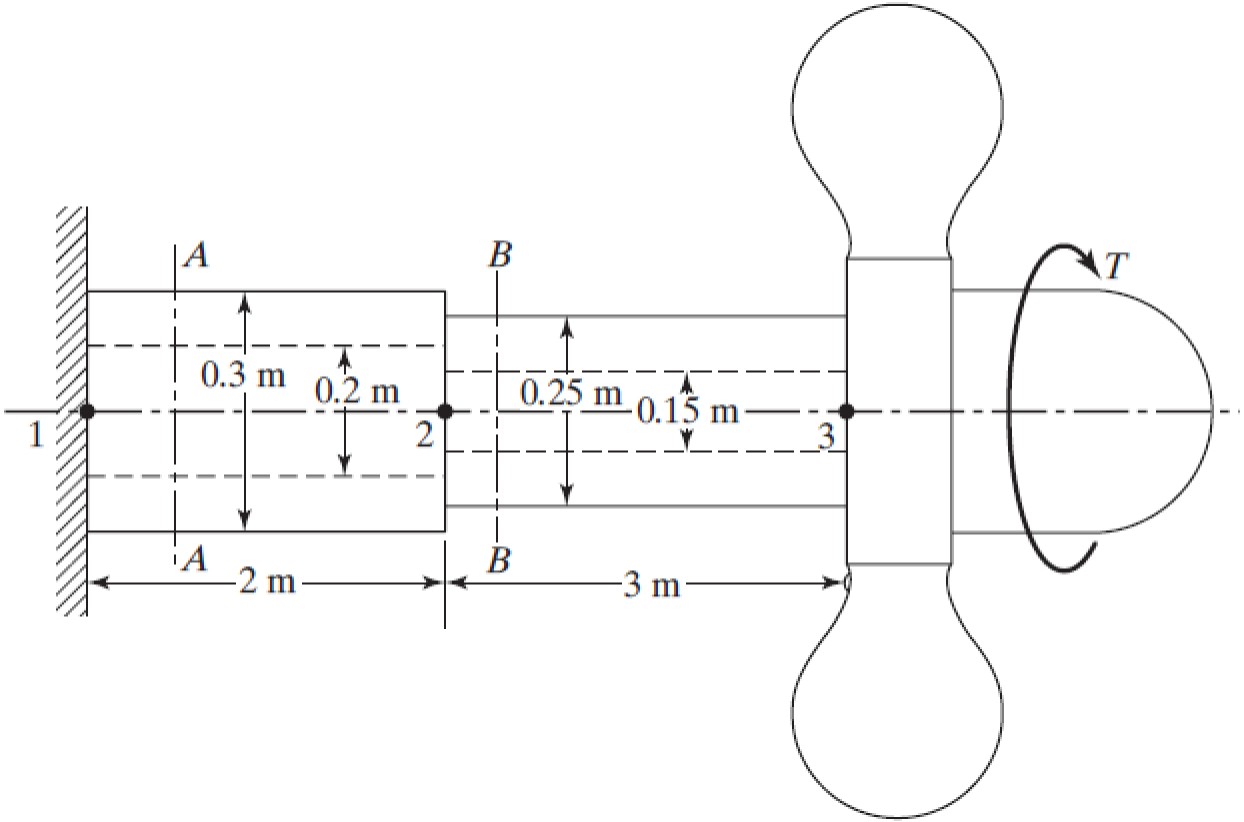
A large number of practical systems can be described using a finite number of DOFs. Systems with a finite number of DOFs are called *discrete* or *lumped parameter systems.*

Some systems, especially those involving continuous elastic members, have an infinite number of DOFs. Those systems are called *continuous or distributed systems.*

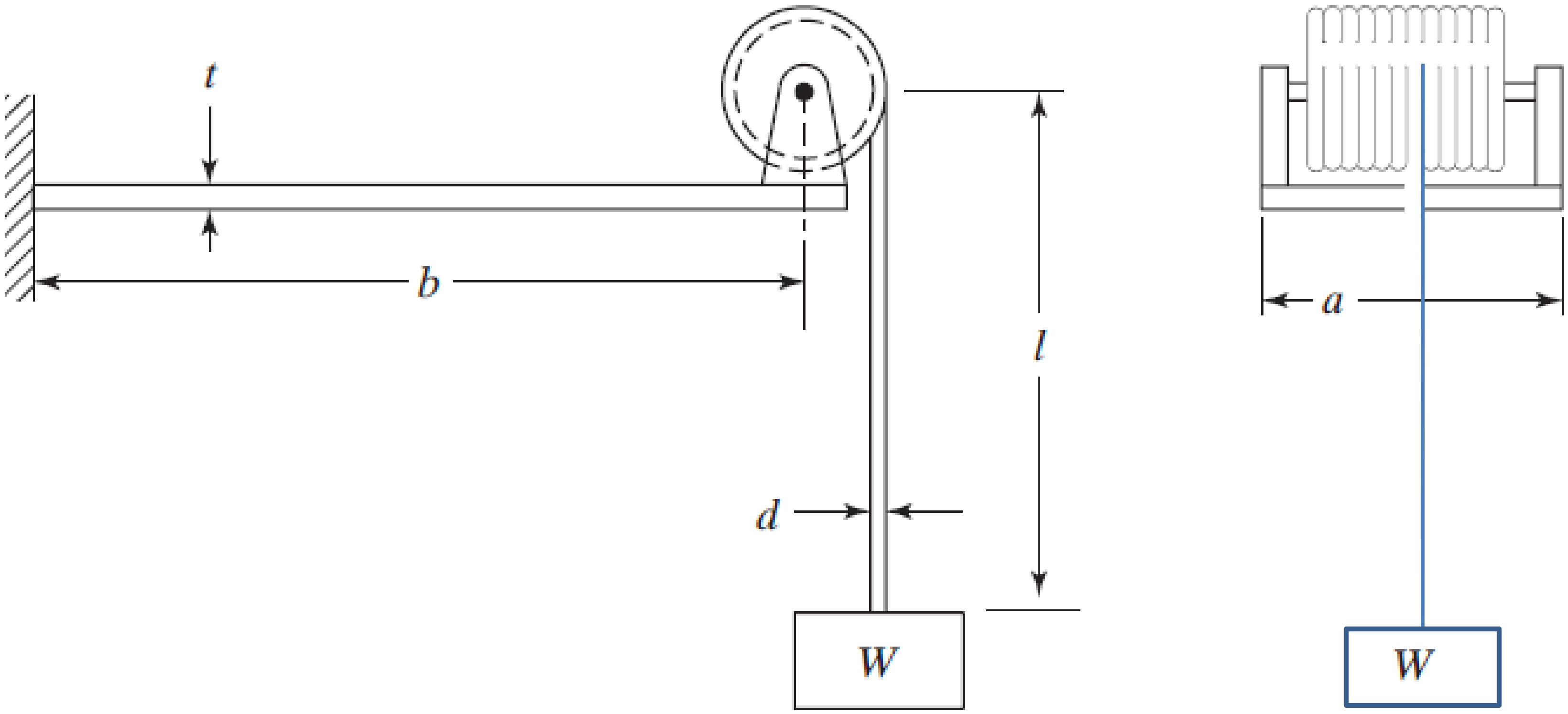
Parallel arrangement of springs in a freight truck



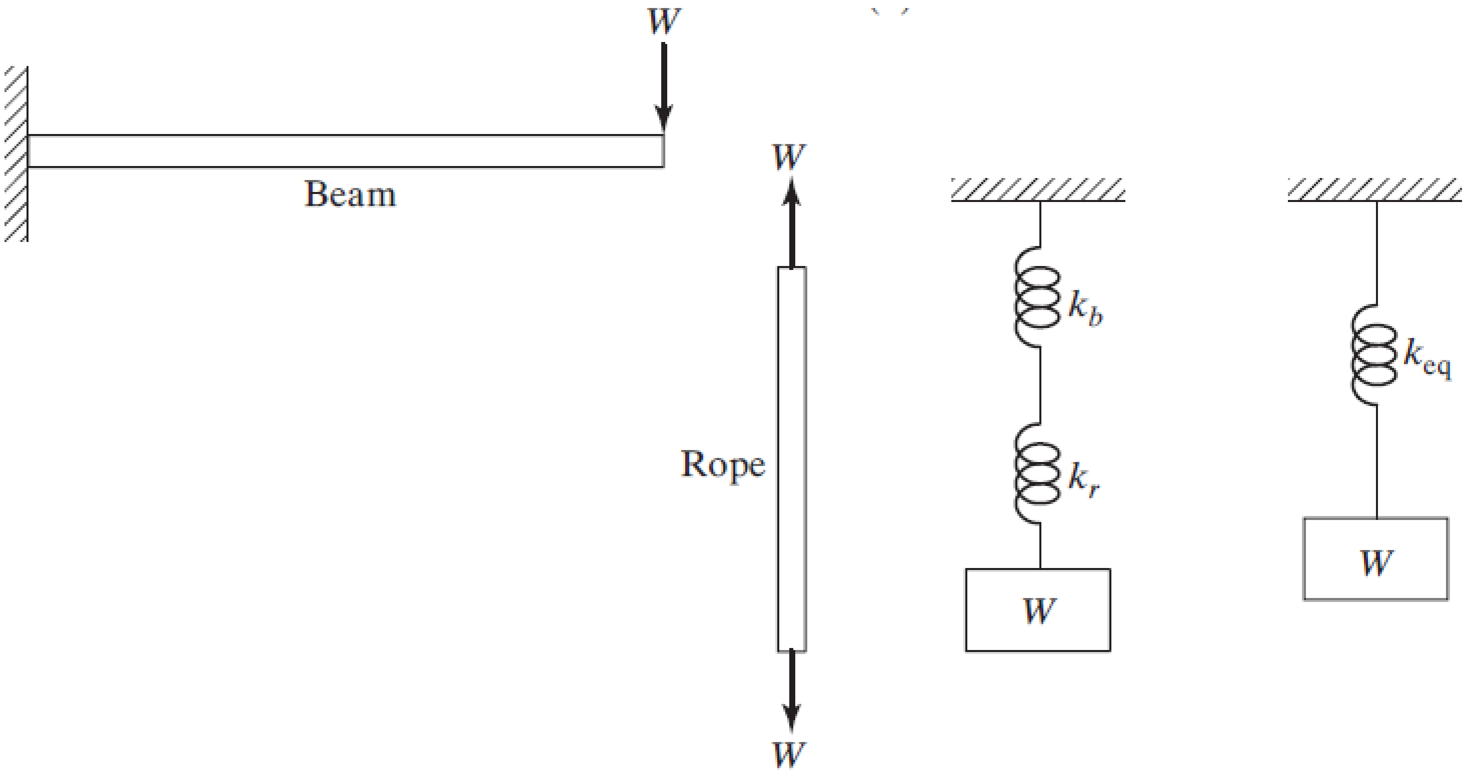
Torsional Spring Constant of a Propeller Shaft



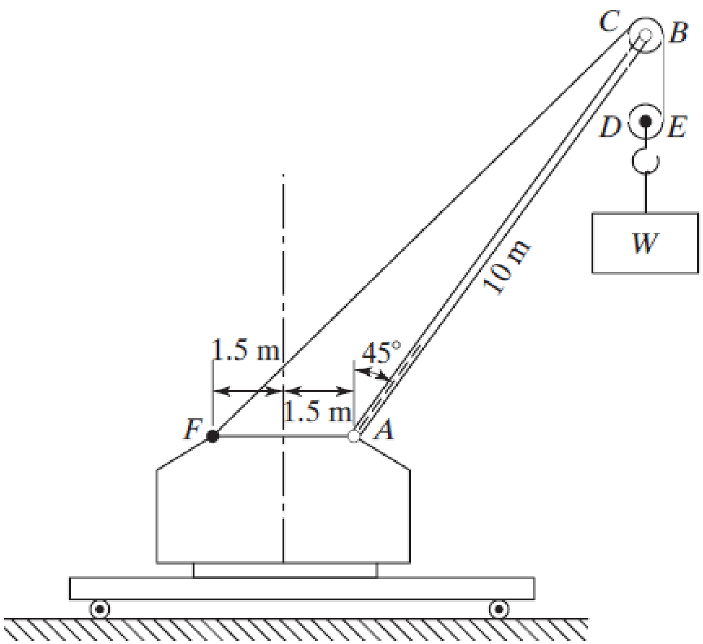
Equivalent k of Hoisting Drum

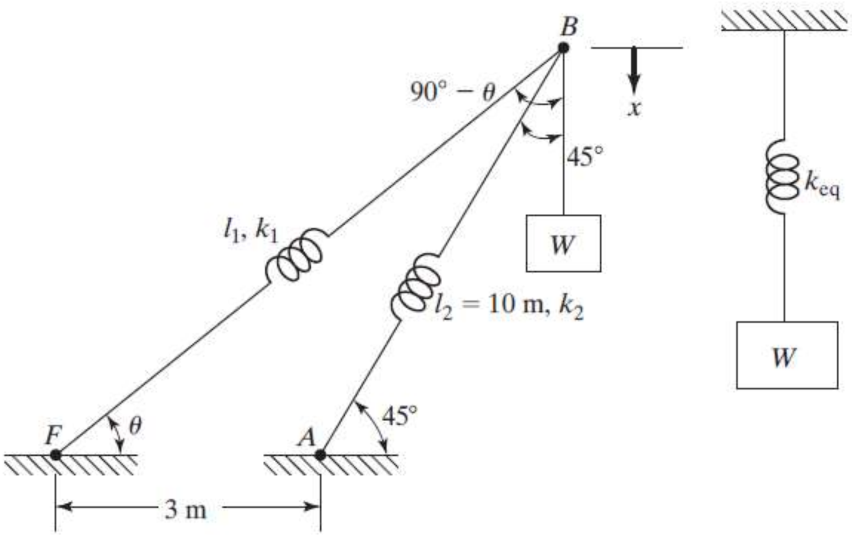


Equivalent k of Hoisting Drum



Equivalent k of a Crane





1.4 Dynamic Loads on Flight Vehicle Structures

Unsteady air loads – Atmospheric turbulence, gust, engine vibration

Pilots input to control surfaces for manoeuver

Landing impact

Runway unevenness'

Blast pressure

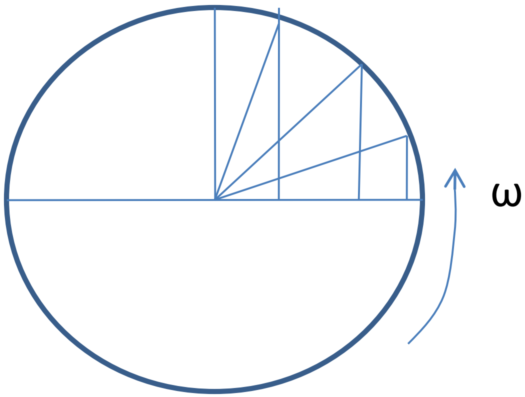
Acoustic loads

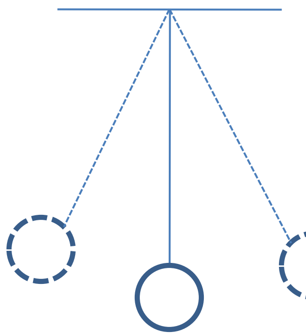
1.4 Spring, Damper and Mass elements

FIGURE 9.17 (a) Undamped spring mount; (b) damped spring mount; (c) pneumatic rubber mount. (Courtesy of *Sound and Vibration*.)

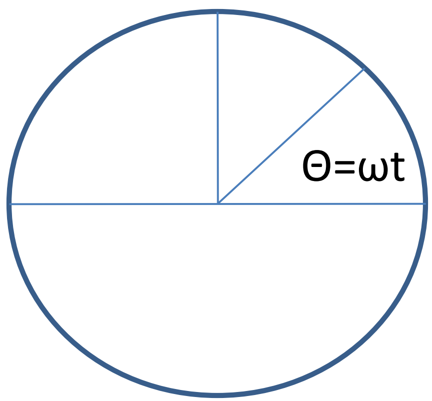
1.4.1 Simple Harmonic Motion (SHM)

A particle moves to and fro in such a way that the acceleration is always proportional to the displacement and directed towards origin, the motion is called SHM





A particle is moving along a circular path with constant velocity ω rad/sec



1.4.1 Simple Harmonic Motion (SHM)

$$x(t) = A\sin \omega t$$

$$\omega = \frac{2\pi}{\tau} = 2\pi f$$

$$\dot{x} = \omega A\cos \omega t$$

$$\ddot{x} = -\omega^2 A\sin \omega t = -\omega^2 x$$

$$\ddot{x} = -\omega^2 x$$

$$\ddot{x} + \omega^2 x = 0 \qquad (1.1)$$

1.4.2 Energy Method

- Application of conservation of energy
- For free vibration of undamped system, the energy is partly potential and partly kinetic
- Their sum is always constant

$$T + U = \text{constant} \qquad (1.2)$$

$$\frac{d}{dt}\left(T + U\right) = 0 \qquad (1.3)$$

- From principle of conservation of energy we can write

$$T_1 + U_1 = T_2 + U_2 \qquad (1.4)$$

- Let 1 and 2 are two instances of time
- Let 1 corresponds to equilibrium position, $U_1 = 0$
- Let 2 corresponds to maximum displacement, $T_2 = 0$
- Therefore, $T_1 + 0 = 0 + U_2 \qquad (1.5)$

1.4.2 Energy Method

- Since system is undergoing harmonic motion, then T_1 and U_2 are maximum values, hence

$$T_{max} = U_{max} \qquad (1.6)$$

- For a spring-mass system, kinetic energy is given by

$$T = \tfrac{1}{2} m\dot{x}^2$$

- Potential energy is given by

$$U = \tfrac{1}{2} kx^2$$

- Let $x = A\sin \omega t$, then one can write $\dot{x} = A\omega; \quad \dot{x}^2 = A^2\omega^2$

- Substituting for x and dx/xt in the expression for U and T one can write

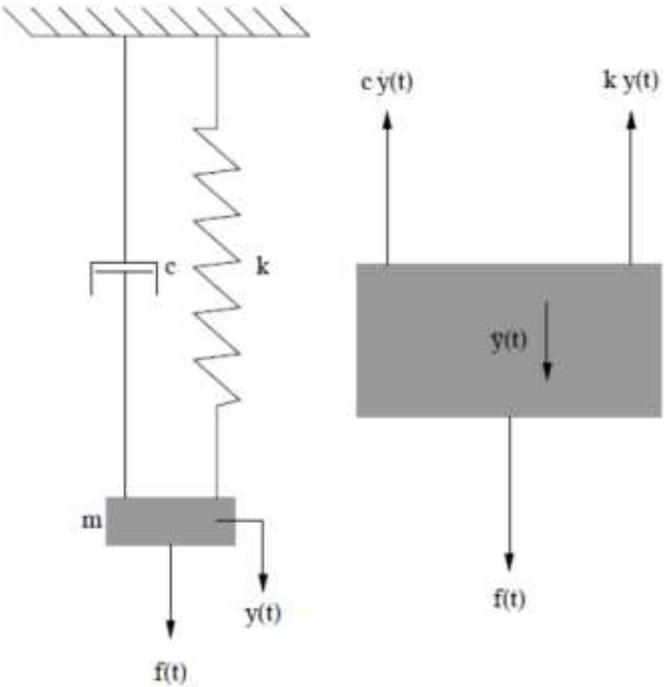
$$T_{max} = \tfrac{1}{2} mA^2\omega^2$$

$$U_{max} = \tfrac{1}{2} kA^2$$

$$\tfrac{1}{2} mA^2\omega^2 = \tfrac{1}{2} kA^2$$

$$\omega = \sqrt{\tfrac{k}{m}} \qquad (1.7)$$

1.5 Equations of motion



$$y(t) = A_1\, e^{p_1 t} + A_2\, e^{p_2 t} = e^{-(c/2m)t}\left(A_1\, e^{\left(\sqrt{(c/2m)^2-\omega^2}\right)t} + A_2\, e^{-\left(\sqrt{(c/2m)^2-\omega^2}\right)t}\right)$$

(a) Critical damping: $(c/2m)^2 = \omega^2 \implies c_c = 2m\omega$

(b) Overdamped system: $(c/2m)^2 > \omega^2$

(c) Underdamped or lightly damped system: $(c/2m)^2 < \omega^2$

1.5 Equations of motion

Introducing the damping ratio,

$$\xi = \frac{c}{c_c} = \frac{c}{2m\omega}$$

Therefore,

$$p_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \omega^2} = -\xi\,\omega \pm \sqrt{(\xi\,\omega)^2 - \omega^2} = \omega\left(-\xi \pm \sqrt{\xi^2 - 1}\right)$$

$$y(t) = e^{-\xi\omega t}\left(A_1\,e^{\left(\omega\sqrt{\xi^2-1}\right)t} + A_2\,e^{-\left(\omega\sqrt{\xi^2-1}\right)t}\right)$$

Finally, we have

a) Critical damping: $\xi = 1$

b) Overdamped system: $\xi > 1$

c) Underdamped or lightly damped system: $0 < \xi < 1$

1.5 Equations of motion

The above can be classified as critically damped motion; nonoscillatory motion; and oscillatory motion.

- Underdamped or lightly-damped motion: $0 < \xi < 1$

$$y(t) = e^{-\xi\omega t}\left(u_0 \cos\omega_d t + \frac{\xi\omega u_0 + v_0}{\omega_d}\sin\omega_d t\right)$$

$$y(t) = e^{-\xi\omega t}(Y\sin\theta\cos\omega_d t + Y\cos\theta\sin\omega_d t) \equiv e^{-\xi\omega t}\,Y\sin(\omega_d t + \theta)$$

where

$$Y = \sqrt{u_0^2 + \left(\frac{\xi\omega u_0 + v_0}{\omega_d}\right)^2}$$

$$\theta = \tan^{-1}\left(u_0 \Big/ \left(\frac{\xi\omega u_0 + v_0}{\omega_d}\right)\right)$$

1.5 Equations of motion

Overdamped (Nonoscillatory) motion: $\xi > 1$

$$y(t) = e^{-\xi\omega t}\left(\frac{\xi\omega u_0+\sqrt{\xi^2-1}\;\omega u_0+v_0}{2\sqrt{\xi^2-1}\;\omega}e^{\left(\omega\sqrt{\xi^2-1}\right)t} - \frac{\xi\omega u_0-\sqrt{\xi^2-1}\;\omega u_0+v_0}{2\sqrt{\xi^2-1}\;\omega}e^{-\left(\omega\sqrt{\xi^2-1}\right)t}\right)$$

Critically damped motion: $\xi = 1$

Logarithmic Decrement

Logarithmic decrement: If there are the displacements at two consecutive peaks at t_1 and t_1+T_d

$$y(t_1) \equiv y_1 = e^{-\xi \omega t_1} Y \sin(\omega_d t_1 + \theta)$$

$$y(t_2) \equiv y_2 = e^{-\xi \omega (t_1+T_d)} Y \sin(\omega_d (t_1 + T_d) + \theta)$$

The *logarithmic decrement* is defined as

$$\delta = \ln\left(\frac{y_1}{y_2}\right) = \ln\left(\frac{e^{-\xi \omega t_1} Y \sin(\omega_d t_1 + \phi)}{e^{-\xi \omega (t_1+T_d)} Y \sin(\omega_d (t_1+T_d)+\phi)}\right)$$

$$\delta = \ln\left(\frac{e^{-\xi \omega t_1}}{e^{-\xi \omega (t_1+T_d)}}\right) = \ln\left(\frac{1}{e^{-\xi \omega T_d}}\right) = \ln\left(e^{\xi \omega T_d}\right) \equiv \xi \omega T_d$$

$$\delta = \xi \omega \left(\frac{2\pi}{\omega \sqrt{1-\xi^2}}\right) = \frac{2\pi \xi}{\sqrt{1-\xi^2}}$$

Logarithmic Decrement

The relationship between the logarithmic decrement and the damping ratio

$$\xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}}$$

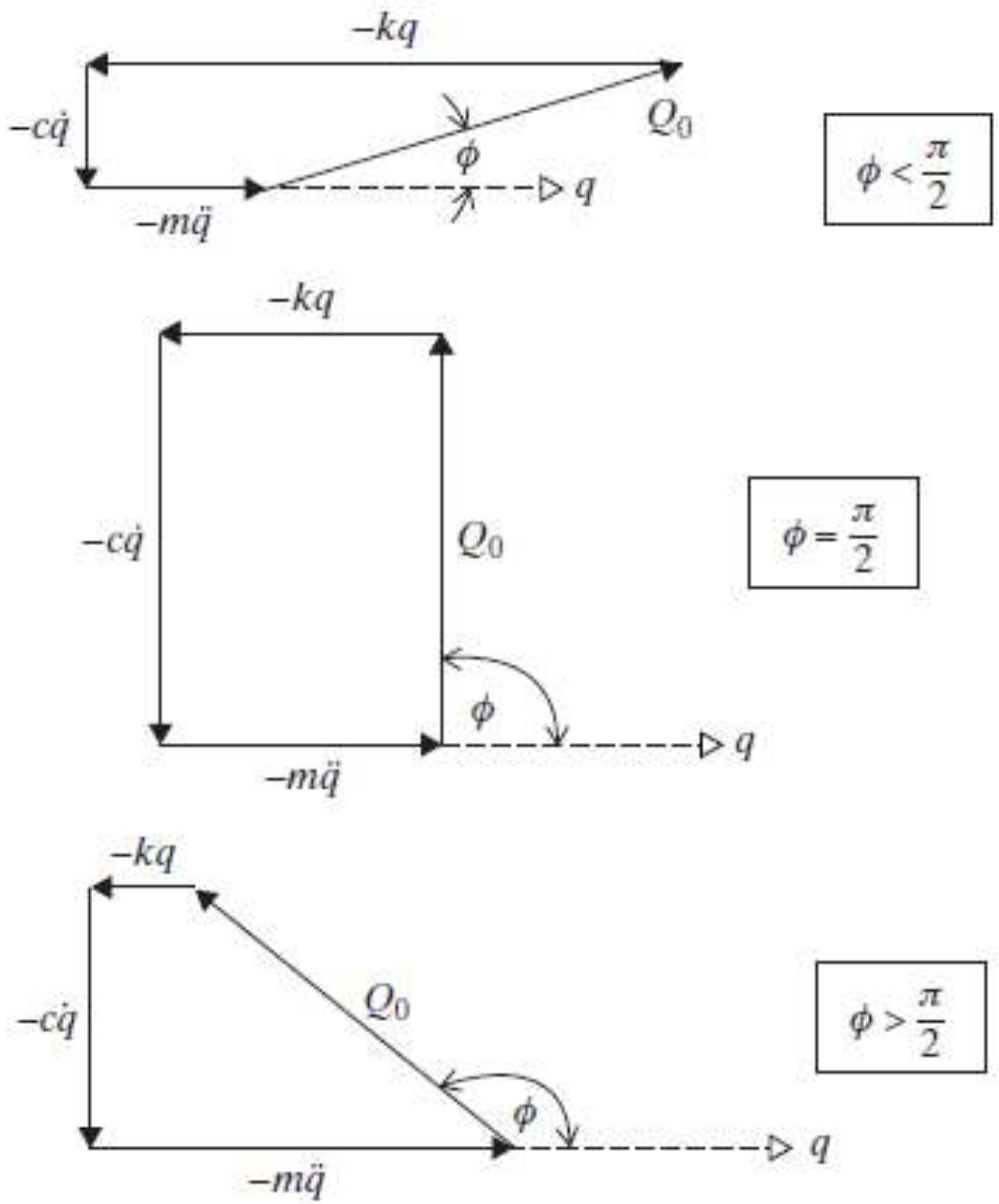
For lightly damped systems, the difference between two successive peaks may be too small to measure accurately. Since the logarithmic decrement between any two successive peaks is constant, we can determine the decrement from the first peak and the peak n cycles later.

$$\delta = \frac{1}{n}\ln\left(\frac{y_0}{y_n}\right)$$

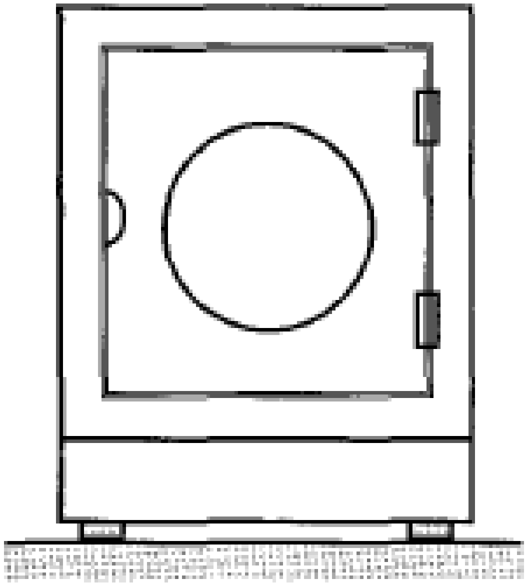
1.7 Damped forced vibration

1.7.1 Resonance

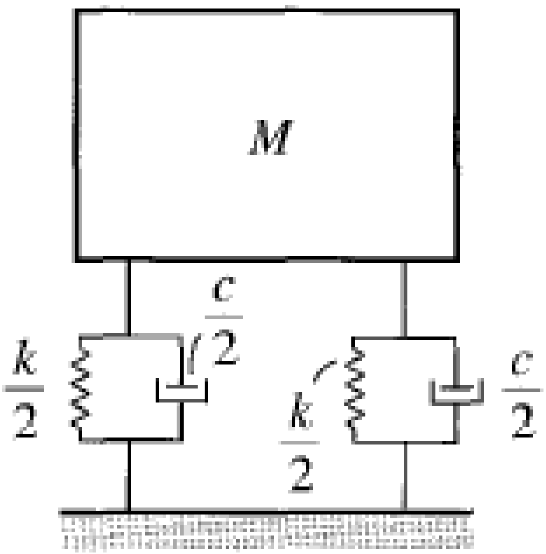
Phase relationships among the applied, spring, damping, and inertia forces for harmonic motion for frequency ratio values less than one-half, equal to one, and equal to one and a half.



Modeling Mechanical Systems



a.



b.

FIGURE 1.28
a. Washing machine, b. Model of washing machine

Modeling Structural Dynamic Systems

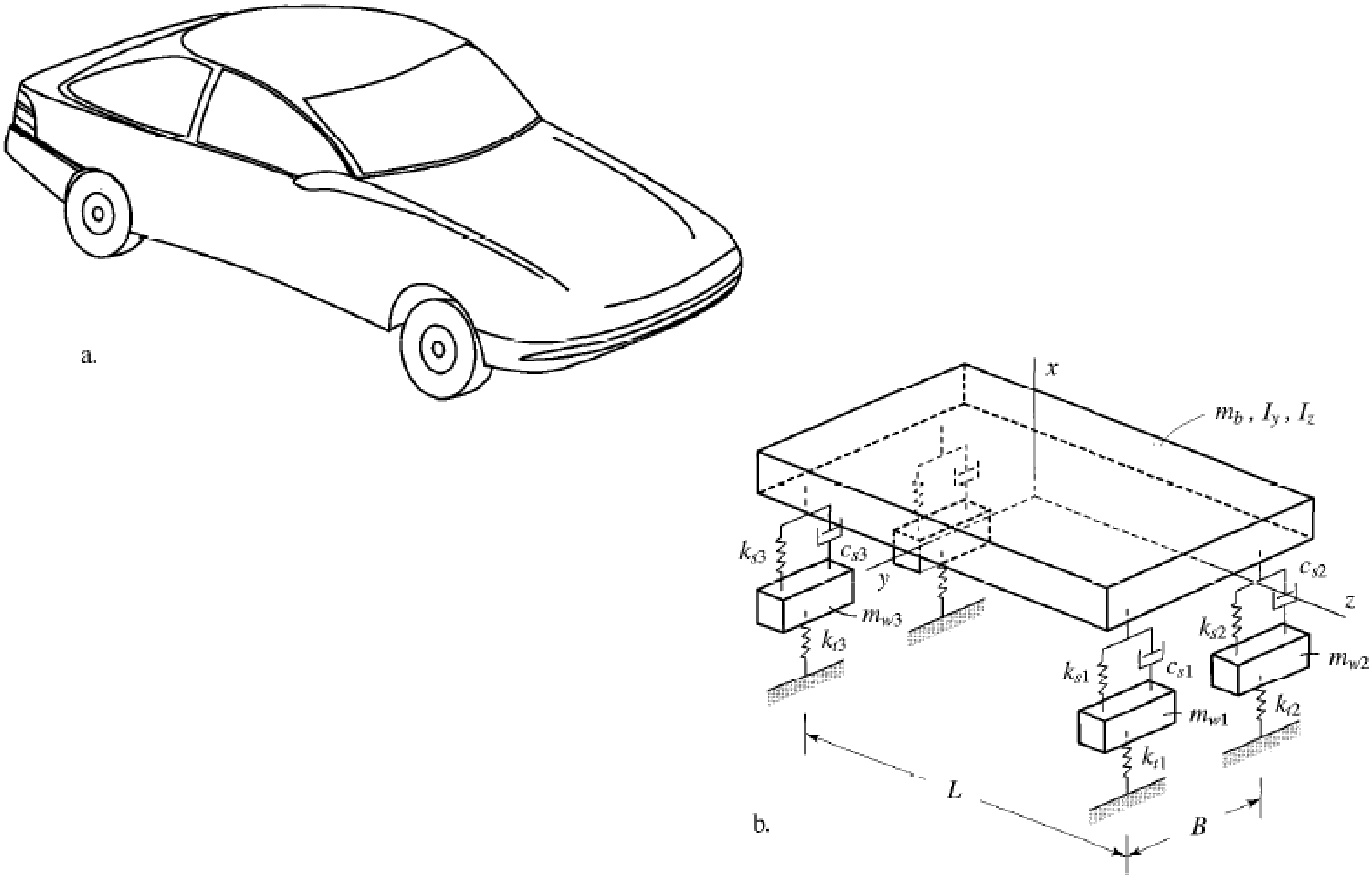


FIGURE 1.29
a. Automobile, b. Model of automobile

Modeling Structural Dynamic Systems

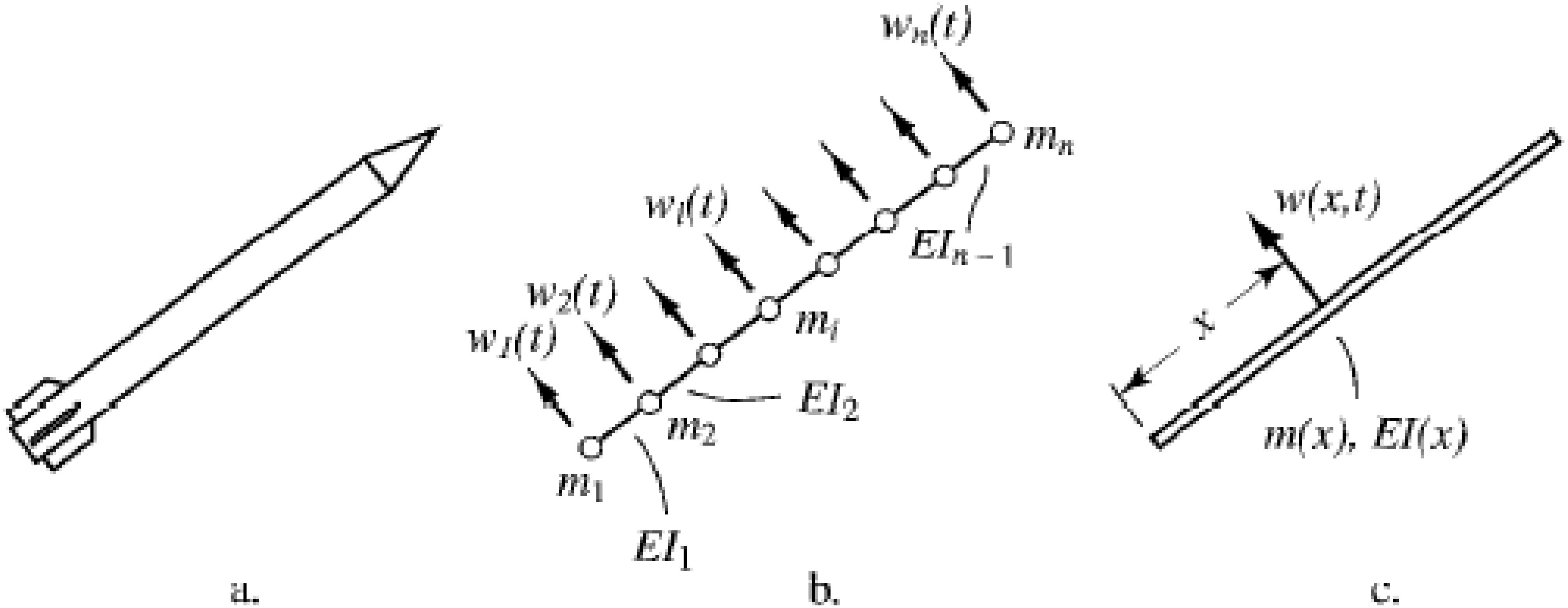


FIGURE 1.30

a. Missile in free flight, b. Discrete model, c. Distributed-parameter model

Modeling Structural Dynamic Systems

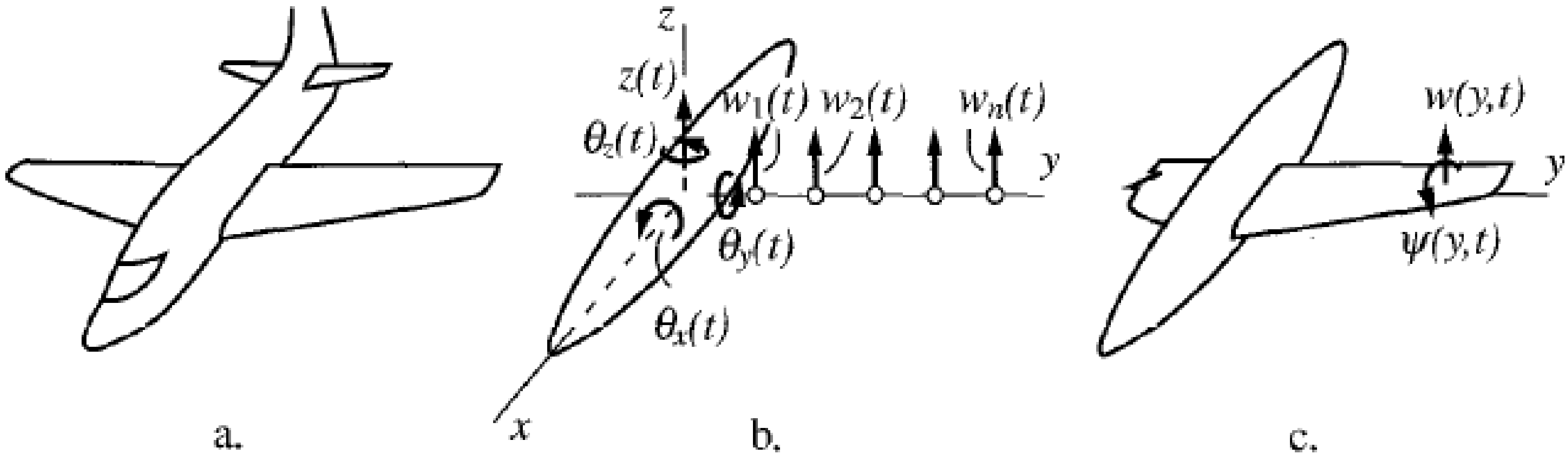


FIGURE 1.31
a. Aircraft in flight, b. Discrete model, c. Distributed-parameter model

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Mechanical Vibration and Structural Dynamics

Unit 2: Vibration of discrete System

Contents

2.0 Discrete and continuous system

- A large number of practical systems can be described using a finite number of degrees of freedom, such as the simple systems shown in slides 5 to 7.

- Some systems, especially those involving continuous elastic members, have an infinite number of degrees of freedom.

- As a simple example, consider the cantilever beam shown in slide 8.

- Since the beam has an infinite number of mass points, we need an infinite number of coordinates to specify its deflected configuration.

- The infinite number of coordinates defines its elastic deflection curve.

- Thus the cantilever beam has an infinite number of degrees of freedom.

- Most structural and machine systems have deformable (elastic) members and therefore have an infinite number of degrees of freedom

- Systems with a finite number of degrees of freedom are called *discrete or lumped parameter systems,* and those with an infinite number of degrees of freedom are called *continuous or distributed systems.*

2.0 Discrete and continuous system (cont…)

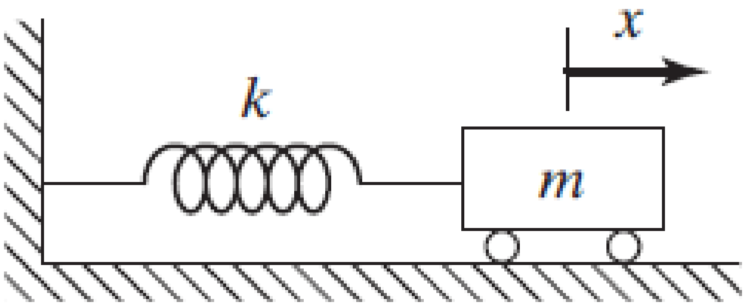
- Most of the time, continuous systems are approximated as discrete systems, and solutions are obtained in a simpler manner.

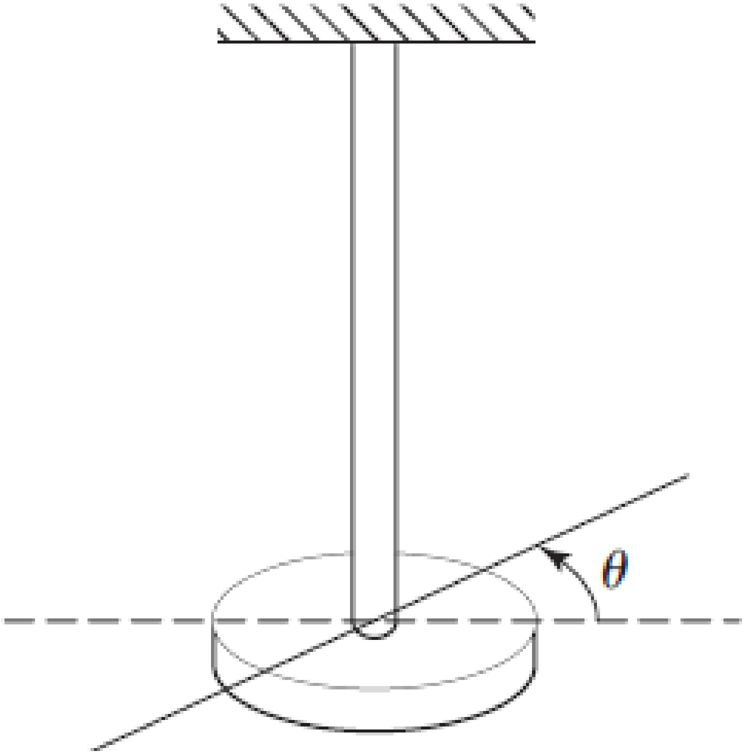
- Although treatment of a system as continuous gives exact results, the analytical methods available for dealing with continuous systems are limited to a narrow selection of problems, such as uniform beams, slender rods, and thin plates.

- Hence most of the practical systems are studied by treating them as finite lumped masses, springs, and dampers.

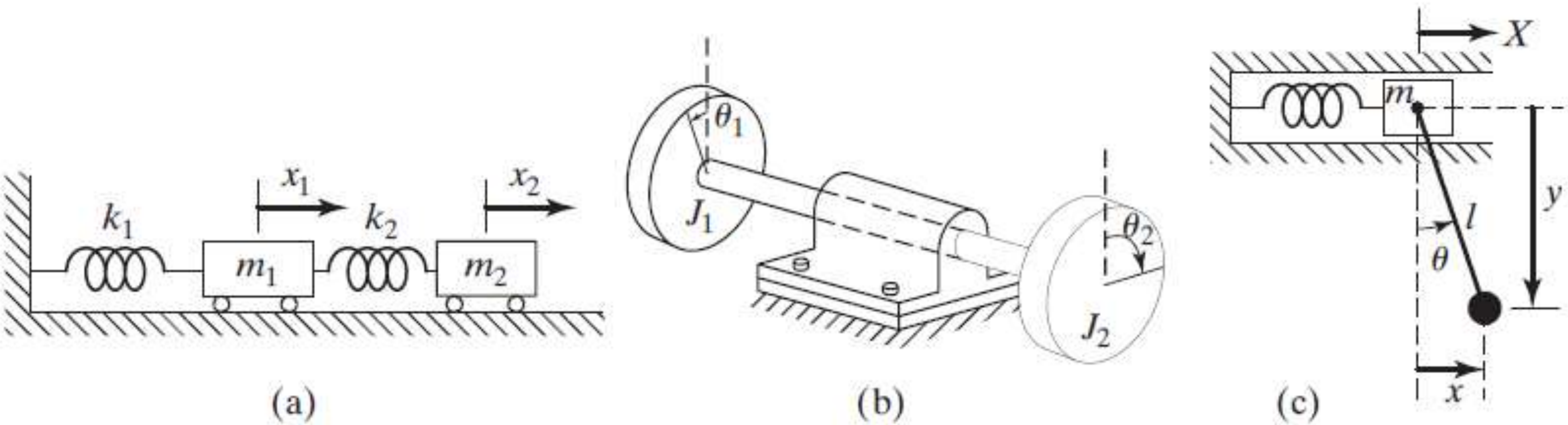
- In general, more accurate results are obtained by increasing the number of masses, springs, and dampers - that is, by increasing the number of degrees of freedom.

2.1 Two/Three-degree-of-freedom (MDOF) system



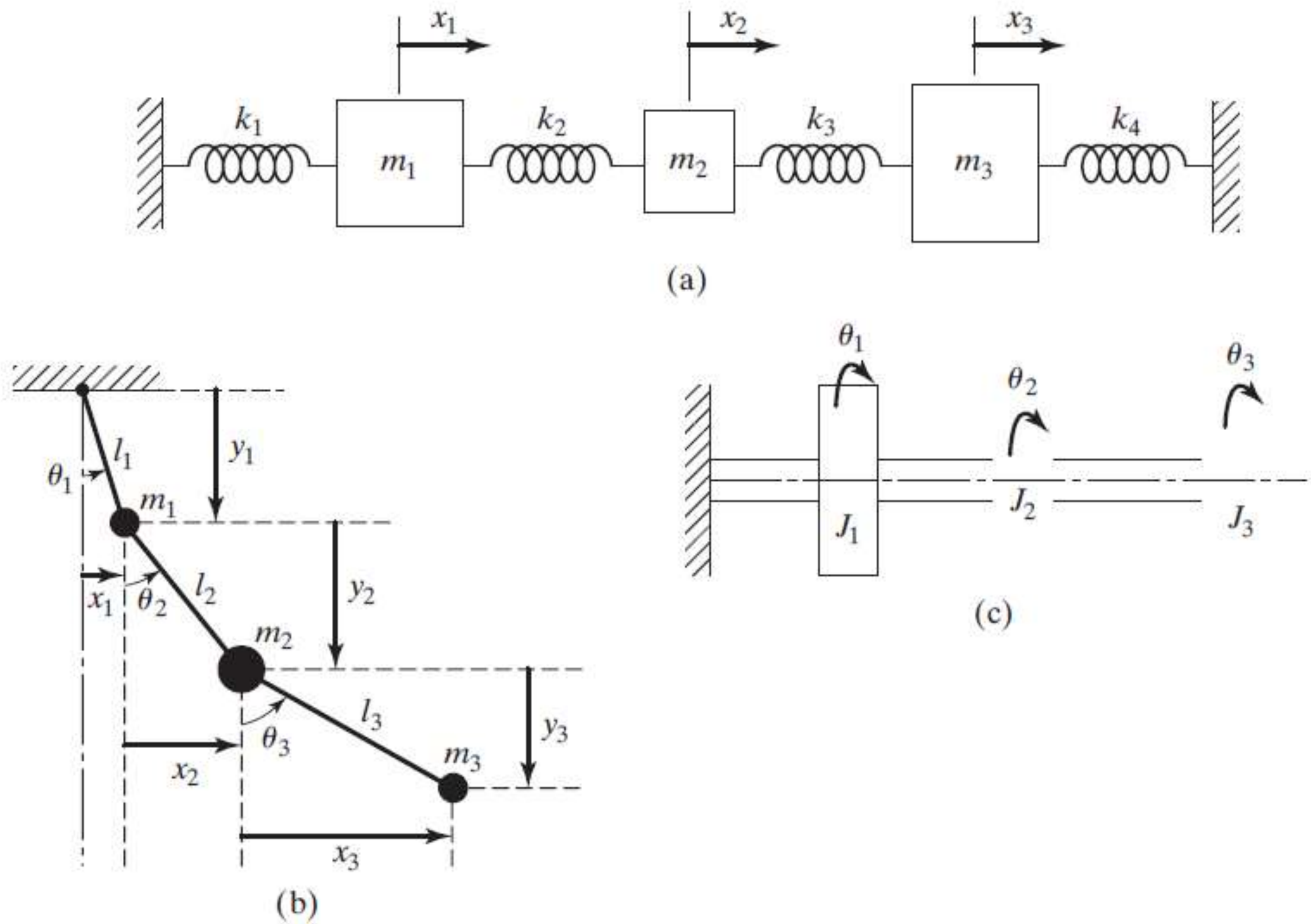


2.1 Two/Three-degree-of-freedom (MDOF) system



(a) (b) (c)

2.1 Two/Three-degree-of-freedom (MDOF) system



2.1 Two/Three-degree-of-freedom (MDOF) system

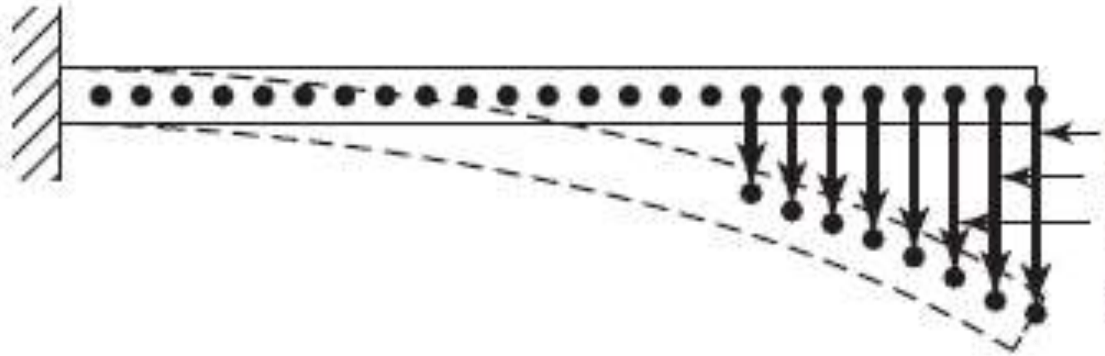
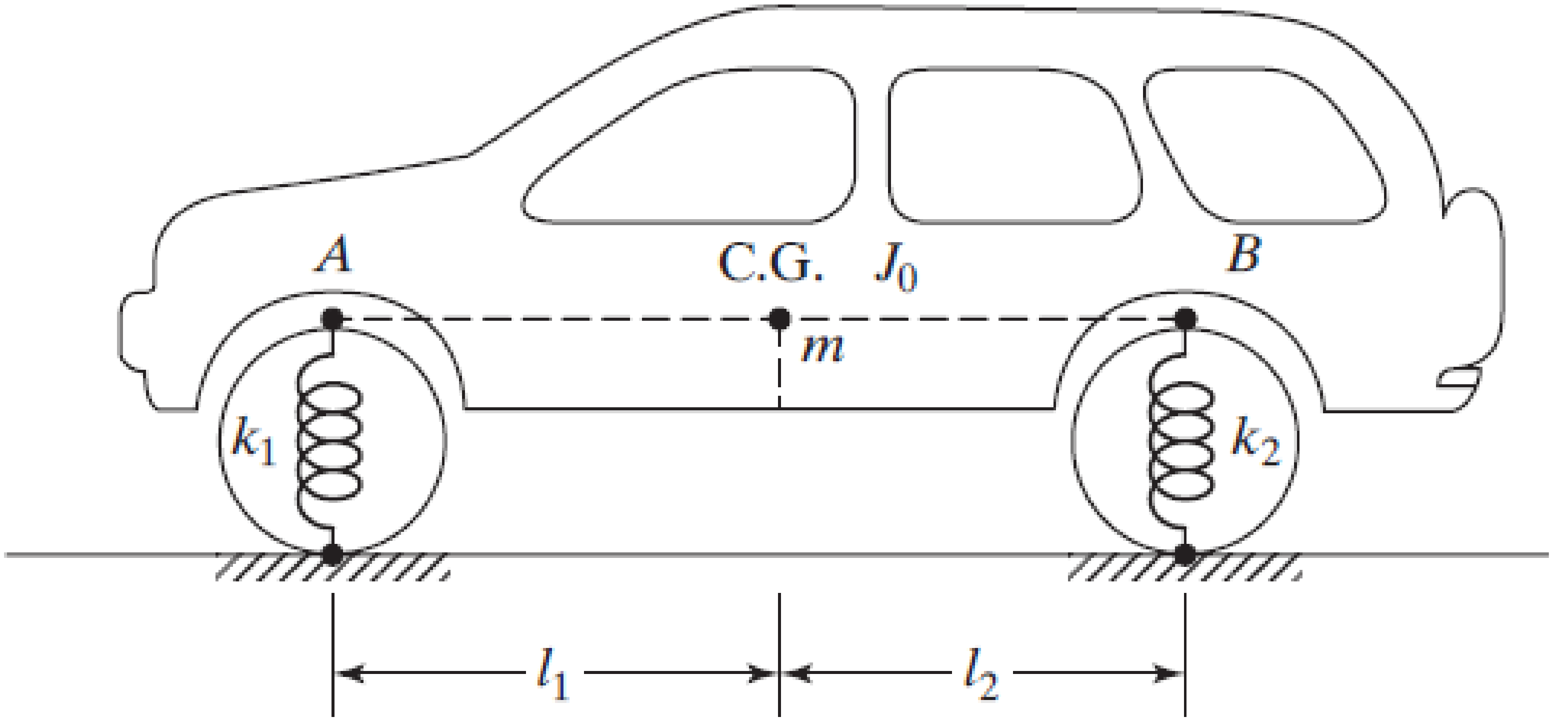
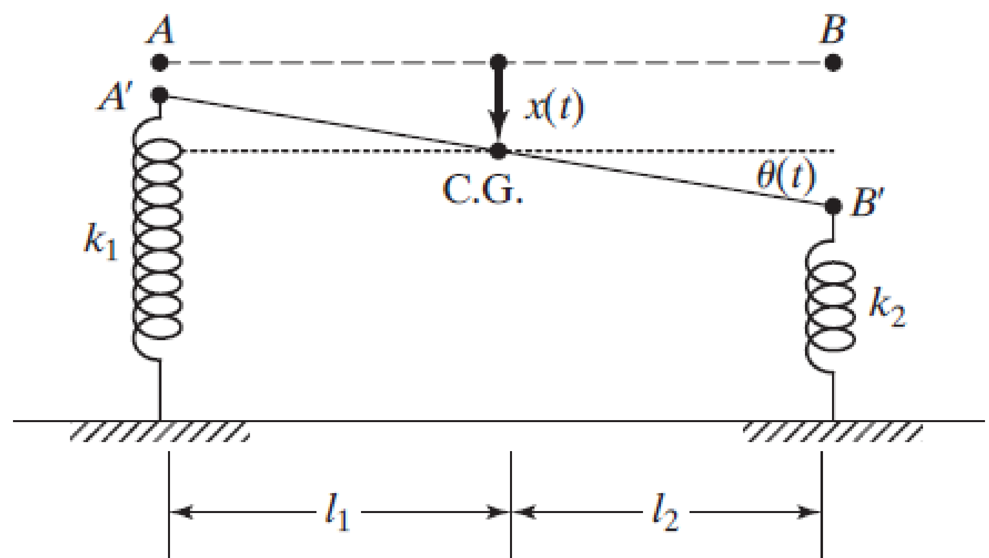


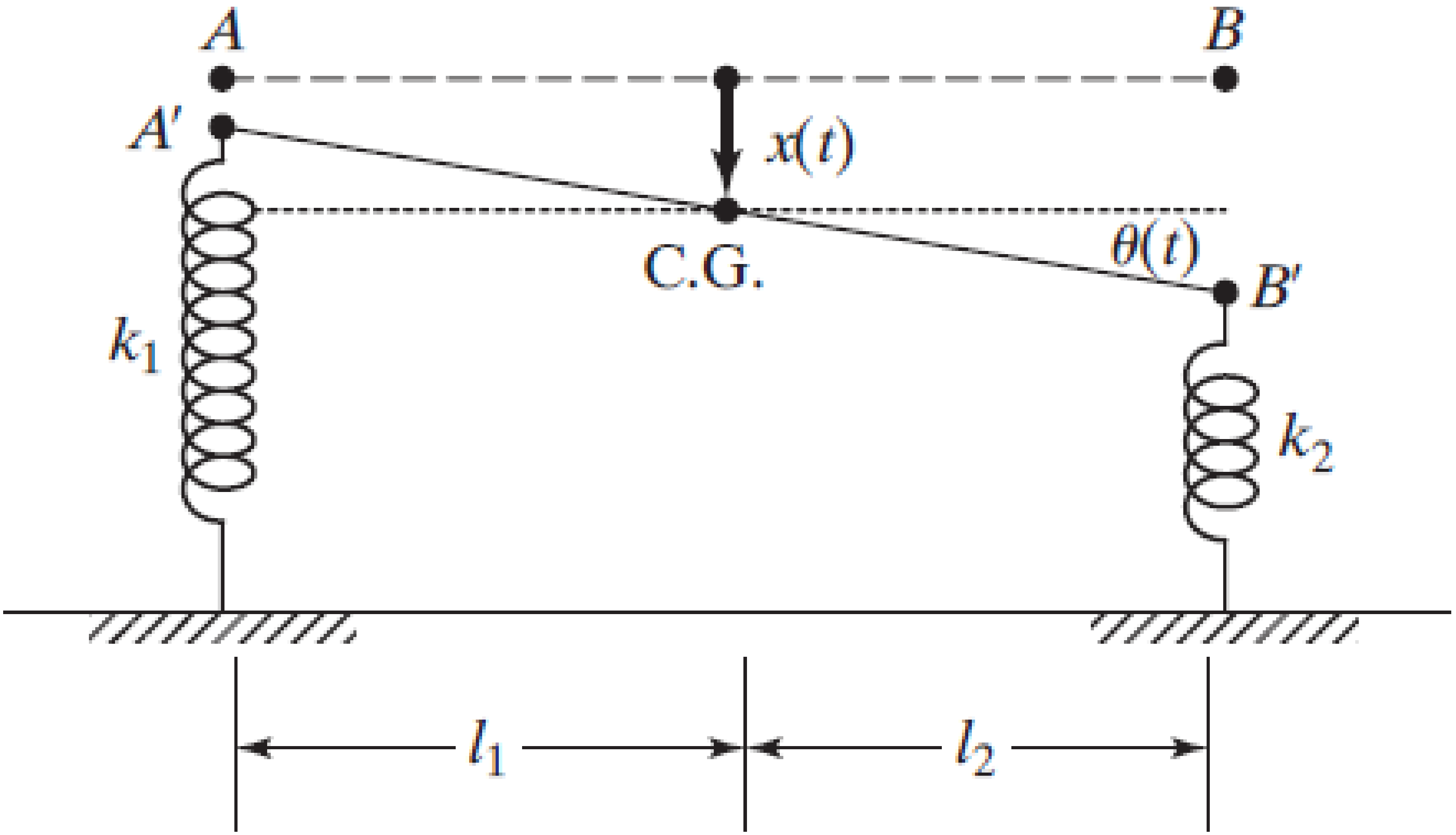
FIGURE 1.14 A cantilever beam (an infinite-number-of-degrees-of-freedom system).

2.2 Static and Dynamic couplings

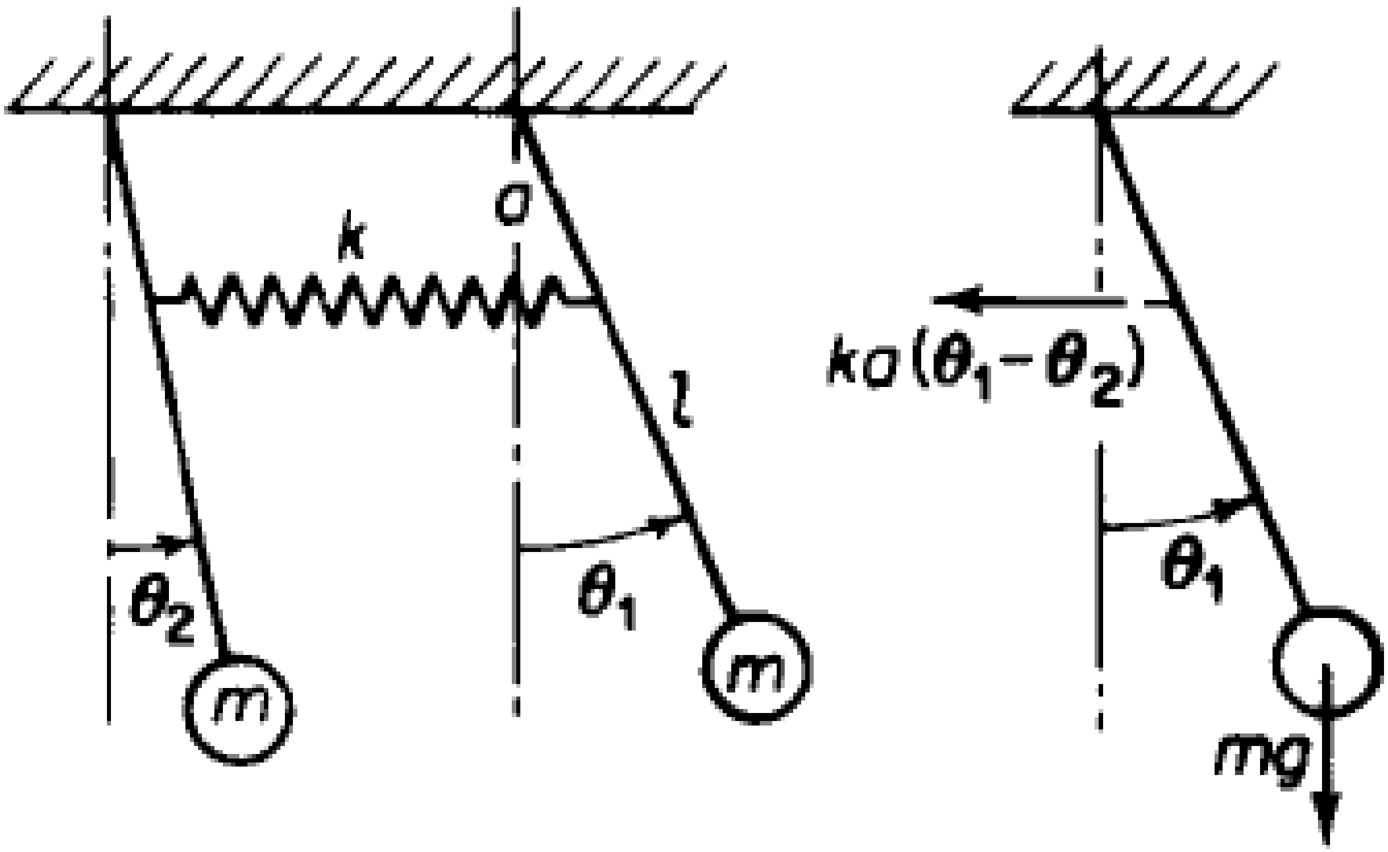




2.2 Static and Dynamic couplings



2.2 Static and Dynamic couplings



2.2 Static and Dynamic couplings

$$ml^2\ddot{\theta}_1 = -mgl\theta_1 - ka^2(\theta_1 - \theta_2)$$

$$ml^2\ddot{\theta}_2 = -mgl\theta_2 + ka^2(\theta_1 - \theta_2)$$

Assuming the normal mode solutions as

$$\theta_1 = A_1 \cos \omega t$$

$$\theta_2 = A_2 \cos \omega t$$

the natural frequencies and mode shapes are found to be

$$\omega_1 = \sqrt{\frac{g}{l}} \qquad \omega_2 = \sqrt{\frac{g}{l} + 2\frac{k}{m}\frac{a^2}{l^2}}$$

$$\left(\frac{A_1}{A_2}\right)^{(1)} = 1.0 \qquad \left(\frac{A_1}{A_2}\right)^{(2)} = -1.0$$

2.2 Static and Dynamic couplings

Figure below shows a rigid bar with its centre of mass not coinciding with its geometric centre, ie, $l_1 \neq l_2$, and supported by two springs, k_1 and k_2.

It represents a two degree of freedom since two coordinates are necessary to describe its motion

The choice of the coordinates will define the type of coupling which can be immediately determine from the mass and stiffness matrices.

Mass or **dynamic coupling** exists if the mass matrix is non-diagonal, whereas stiffness or **static coupling** exists if the stiffness matrix is non-diagonal.

It is possible to have both forms of coupling.

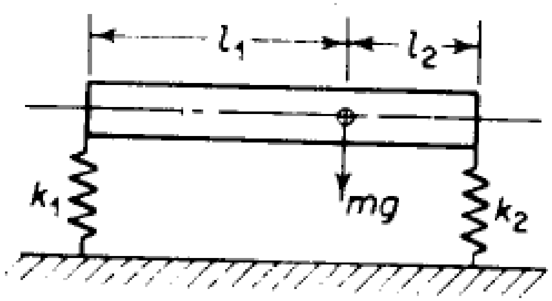


Figure 5.2-1.

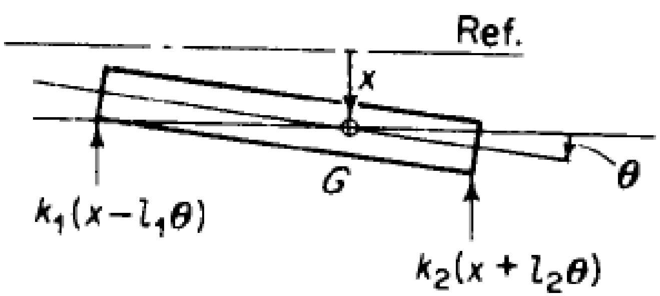


Figure 5.2-2. Coordinates leading to static coupling.

2.2 Static and Dynamic couplings

Static Coupling

Choosing coordinates x and θ shown in the figure below, where x is the linear displacement of the center of mass, the system will have static coupling as shown by the matrix equation

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix}+\begin{bmatrix} (k_1+k_2) & (k_2l_2-k_1l_1) \\ (k_2l_2-k_1l_1) & (k_1l_1^2+k_2l_2^2) \end{bmatrix}\begin{Bmatrix} x \\ \theta \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

If $k_1l_1=k_2l_2$, the coupling disappears, and we obtain uncoupled x and θ vibrations

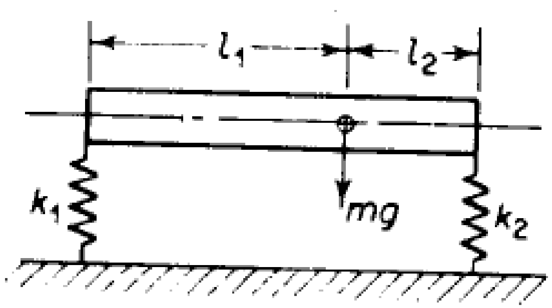


Figure 5.2-1.

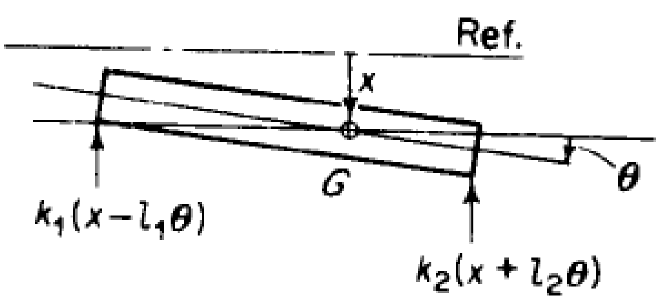


Figure 5.2-2. Coordinates leading to static coupling.

2.2 Static and Dynamic couplings

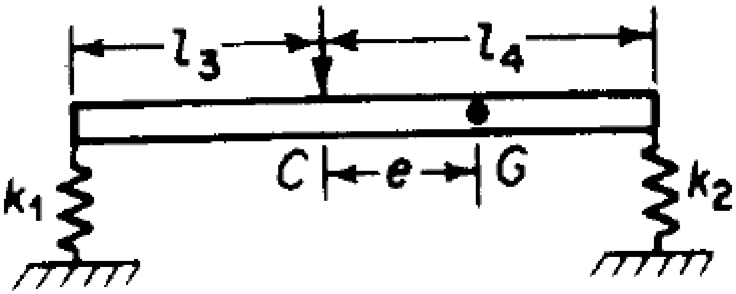
Dynamic Coupling

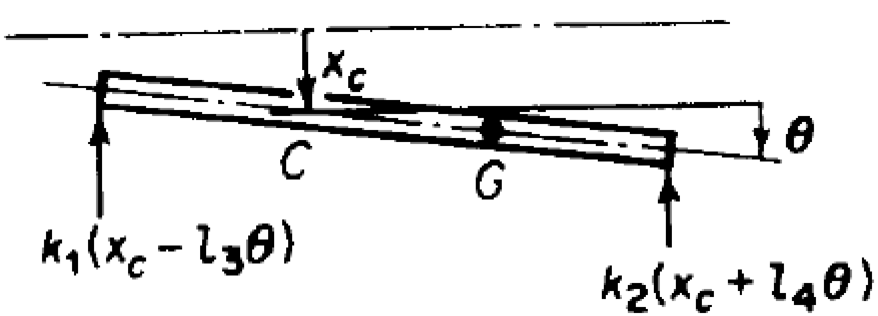
There is some point C along the bar where a force applied normal to the bar produces pure translation; i.e.,

The equations of motion in terms of x_c and θ can be shown to be

$$\begin{bmatrix} m & me \\ me & J \end{bmatrix} \begin{Bmatrix} \ddot{x}_c \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} (k_1 + k_2) & 0 \\ 0 & (k_1 l_3^2 + k_2 l_4^2) \end{bmatrix} \begin{Bmatrix} x_c \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Which shows that the coordinates chosen eliminated the static coupling and introduced dynamic coupling





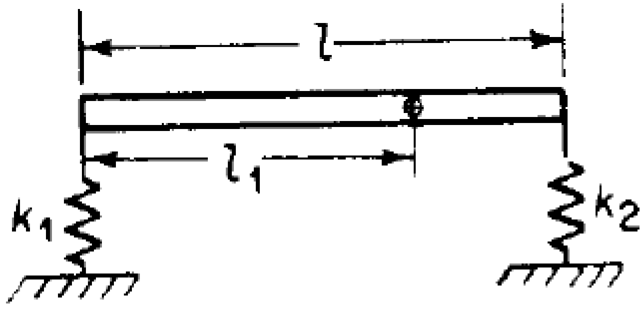
2.2 Static and Dynamic couplings

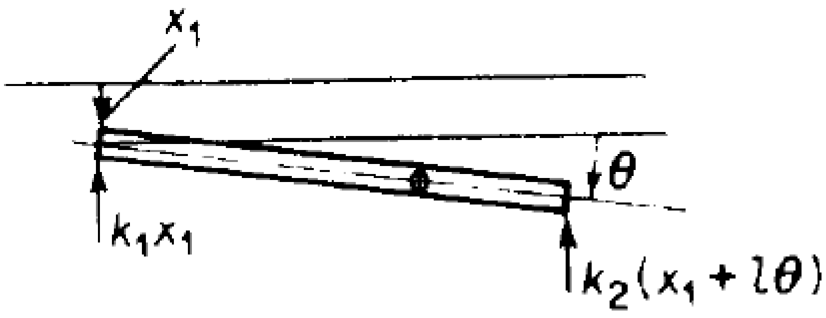
Static and Dynamic Coupling

If we choose x=x1 at the end of the bar, as shown in figure below, the equations of motion become

$$\begin{bmatrix} m & ml_1 \\ ml_1 & J_1 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} (k_1 + k_2) & k_2 l \\ k_2 l & k_2 l^2 \end{bmatrix} \begin{Bmatrix} x_1 \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

and both static and dynamic coupling are now present





2.2 Static and Dynamic couplings

$$T = \frac{1}{2} m\dot{x}^2$$

2.2 Forced vibration of 2-DOF System

The equations of motion of a general two-degree-of-freedom system under external forces can be written as

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \qquad (2.3)$$

We shall consider the external forces to be harmonic:

$$F_j(t) = F_{j0} e^{i\omega t} \qquad j = 1,2 \qquad (2.4)$$

where ω is the forcing frequency.

We can write the steady-state solution as

$$x_j(t) = X_j e^{i\omega t} \qquad j = 1,2 \qquad (2.5)$$

where X_1 and X_2 are, in general, complex quantities that depend on and the system parameters.

2.2 Forced vibration of 2-DOF System

Substitution of Eqs. (2.4) and (2.5) into Eq. (2.3) leads to

$$\begin{bmatrix} \left(-\omega^2 m_{11} + i\omega c_{11} + k_{11}\right) & \left(-\omega^2 m_{12} + i\omega c_{12} + k_{12}\right) \\ \left(-\omega^2 m_{12} + i\omega c_{12} + k_{12}\right) & \left(-\omega^2 m_{22} + i\omega c_{22} + k_{22}\right) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F_{10} \\ F_{20} \end{Bmatrix} \qquad (2.6)$$

we define the mechanical impedance, $Z_{rs}(i\omega)$ as

$$Z_{rs}(i\omega) = -\omega^2 m_{rs} + i\omega c_{rs} + k_{rs} \qquad r, s = 1,2 \qquad (2.7)$$

and write Eq. (2.6) as

$$[Z(i\omega)]\vec{X} = \vec{F}_0 \qquad (2.8)$$

2.2 Forced vibration of 2-DOF System

where

$$[Z(i\omega)] = \begin{bmatrix} Z_{11}(i\omega) & Z_{12}(i\omega) \\ Z_{12}(i\omega) & Z_{22}(i\omega) \end{bmatrix} = \text{Impedance} \quad \text{matrix}$$

and

$$\vec{X} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$$

$$\vec{F}_0 = \begin{Bmatrix} F_{10} \\ F_{20} \end{Bmatrix}$$

Equation (5.32) can be solved to obtain

$$\vec{X} = [Z(i\omega)]^{-1} \vec{F}_0 \qquad (2.9)$$

2.2 Forced vibration of 2-DOF System

where the inverse of the impedance matrix is given by

$$[Z(i\omega)]^{-1} = \frac{1}{Z_{11}(i\omega)Z_{22}(i\omega) - Z_{12}^2(i\omega)} \begin{bmatrix} Z_{22}(i\omega) & -Z_{12}(i\omega) \\ -Z_{12}(i\omega) & Z_{11}(i\omega) \end{bmatrix} \qquad (2.10)$$

Equations (2.9) and (2.10) lead to the solution

$$X_1(i\omega) = \frac{Z_{22}(i\omega)F_{10} - Z_{12}(i\omega)F_{20}}{Z_{11}(i\omega)Z_{22}(i\omega) - Z_{12}^2(i\omega)}$$

$$X_2(i\omega) = \frac{-Z_{12}(i\omega)F_{10} - Z_{11}(i\omega)F_{20}}{Z_{11}(i\omega)Z_{22}(i\omega) - Z_{12}^2(i\omega)} \qquad (2.11)$$

By substituting Eq. (2.11) into Eq. (2.5) we can find the complete solution.

2.4 Multiple-degree-of-freedom Linear System

Equations of Motion

2.4.1 Position Vector

Let P_0 be the space coordinates of a point of an elastic mechanical system at a time t_o.

Because of the application of an external force at $t = t_o$, the point in consideration will occupy a new position P at a time t.

The vector PP_o will thus represent the displacement of the point with initial position P_0.

If we now consider a discrete system, or a continuum that has been approximated as a discrete system using a set of generalized coordinates q, we can write

$$P = F(q) \qquad (2.5)$$

where q is the set of the generalized coordinates that define completely the mechanical system and F is the transformation operator.

For a linear system, the transformation operator F does not depend on the generalized coordinates q, and thus we can write for any point j of the mechanical system

2.4 Multiple-degree-of-freedom Linear System

$$P_j = \begin{bmatrix} \dfrac{\partial P_j}{\partial q_1} & \dfrac{\partial P_j}{\partial q_2} & \cdots & \dfrac{\partial P_j}{\partial q_n} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{Bmatrix} \tag{2.6}$$

where $\partial P_j / \partial q_i$ are constants that do not depend on the generalized coordinates for a linear system and that represent the variation in the displacement at the point in consideration due to a unit variation in the generalized coordinate q_i.

In this section, to simplify the notation, we will use Einstein's summation notation for repeated indices, and we write Eq. (2.6) as

$$P_j = \sum_{i=1}^{n} \left[\frac{\partial P_j}{\partial q_i} \right] q_i = \frac{\partial P_j}{\partial q_i} q_i \tag{2.7}$$

2.4 Multiple-degree-of-freedom Linear System

2.4.2 Velocity Vector

The velocity at any point j of the mechanical elastic system at a time t can be written as

$$V_j = \frac{dP_j}{dt} \qquad (2.8)$$

Using Eq. (2.6), we can write the velocity vector as

$$V_j = \frac{dP_j}{dt} = \frac{dP_j}{\partial q_i}\frac{dq_i}{dt} = \frac{dP_j}{\partial q_i}q_i^{'} \qquad (2.9)$$

where $q_i^{'} = dq_i / dt$

2.4.3 Kinetic Energy Functional

The kinetic energy functional of the elastic mechanical system reads

$$T = \frac{1}{2}\int_v \rho(P)V(P).V(P)dv \qquad (2.10)$$

2.4 Multiple-degree-of-freedom Linear System

Where ρ(P) is the material density at a point P, V(P) is the velocity vector at point P, and v is the volume of the elastic mechanical system.

For a discrete system we can use Eqs. (2.9) and (2.10) and write kinetic energy functional as

$$T = \frac{1}{2} q_j^{'} \left[\int_v \rho \frac{\partial P}{\partial q_j} \cdot \frac{\partial P}{\partial q_i} dv \right] q_i^{'} \qquad (2.11)$$

Or, in matrix notation, we can write

$$T = \tfrac{1}{2} \{q'\}^T [M] \{q'\} \qquad (2.12)$$

We call [M] the mass matrix of the mechanical system.

The elements of the mass matrix are given by

$$M_{ij} = \int_v \rho \frac{\partial P}{\partial q_i} \frac{\partial P}{\partial q_j} dv \qquad (2.13)$$

2.4 Multiple-degree-of-freedom Linear System

We conclude from Eq. (2.13) that the mass matrix is a symmetrical real matrix and because the expression $\{q'\}^T[M]\{q'\}$ represents an energy expression for any vector {q'} different from the null vector, we further conclude that

$$\{x\}^T[M]\{x\} > 0 \qquad \forall\{x\} \neq \{0\} \qquad (2.14)$$

Therefore, [M] is a positive definite matrix

2.4.4 Strain Energy Functional

The stress-strain relationship for an elastic linear continuum can be written as

$$\{\sigma\} = [C]\{\varepsilon\} \qquad (2.15)$$

where [C] is the material constitutive matrix and is a symmetric matrix because the stress and strain tensors are symmetric tensors.

Writing now the strain-displacement relationship as

$$\{\varepsilon\} = [d]\{P\} \qquad (2.16)$$

2.4 Multiple-degree-of-freedom Linear System

where [d] is the differential operator relating the strains to the displacements, and substituting Eq. (2.7) into Eq. (2.16), we obtain

$$\{\varepsilon\}=[d][N]\{P\} \tag{2.17}$$

where [N] has been used to denote the transformation matrix of the displacements to the generalized coordinates. The strain energy functional of the elastic mechanical system reads

$$U=\frac{1}{2}\int_v \{\sigma\}^T\{\varepsilon\}dv \tag{2.18}$$

Using now the relation of Eqs. (2.15) and (2.17) and Eq. (2.18), we can write the strain energy functional as

$$U=\tfrac{1}{2}\{q\}^T\int_v [N]^T[d]^T[C][d][N]dv\{q\} \tag{2.19}$$

or

$$U=\tfrac{1}{2}\{q\}^T[K]\{q\} \tag{2.20}$$

where

$$[K]=\int_v [N]^T[d]^T[C][d][N]dv \tag{2.21}$$

We call [K] the stiffness matrix of the elastic mechanical system.

Again, we observe that [K] is a real symmetrical matrix because the constitutive material matrix is a symmetric matrix and is real.

Furthermore, from energy consideration concepts, we conclude from Eq. (2.20) that [K] is a positive definite matrix for a constrained mechanical elastic system or a semi-positive definite matrix for an elastic mechanical free body.

2.4.5 Expression of the Dissipation Function

We consider in this section that the damping forces of the elastic mechanical system are of viscous nature and are linearly related to the velocity vector, and we write

$$F_D(P) = \frac{\partial F_D(P)}{\partial q_i'} q_i' \qquad (2.22)$$

where $F_D(P)$ is the damping force of the elastic mechanical system at point P.

The variation in the virtual work of the damping forces in a virtual displacement δP reads

2.4 Multiple-degree-of-freedom Linear System

$$T = \frac{1}{2}\int_{v} \rho(P)V(P).V(P)dv \qquad (2.10)$$

$$V_j = \frac{dP_j}{dt} \qquad (2.8)$$

2.4 Multiple-degree-of-freedom Linear System

$$T = \frac{1}{2}\int_{v} \rho(P) V(P).V(P) dv \qquad (2.10)$$

$$V_j = \frac{dP_j}{dt} \qquad (2.8)$$

2.4 Multiple-degree-of-freedom Linear System

$$T = \frac{1}{2}\int_{v} \rho(P) V(P).V(P) dv \qquad (2.10)$$

$$V_j = \frac{dP_j}{dt} \qquad (2.8)$$

2.4 Multiple-degree-of-freedom Linear System

$$T = \frac{1}{2}\int_v \rho(P)V(P).V(P)dv \qquad (2.10)$$

$$V_j = \frac{dP_j}{dt} \qquad (2.8)$$

2.4 Multiple-degree-of-freedom Linear System

$$T = \frac{1}{2}\int_v \rho(P)V(P).V(P)dv \qquad (2.10)$$

$$V_j = \frac{dP_j}{dt} \qquad (2.8)$$

2.4 Multiple-degree-of-freedom Linear System

$$T = \frac{1}{2}\int_{v} \rho(P)V(P).V(P)dv \qquad (2.10)$$

$$V_j = \frac{dP_j}{dt} \qquad (2.8)$$

2.4 Multiple-degree-of-freedom Linear System

$$T = \frac{1}{2}\int_{v} \rho(P) V(P).V(P) dv \qquad (2.10)$$

$$V_j = \frac{dP_j}{dt} \qquad (2.8)$$

2.4 Multiple-degree-of-freedom Linear System

$$T = \frac{1}{2}\int_v \rho(P)V(P).V(P)dv \qquad (2.10)$$

$$V_j = \frac{dP_j}{dt} \qquad (2.8)$$

2.5 Coordinate Coupling and Principle coordinates

As stated earlier, an n-degree-of-freedom system requires n independent coordinates to describe its configuration.

Usually, these coordinates are independent geometrical quantities measured from the equilibrium position of the vibrating body.

However, it is possible to select some other set of n coordinates to describe the configuration of the system.

The latter set may be, for example, different from the first set in that the coordinates may have their origin away from the equilibrium position of the body.

There could be still other sets of coordinates to describe the configuration of the system. Each of these sets of n coordinates is called **the generalized coordinates**

2.7 Vibration Absorber

The vibration absorber, also called dynamic vibration absorber, is a mechanical device used to reduce or eliminate unwanted vibration.

It consists of another mass and stiffness attached to the main (or original) mass that needs to be protected from vibration.

Thus the main mass and the attached absorber mass constitute a two-degree-of-freedom system, hence the vibration absorber will have two natural frequencies.

The vibration absorber is commonly used in machinery that operates at constant speed, because the vibration absorber is tuned to one particular frequency and is effective only over a narrow band of frequencies.

Common applications of the vibration absorber include reciprocating tools, such as sanders, saws, and compactors, and large reciprocating internal combustion engines which run at constant speed (for minimum fuel consumption).

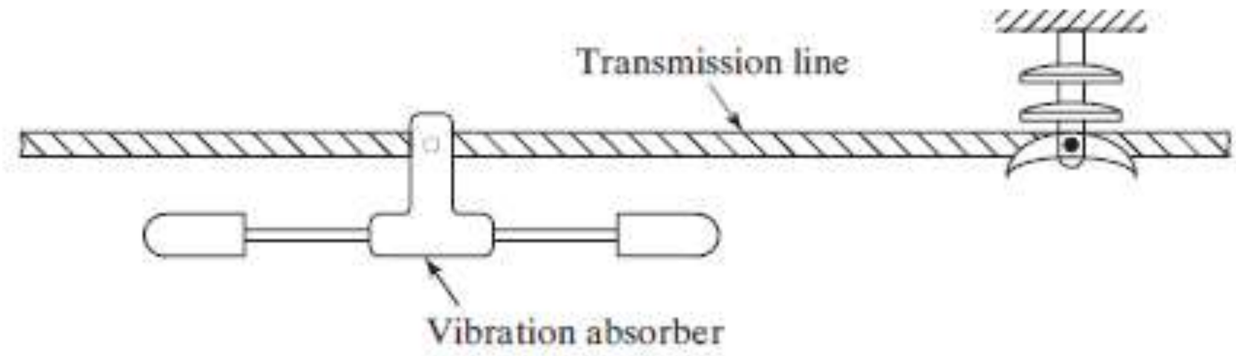
2.7 Vibration Absorber

In these systems, the vibration absorber helps balance the reciprocating forces.

Without a vibration absorber, the unbalanced reciprocating forces might make the device impossible to hold or control.

Vibration absorbers are also used on high-voltage transmission lines.

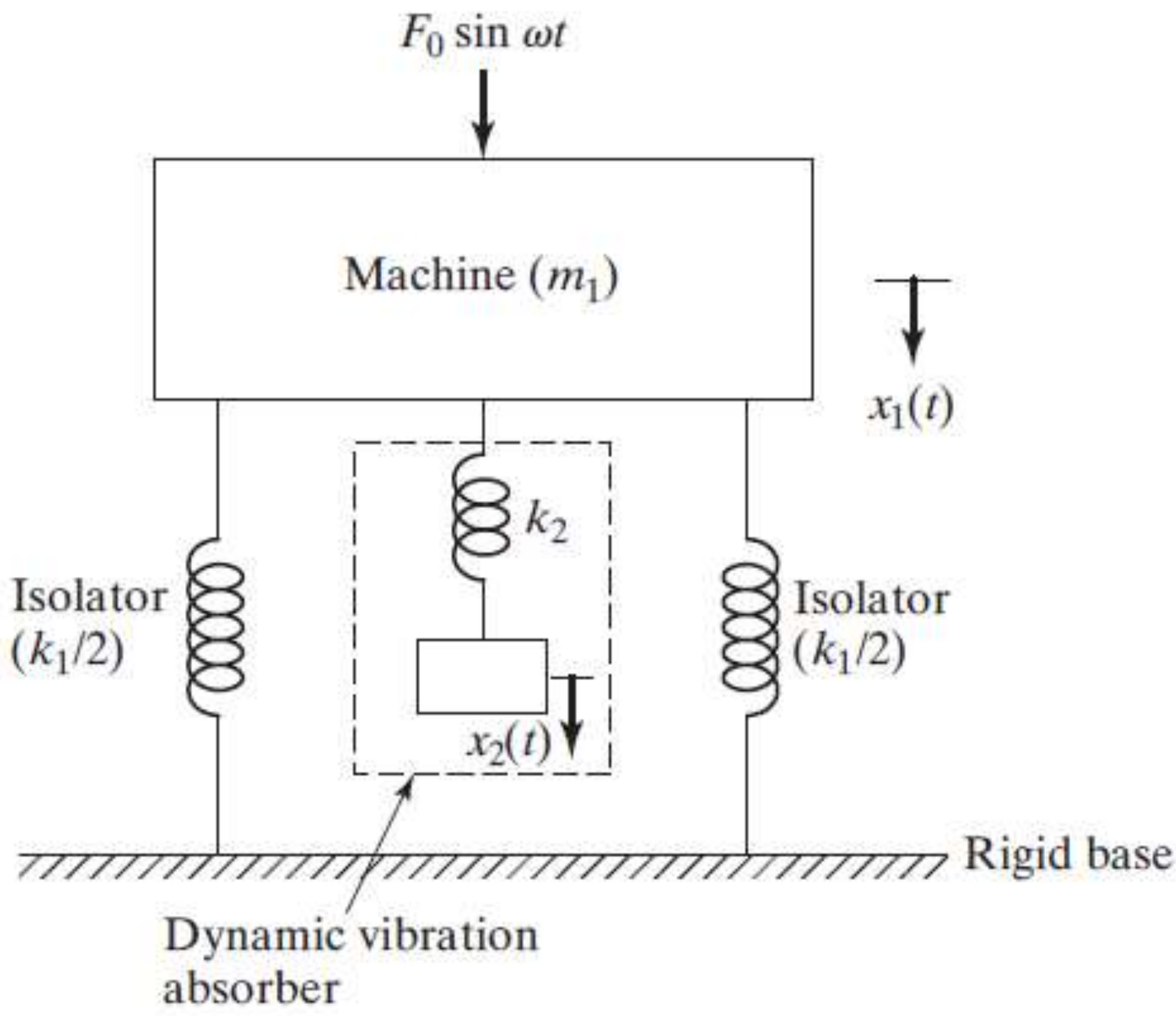
In this case, the dynamic vibration absorbers, in the form of dumbbell-shaped devices (Figure below), are hung from transmission lines to mitigate the fatigue effects of wind induced vibration.



2.7 Vibration absorber

2.7 Vibration absorber

2.7 Dynamic Vibration Absorber



2.7 Dynamic Vibration Absorber

When we attach an auxiliary mass m_2 to a machine of mass m_1 through a spring of stiffness k_2 the resulting two-degree-of-freedom system will look as shown in Figure in next slide.

The equations of motion of the masses m_1 and m_2 are

$$m_1\ddot{x}_1 + k_1 x_1 + k_2\left(x_1 - x_2\right) = F_0 \sin \omega t$$

$$m_2\ddot{x}_2 + k_2\left(x_2 - x_1\right) = 0 \qquad (2.30)$$

By assuming harmonic solution,

$$x_j\left(t\right) = X_j \sin \omega t \qquad j = 1,2 \qquad (2.31)$$

2.7 Dynamic Vibration Absorber

we can obtain the steady-state amplitudes of the masses m_1 and m_2 as

$$X_1 = \frac{\left(k_2 - m_2\omega^2\right)F_0}{\left(k_1 + k_2 - m_1\omega^2\right)\left(k_2 - m_2\omega^2\right) - k_2^2} \qquad (2.32)$$

$$X_2 = \frac{k_2 F_0}{\left(k_1 + k_2 - m_1\omega^2\right)\left(k_2 - m_2\omega^2\right) - k_2^2} \qquad (2.33)$$

We are primarily interested in reducing the amplitude of the machine (X_1)

In order to make the amplitude of m1 zero, the numerator of Eq. (2.32) should be set equal to zero.

This gives

$$\omega^2 = \frac{k_2}{m_2} \qquad (2.34)$$

2.7 Dynamic Vibration Absorber

If the machine, before the addition of the dynamic vibration absorber, operates near its resonance, $\omega^2 \cong \omega_1^2 = k_1 / m_1$

Thus if the absorber is designed such that

$$\omega^2 = \frac{k_2}{m_2} = \frac{k_1}{m_1} \qquad (2.35)$$

the amplitude of vibration of the machine, while operating at its original resonant frequency, will be zero. By defining

$$\delta_{st} = \frac{F_0}{k_1}; \qquad \omega_1 = \left(\frac{k_1}{m_1}\right)^{\frac{1}{2}}$$

as the natural frequency of the machine or main system, and

$$\omega_2 = \left(\frac{k_2}{m_2}\right)^{\frac{1}{2}} \qquad (2.36)$$

2.7 Dynamic Vibration Absorber

as the natural frequency of the absorber or auxiliary system, Eqs. (2.32) and (2.33) can be rewritten as

$$\frac{X_1}{\delta_{st}} = \frac{1-\left(\dfrac{\omega}{\omega_2}\right)^2}{\left[1+\dfrac{k_2}{k_1}-\left(\dfrac{\omega}{\omega_1}\right)^2\right]\left[1-\left(\dfrac{\omega}{\omega_2}\right)^2\right]-\dfrac{k_2}{k_1}} \qquad (2.37)$$

$$\frac{X_2}{\delta_{st}} = \frac{1}{\left[1+\dfrac{k_2}{k_1}-\left(\dfrac{\omega}{\omega_1}\right)^2\right]\left[1-\left(\dfrac{\omega}{\omega_2}\right)^2\right]-\dfrac{k_2}{k_1}} \qquad (2.38)$$

2.7 Dynamic Vibration Absorber

Figure in next slide shows the variation of the amplitude of vibration of the machine (X_1/δ_{st}) with the machine speed (ω/ω_1).

The two peaks correspond to the two natural frequencies of the composite system.

As seen before, $X_1 = 0$ at $\omega = \omega_1$

At this frequency, Eq. (2.38) gives

$$X_2 = -\frac{k_1}{k_2}\delta_{st} = -\frac{F_0}{k_2} \qquad (2.39)$$

This shows that the force exerted by the auxiliary spring is opposite to the impressed force and neutralizes it, thus reducing to zero.

The size of the dynamic vibration absorber can be found from Eqs. (9.142) and (9.138):

2.7 Dynamic Vibration Absorber

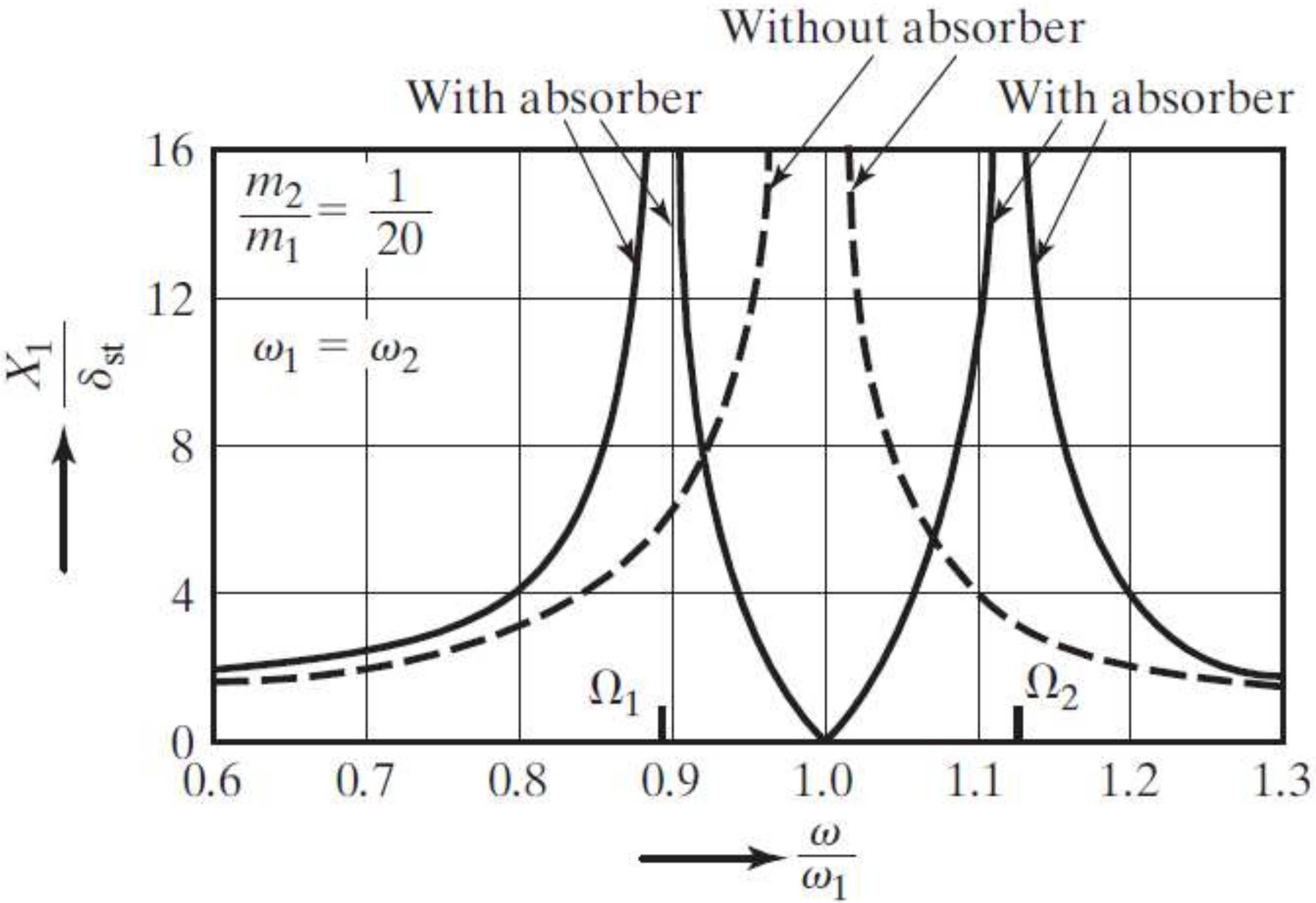
This shows that the force exerted by the auxiliary spring is opposite to the impressed force ($k_2X_2 = -F_0$) and neutralizes it, thus reducing X_1 to zero.

The size of the dynamic vibration absorber can be found from Eqs. (2.39) and (2.35):

$$k_2 X_2 = m_2 \omega^2 X_2 = -F_0 \qquad (2.40)$$

Thus the values of k_2 and m_2 depend on the allowable value of X_2.

2.7 Dynamic Vibration Absorber



Effect of undamped vibration absorber on the response of machine

2.7 Dynamic Vibration Absorber

It can be seen from Figure in previous page that the dynamic vibration absorber, while eliminating vibration at the known impressed frequency ω, introduces two resonant frequencies Ω_1 and Ω_2 at which the amplitude of the machine is infinite.

In practice, the operating frequency ω must therefore be kept away from the frequencies Ω_1 and Ω_2.

The values of Ω_1 and Ω_2 can be found by equating the denominator of Eq. (2.37) to zero.

Noting that

$$\frac{k_2}{k_1} = \frac{k_2}{m_2}\frac{m_2}{m_1}\frac{m_1}{k_1} = \frac{m_2}{m_1}\left(\frac{\omega_2}{\omega_1}\right)^2 \qquad (2.41)$$

and setting the denominator of Eq. (2.37) to zero leads to

$$\left(\frac{\omega}{\omega_2}\right)^4\left(\frac{\omega_2}{\omega_1}\right)^2 - \left(\frac{\omega}{\omega_2}\right)^2\left[1+\left(1+\frac{m_2}{m_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^2\right]+1=0 \qquad (2.42)$$

2.7 Dynamic Vibration Absorber

The two roots of this equation are given by

$$\left.\begin{aligned}\left(\frac{\Omega_1}{\omega_2}\right)^2\\\left(\frac{\Omega_2}{\omega_2}\right)^2\end{aligned}\right\}=\frac{\left\{\left[1+\left(1+\frac{m_2}{m_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^2\right]\mp\left\{\left[1+\left(1+\frac{m_2}{m_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^2\right]^2-4\left(\frac{\omega_2}{\omega_1}\right)^2\right\}^{\frac{1}{2}}\right\}}{2\left(\frac{\omega_2}{\omega_1}\right)^2} \qquad (2.43)$$

which can be seen to be functions of (m_2/m_1) and (ω_2/ω_1).

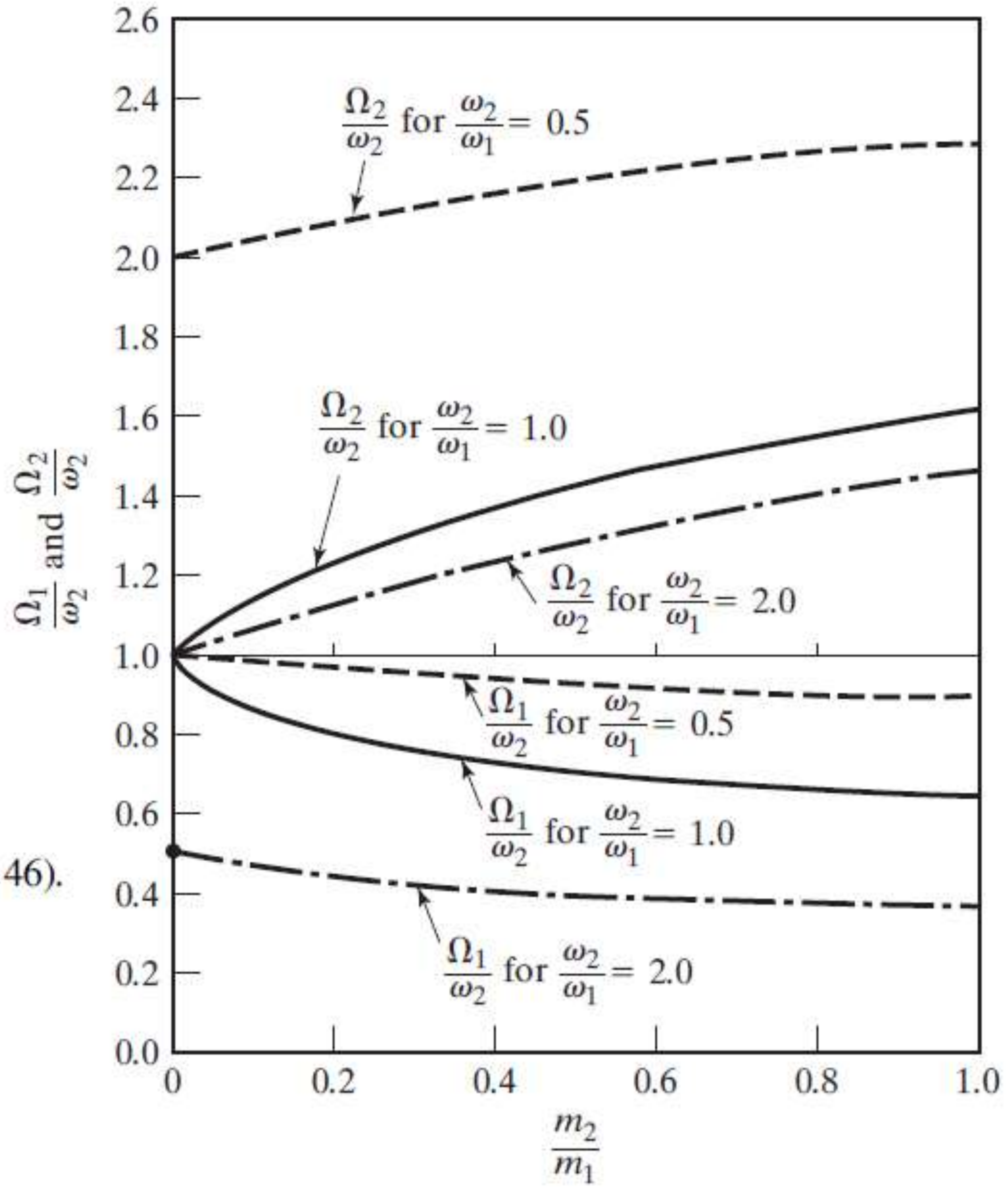
2.7 Dynamic Vibration Absorber

1. It can be seen, from Eq. (9.146), that is less than and is greater than the operating speed (which is equal to the natural frequency,) of the machine. Thus the machine must pass through during start-up and stopping. This results in large amplitudes.

2. Since the dynamic absorber is tuned to one excitation frequency the steady-state amplitude of the machine is zero only at that frequency. If the machine operates at other frequencies or if the force acting on the machine has several frequencies, then the amplitude of vibration of the machine may become large.

3. The variations of and as functions of the mass ratio are shown in Fig. 9.35 for three different values of the frequency ratio It can be seen that the difference between and increases with increasing values of m2/m1.

2.7 Dynamic Vibration Absorber



Variations of Ω_1 and Ω_2 given by Eq. (9.146).

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Mechanical Vibration and Structural Dynamics

Unit 3: Vibration of continuous system

Contents

III	**Vibration of Continuous system**
3.1	Introduction to Hamilton Principle
3.2	Longitudinal, transverse and torsional vibration of cylindrical shaft - extension to taper shaft
3.3	Dynamic equations of equilibra of general elastic body

3.1 What is continuous system?

A structural member consisting of a single piece of a particular material(s) without any visible discontinuity is a continuous structure or continuous system

Example: Rods, Beams, shafts, panels/plates, and shells

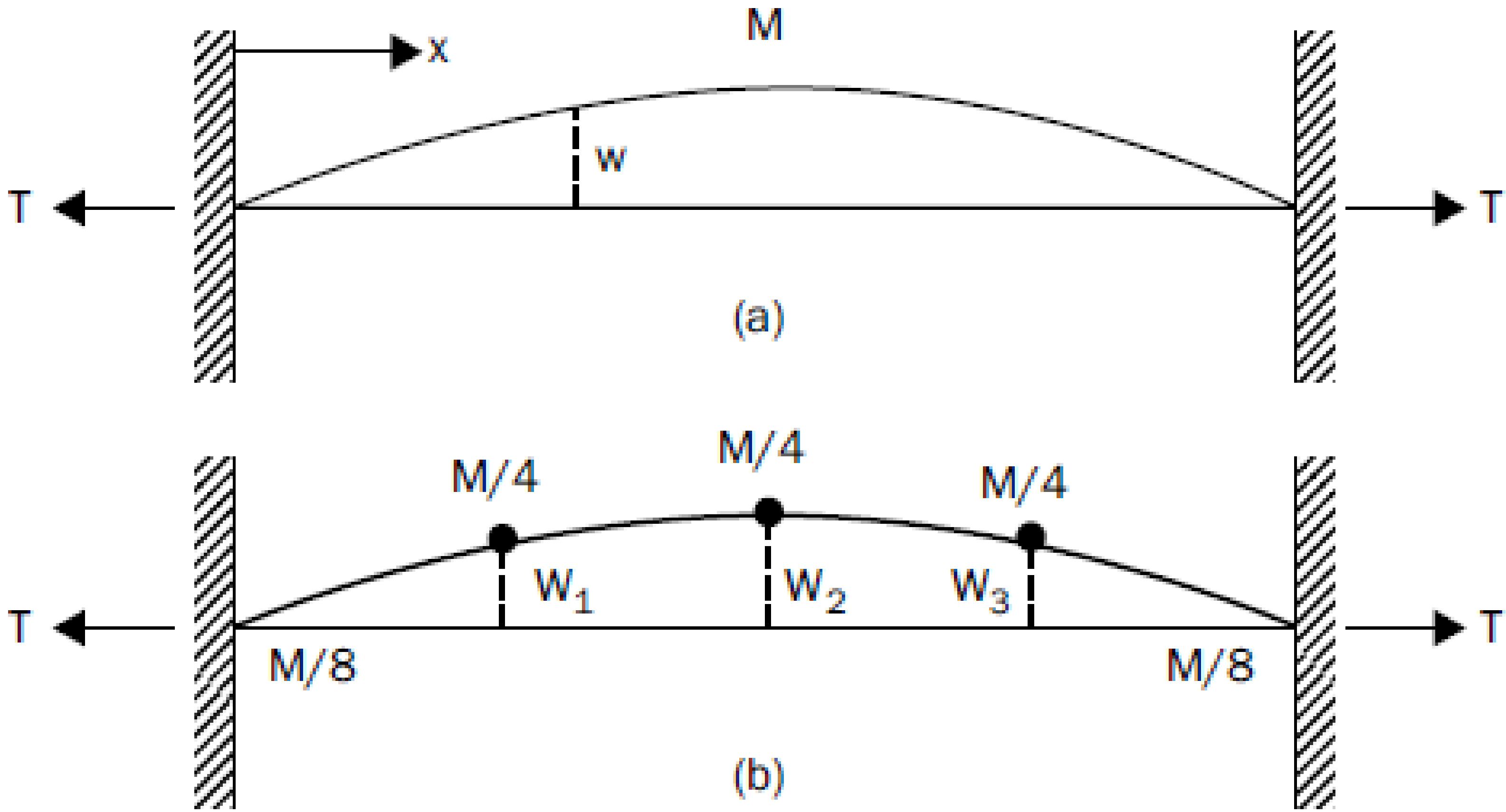
A single piece of above kind of continuous structure made of composites materials is essentially a continuous system

Smart structures are also modeled a continuous structures

Sometimes discontinuous structure, behaves like continuous structure when properly joined with bolts, rivets or weld

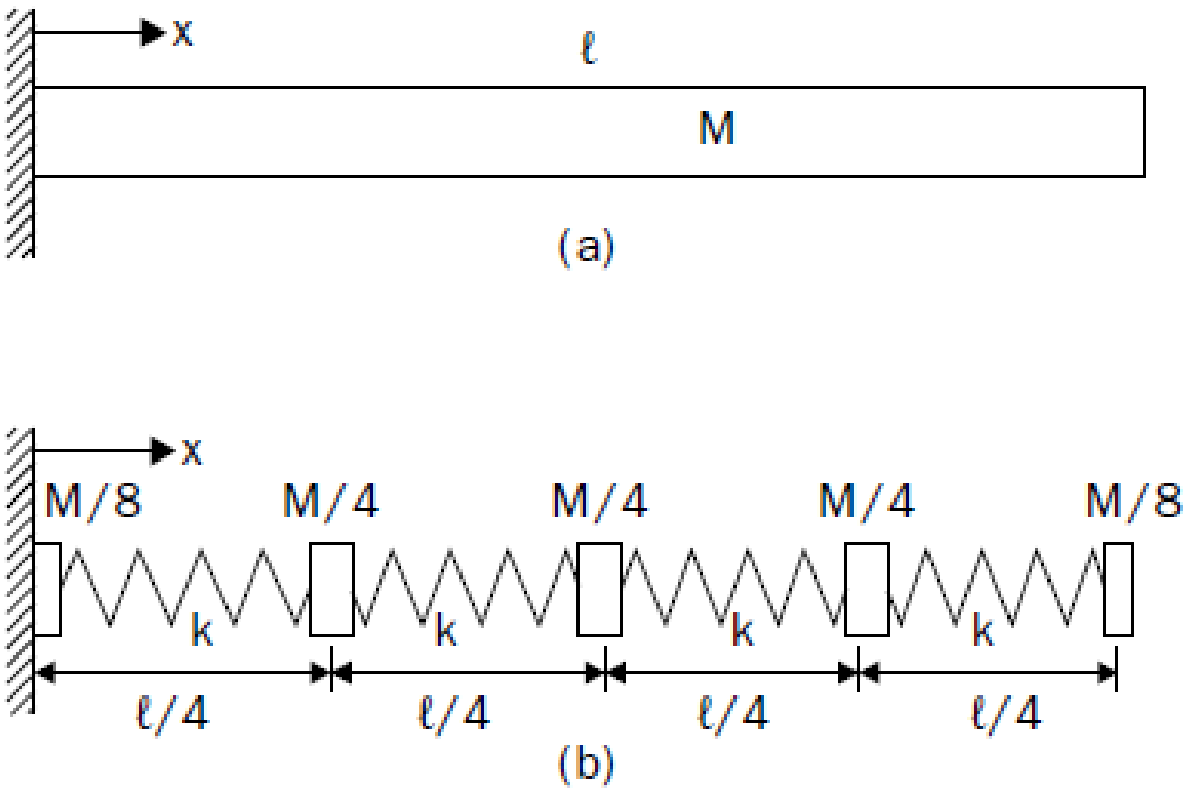
Vehicle structures (surface, air and space) appear and behave like a continuous structures

3.1 What is continuous system?



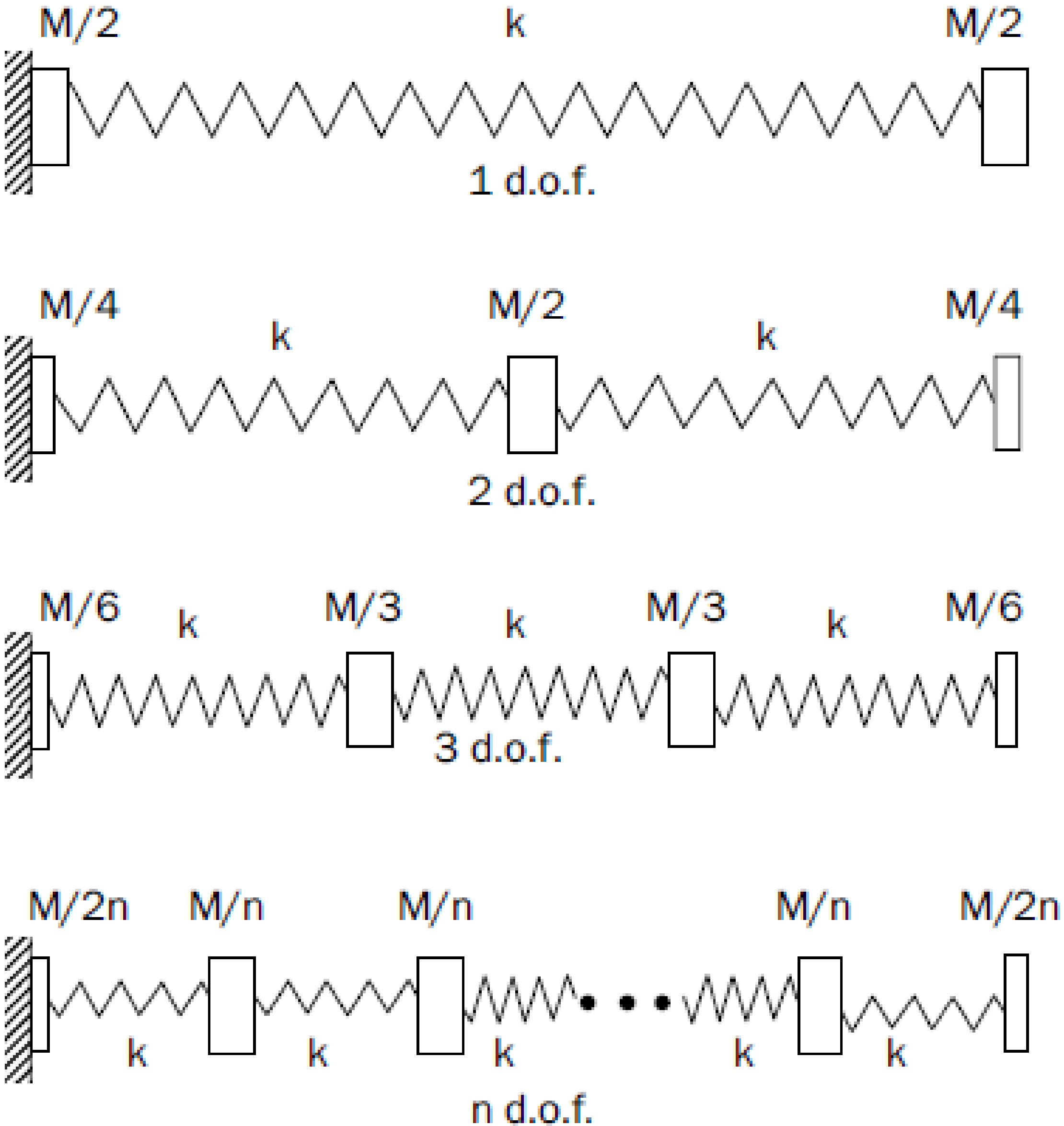
(a) A continuous string of mass M, displaced transversely;
(b) a discrete model of the string.

3.1 What is continuous system? (cont…)



(a) A continuous bar of mass M;
(b) A discrete model of the bar.

3.1 What is continuous system?



3.1 What is continuous system?

n	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
1	1.4142	–	–	–	–
2	1.5307	3.6955	–	–	–
3	1.5529	4.2426	5.7956	–	–
4	1.5607	4.4446	6.6518	7.8463	–
5	1.5643	4.5399	7.0711	8.9101	9.8769
7	1.5675	4.6239	7.4484	9.8995	11.8541
10	1.5692	4.6689	7.6537	10.4500	12.9890
15	1.5701	4.6930	7.7646	10.7510	13.6197
20	1.5704	4.7015	7.8036	10.8576	13.8447
∞ (exact)	1.5708	4.7124	7.8540	10.9956	14.1372

Nondimensional Frequencies $\omega^* = \omega \sqrt{(MI/AE)}$ *for n d.o.f. Discrete* Models of Longitudinal Vibrations of a Fixed-Free Bar, as Described in Figure in the previous slide

3.1 Introduction to continuous system

- The displacement, velocity and acceleration are describe as a function of space (x,y,z) and time (t)
- Coordinate System (rectangular, cylindrical and spherical)
- In analytical dynamics generalized coordinate system
- Application of variation principles
- Derivation of energy expressions (KE, PE , Virtual work, etc)
- Application of Lagrange's equation or Hamilton's principle

Continuous systems	Dimensionality	Differential order
String	1	2
Bar	1	2
Beam	1	4
Membrane	2	2
Plate	2	4
Shell	2	8
Three dimensional	3	6

3.2 Hamilton's Principle

Hamilton's Principle is used for the development of equations of motion in vectorial form using scalar energy quantities in a variational form

$$\int_{t_1}^{t_2} \delta(T - V)dt + \int_{t_1}^{t_2} \delta W_{nc} dt = 0$$

Where T = total kinetic energy of system

V=potential energy of system, including both strain energy and potential of any conservative external forces

W_{nc}= work done by non-conservative forces acting on system, including damping and any arbitrary external loads

δ = variation taken during indicated time interval

Hamilton's principle states that the variation of kinetic and potential energy plus the variation of the work done by the non-conservative forces considered during interval t_1 to t_2 must equal to zero

The application of this principle leads directly to the equations of motion for any given system

3.3 Solutions of vibration problems using Variational Principles

3.1 Introduction to continuous system
3.2 Discreatize models of continuous systems
3.3 Solutions of vibration problems using Variational Principles
3.4 Vibrations of strings, bars, shafts and beams

3.3.1 Rayleigh – Ritz Method

Table 9.6 Scalar products for Rayleigh-Ritz method

Structural element	Case	$(u, v)_T$	$(u, v)_V$
Torsional shaft	No added disks or springs	$\int_0^L \rho J u(x) v(x)\, dx$	$\int_0^L GJ \frac{du}{dx}\frac{dv}{dx}\, dx$
	Added disk at $x = \tilde{x}$	$\int_0^L \rho J u(x) v(x)\, dx + I_D u(\tilde{x}) v(\tilde{x})$	$\int_0^L GJ \frac{du}{dx}\frac{dv}{dx}\, dx$
	Torsional spring at $x = \tilde{x}$	$\int_0^L \rho J u(x) v(x)\, dx$	$\int_0^L GJ \frac{du}{dx}\frac{dv}{dx}\, dx + k_t u(\tilde{x}) v$
Longitudinal bar	No added masses or springs	$\int_0^L \rho A u(x) v(x)\, dx$	$\int_0^L EA \frac{du}{dx}\frac{dv}{dx}\, dx$
	Added mass at $x = \tilde{x}$	$\int_0^L \rho A u(x) v(x)\, dx + m u(\tilde{x}) v(\tilde{x})$	$\int_0^L EA \frac{du}{dx}\frac{dv}{dx}\, dx$
	Spring at $x = \tilde{x}$	$\int_0^L \rho A u(x) v(x)\, dx$	$\int_0^L EA \frac{du}{dx}\frac{dv}{dx}\, dx + k u(\tilde{x}) v$

3.3.1 Rayleigh – Ritz Method

Structural element	Case	$(u, v)_T$	$(u, v)_V$
Beam	No added masses, disks, or springs	$\int_0^L \rho A u(x) v(x)\, dx$	$\int_0^L EI \frac{d^2u}{dx^2}\frac{d^2v}{dx^2} dx$
	Added mass at $x = \tilde{x}$	$\int_0^L \rho A u(x) v(x)\, dx + m u(\tilde{x}) v(\tilde{x})$	$\int_0^L EI \frac{d^2u}{dx^2}\frac{d^2v}{dx^2} dx$
	Added spring at $x = \tilde{x}$	$\int_0^L \rho A u(x) v(x)\, dx$	$\int_0^L EI \frac{d^2u}{dx^2}\frac{d^2v}{dx^2} dx + k u(\tilde{x}$
	Added disk (I_D) at $x = \tilde{x}$	$\int_0^L \rho A u(x) v(x)\, dx + I_D \frac{du(\tilde{x})}{dx}\frac{dv(\tilde{x})}{dx}$	$\int_0^L EI \frac{d^2u}{dx^2}\frac{d^2v}{dx^2} dx$

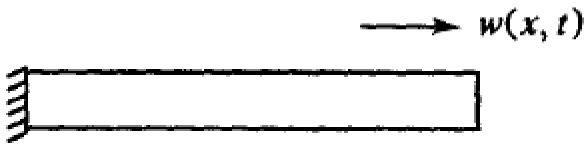
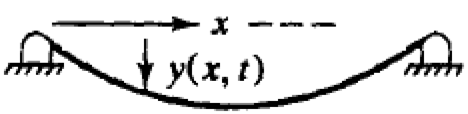
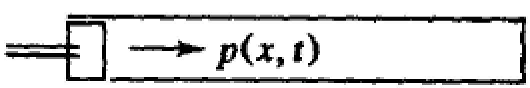
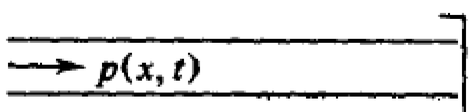
3.4.3 Torsional Vibrations of shafts

Table 9.1 Boundary conditions for torsional oscillations of a circular shaft

End condition	Boundary condition	Remarks
Fixed, $x = 0$ or $x = 1$	$\theta = 0$	
Free, $x = 0$ or $x = 1$	$\frac{\partial \theta}{\partial x} = 0$	
Torsional spring, $x = 0$	$\frac{\partial \theta}{\partial x} = \beta \theta$	$\beta = \frac{k_t L}{JG}$
Torsional spring, $x = 1$	$\frac{\partial \theta}{\partial x} = -\beta \theta$	$\beta = \frac{k_t L}{JG}$
Torsional damper, $x = 0$	$\frac{\partial \theta}{\partial x} = \beta \frac{\partial \theta}{\partial t}$	$\beta = c_t \sqrt{\frac{J}{\rho G}}$
Torsional damper, $x = 1$	$\frac{\partial \theta}{\partial x} = -\beta \frac{\partial \theta}{\partial t}$	$\beta = c_t \sqrt{\frac{J}{\rho G}}$
Attached disk, $x = 0$	$\frac{\partial \theta}{\partial x} = \beta \frac{\partial^2 \theta}{\partial t^2}$	$\beta = \frac{I_D}{\rho JL}$
Attached disk, $x = 1$	$\frac{\partial \theta}{\partial x} = -\beta \frac{\partial^2 \theta}{\partial t^2}$	$\beta = \frac{I_D}{\rho JL}$

3.4.3 Torsional Vibrations of shafts

Table 9.2 Physical problems governed by the wave equation

Problem	Schematic	Nondimensional wave equation	Wave speed	
Torsional oscillations of circular cylinder		$\frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 \theta}{\partial t^2}$	$c = \sqrt{\frac{G}{\rho}}$	G = shear modulus ρ = mass density
Longitudinal oscillations of bar		$\frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial t^2}$	$c = \sqrt{\frac{E}{\rho}}$	E = elastic modulus ρ = mass density
Transverse vibrations of taut string		$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$	$c = \sqrt{\frac{T}{\mu}}$	T = tension μ = linear density
Pressure waves in an ideal gas		$\frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial t^2}$	$c = \sqrt{kRT}$	k = ratio of specific heats R = gas constant T = temperature
Waterhammer waves in rigid pipe		$\frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial t^2}$	$c = \sqrt{\frac{k}{\rho}}$	k = bulk modulus of fluid ρ = mass density

3.4.4 Vibrations of beams

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} = f(x,t) \qquad [9.68]$$

Equation (9.68) is nondimensionalized by introducing

$$x^* = \frac{x}{L} \qquad t^* = t\sqrt{\frac{EI}{\rho A L^4}} \qquad w^* = \frac{w}{L} \qquad f^* = \frac{f}{f_m} \qquad [9.69]$$

where f_m is the maximum value of f. The resulting nondimensional form of Eq. (9.68) is

$$\frac{\partial^2 w}{\partial t^2} + \frac{\partial^4 w}{\partial x^4} = \frac{f_m L^3}{EI} f(x,t) \qquad [9.70]$$

3.4.4 Vibrations of beams

Table 9.3 Boundary conditions for transverse vibrations of a beam

End condition	Boundary condition A	Boundary condition B	Remarks
Free, $x = 0$ or $x = 1$	$\frac{\partial^2 w}{\partial x^2} = 0$	$\frac{\partial^3 w}{\partial x^3} = 0$	
Pinned, $x = 0$ or $x = 1$	$w = 0$	$\frac{\partial^2 w}{\partial x^2} = 0$	
Fixed, $x = 0$ or $x = 1$	$w = 0$	$\frac{\partial w}{\partial x} = 0$	
Linear spring, $x = 0$	$\frac{\partial^2 w}{\partial x^2} = 0$	$\frac{\partial^3 w}{\partial x^3} = -\beta w$	$\beta = \frac{kL^3}{EI}$
Linear spring, $x = 1$	$\frac{\partial^2 w}{\partial x^2} = 0$	$\frac{\partial^3 w}{\partial x^3} = \beta w$	$\beta = \frac{kL^3}{EI}$

3.4.4 Vibrations of beams

End condition	Boundary condition A	Boundary condition B	Remarks
Viscous damper, $x = 0$	$\frac{\partial^2 w}{\partial x^2} = 0$	$\frac{\partial^3 w}{\partial x^3} = -\beta \frac{\partial w}{\partial t}$	$\beta = \frac{cL}{\sqrt{\rho EIA}}$
Viscous damper, $x = 1$	$\frac{\partial^2 w}{\partial x^2} = 0$	$\frac{\partial^3 w}{\partial x^3} = \beta \frac{\partial w}{\partial t}$	$\beta = \frac{cL}{\sqrt{\rho EIA}}$
Attached mass, $x = 0$	$\frac{\partial^2 w}{\partial x^2} = 0$	$\frac{\partial^3 w}{\partial x^3} = -\beta \frac{\partial^2 w}{\partial t^2}$	$\beta = \frac{m}{\rho AL}$
Attached mass, $x = 1$	$\frac{\partial^2 w}{\partial x^2} = 0$	$\frac{\partial^3 w}{\partial x^3} = \beta \frac{\partial^2 w}{\partial t^2}$	$\beta = \frac{m}{\rho AL}$
Attached inertia element, $x = 0$	$\frac{\partial^2 w}{\partial x^2} = -\beta \frac{\partial^3 w}{\partial x \partial t^2}$	$\frac{\partial^3 w}{\partial x^3} = 0$	$\beta = \frac{J}{\rho AL^3}$
Attached inertia element, $x = 1$	$\frac{\partial^2 w}{\partial x^2} = \beta \frac{\partial^3 w}{\partial x \partial t^2}$	$\frac{\partial^3 w}{\partial x^3} = 0$	$\beta = \frac{J}{\rho AL^3}$

3.4.4 Vibrations of beams

Frequency equations and eigenfunctions for each of the six cases are summarized below.

Clamped–clamped:

$$\cos\beta \bullet \cosh\beta = 1 \tag{4.30a}$$

$$X = (\cosh\beta\xi - \cos\beta\xi) - \gamma(\sinh\beta\xi - \sin\beta\xi) \tag{4.30b}$$

$\gamma = 0.98250,\ 1.00078,\ 0.99997,\ 1.00000,\ \ldots$

Free–free:

$$\cos\beta \bullet \cosh\beta = 1 \tag{4.31a}$$

$$X = (\cosh\beta\xi + \cos\beta\xi) - \gamma(\sinh\beta\xi + \sin\beta\xi) \tag{4.31b}$$

$\gamma =$ same as clamped-clamped

3.4.4 Vibrations of beams

Clamped–SS:

$$\tan\beta = \tanh\beta \tag{4.32a}$$

$$X = (\cosh\beta\xi - \cos\beta\xi) - \gamma(\sinh\beta\xi - \sin\beta\xi) \tag{4.32b}$$

$\gamma = 1.00078, 1.00000, \ldots$

Free–SS:

$$\tan\beta = \tanh\beta \tag{4.33a}$$

$$X = (\cosh\beta\xi + \cos\beta\xi) - \gamma(\sinh\beta\xi + \sin\beta\xi) \tag{4.33b}$$

$\gamma =$ same as clamped-SS

3.4.4 Vibrations of beams

Clamped–free:

$$\cos\beta \cdot \cosh\beta = -1 \tag{4.34a}$$

$$X = (\cosh\beta\xi - \cos\beta\xi) - \gamma(\sinh\beta\xi - \sin\beta\xi) \tag{4.34b}$$

$\gamma = 0.73410,\ 1.01847,\ 0.99922,\ 1.00003,\ 1.00000,\ \ldots$

SS–SS:

$$\sin\beta = 0 \tag{4.35a}$$

$$X = \sin\beta\xi \tag{4.35b}$$

In the above equations, $\xi = x/\ell$ is measured in each case from the left end of the beam. The values of β are the square roots of the frequency parameters listed in Table in next slide. More accurate values of β and γ are available in the classical study of Young and Felgar .

3.4.4 Vibrations of beams

m	C–C	C–SS	C–F	SS–SS	SS–F	F–F
1	22.373	15.418	3.5160	9.8696	0	0
2	61.673	49.965	22.034	39.478	15.418	0
3	120.903	104.248	61.697	88.826	49.965	22.373
4	199.859	178.270	120.902	157.914	104.248	61.673
5	298.556	272.031	199.860	246.740	178.270	120.903
>5	$(2m+1)^2\pi^2/4$	$(4m+1)^2\pi^2/16$	$(2m-1)^2\pi^2/4$	$m^2\pi^2$	$(4m-3)^2\pi^2/16$	$(2m-3)^2\pi^2/4$

Table 4.1 Frequency Parameters $\beta^2 = \omega \ell^2 \sqrt{\rho A / EI}$ for Beams

3.4.4 Vibrations of beams

Table 9.4 Natural frequencies and mode shapes for beams.

End conditions $X = 0 \quad X = 1$	Characteristic equation	Five lowest natural frequencies $\omega_k = \sqrt{\lambda_k}$	Mode shape	Kinetic energy scalar product $(X_j(x), X_k(x))$
Fixed-fixed	$\cos\lambda^{1/4}\cosh\lambda^{1/4} = 1$	$\omega_1 = 22.37$ $\omega_2 = 61.66$ $\omega_3 = 120.9$ $\omega_4 = 199.9$ $\omega_5 = 298.6$	$C_k\left[\cosh\lambda_k^{1/4}x - \cos\lambda_k^{1/4}x - \alpha_k(\sinh\lambda_k^{1/4}x - \sin\lambda_k^{1/4}x)\right]$ $\alpha_k = \dfrac{\cosh\lambda_k^{1/4} - \cos\lambda_k^{1/4}}{\sinh\lambda_k^{1/4} - \sin\lambda_k^{1/4}}$	$\int_0^1 X_j(x)X_k(x)\,dx$
Pinned-pinned	$\sin\lambda^{1/4} = 0$	$\omega_1 = 9.870$ $\omega_2 = 39.48$ $\omega_3 = 88.83$ $\omega_4 = 157.9$ $\omega_5 = 246.7$	$C_k\sin\lambda_k^{1/4}x$	$\int_0^1 X_j(x)X_k(x)\,dx$
Fixed-free	$\cos\lambda^{1/4}\cosh^{1/4} = -1$	$\omega_1 = 3.51$ $\omega_2 = 22.03$ $\omega_3 = 61.70$ $\omega_4 = 120.9$ $\omega_5 = 199.9$	$C_k\left[\cosh\lambda_k^{1/4}x - \cos\lambda_k^{1/4}x - \alpha_k(\sinh\lambda_k^{1/4}x - \sin\lambda_k^{1/4}x)\right]$ $\alpha_k = \dfrac{\cos\lambda_k^{1/4} + \cosh\lambda_k^{1/4}}{\sin\lambda_k^{1/4} + \sinh\lambda_k^{1/4}}$	$\int_0^1 X_j(x)X_k(x)\,dx$
Free-free	$\cosh\lambda^{1/4}\cos\lambda^{1/4} = 1$	$\omega_1 = 0$ $\omega_2 = 22.37$ $\omega_3 = 61.66$ $\omega_4 = 120.9$ $\omega_5 = 199.9$	$1, \sqrt{3}x\,(k = 1)$ $C_k\left[\cosh\lambda_k^{1/4}x + \cos\lambda_k^{1/4}x + \alpha_k(\sinh\lambda_k^{1/4}x + \sin\lambda_k^{1/4}x)\right]$ $\alpha_k = \dfrac{\cosh\lambda_k^{1/4} - \cos\lambda_k^{1/4}}{\sin\lambda_k^{1/4} - \sinh\lambda_k^{1/4}}$	$\int_0^1 X_j(x)X_k(x)\,dx$
Fixed-linear spring	$\lambda^{3/4}(\cosh\lambda^{1/4}\cos\lambda^{1/4} + 1)$ $- \beta(\cos\lambda^{1/4}\sinh\lambda^{1/4} - \cosh\lambda^{1/4}\sin\lambda^{1/4})$ $= 0$	For $\beta = 0.25$ $\omega_1 = 3.65$ $\omega_2 = 22.08$ $\omega_3 = 61.70$ $\omega_4 = 120.9$ $\omega_5 = 199.9$	$C_k\left[\cos\lambda_k^{1/4}x - \cosh\lambda_k^{1/4}x - \alpha_k(\sin\lambda^{1/4}x - \sinh\lambda_k^{1/4}x)\right]$ $\alpha_k = \dfrac{\cos\lambda_k^{1/4} + \cosh\lambda_k^{1/4}}{\sin\lambda_k^{1/4} + \sinh\lambda_k^{1/4}}$	$\int_0^1 X_j(x)X_k(x)\,dx$

3.4.4 Vibrations of beams

Pinned-linear spring	$\cot\lambda^{1/4} - \coth\lambda^{1/4} = -\frac{2\beta}{\lambda^{3/4}}$	For $\beta = 0.25$ $\omega_1 = 0.8636$ $\omega_2 = 15.41$ $\omega_3 = 49.47$ $\omega_4 = 104.25$ $\omega_5 = 178.27$	$C_k\left[\sin\lambda_k^{1/4}x + \frac{\sin\lambda_k^{1/4}}{\sinh\lambda_k^{1/4}}\sinh\lambda_k^{1/4}x\right]$	$\int_0^1 X_j(x)X_k(x)\,dx$
Fixed-attached mass	$\lambda^{1/4}(\cos\lambda^{1/4}\cosh\lambda^{1/4} + 1)$ $+\beta(\cos\lambda^{1/4}\sinh\lambda^{1/4} - \cosh\lambda^{1/4}\sin\lambda^{1/4})$ $= 0$	For $\beta = 0.25$ $\omega_1 = 3.047$ $\omega_2 = 21.54$ $\omega_3 = 61.21$ $\omega_4 = 120.4$ $\omega_5 = 199.4$	$C_k\left[\cos\lambda_k^{1/4}x - \cosh\lambda_k^{1/4}x + \alpha_k(\sinh\lambda_k^{1/4}x - \sin\lambda_k^{1/4}x)\right]$ $\alpha_k = \frac{\cos\lambda_k^{1/4} + \cosh\lambda_k^{1/4}}{\sin\lambda_k^{1/4} + \sinh\lambda_k^{1/4}}$	$\int_0^1 X_j(x)X_k(x)\,dx$ $+\beta X_j(1)X_k(1)$
Pinned-free	$\tan\lambda^{1/4} = \tanh\lambda^{1/4}$	$\omega_1 = 0$ $\omega_2 = 15.42$ $\omega_3 = 49.96$ $\omega_4 = 104.2$ $\omega_5 = 178.3$	$\sqrt{3}x,\ (k = 1)$ $C_k\left[\sin\lambda_k^{1/4}x + \frac{\sin\lambda_k^{1/4}}{\sinh\lambda_k^{1/4}}\sinh\lambda_k^{1/4}x\right]$ $(k > 1)$	$\int_0^1 X_j(x)X_k(x)\,dx$
Fixed-pinned	$\tan\lambda^{1/4} = \tanh\lambda^{1/4}$	$\omega_1 = 15.42$ $\omega_2 = 49.96$ $\omega_3 = 104.2$ $\omega_4 = 178.3$ $\omega_5 = 272.0$	$C_k\left[\cos\lambda_k^{1/4}x - \cosh\lambda_k^{1/4}x - \alpha_k(\sin\lambda_k^{1/4}x - \sinh\lambda_k^{1/4}x)\right]$ $\alpha_k = \frac{\cos\lambda_k^{1/4} - \cosh\lambda_k^{1/4}}{\sin\lambda_k^{1/4} - \sinh\lambda_k^{1/4}}$	$\int_0^1 X_j(x)X_k(x)\,dx$
Fixed-attached inertia element	$\cos\lambda^{1/4}\cosh\lambda^{1/4}$ $+\beta(\sin\lambda^{1/4}\cosh\lambda^{1/4} + \cos\lambda^{1/4}\sinh\lambda^{1/4})$ $= -1$	For $\beta = 0.25$ $\omega_1 = 4.425$ $\omega_2 = 27.28$ $\omega_3 = 71.41$ $\omega_4 = 135.4$ $\omega_5 = 219.2$	$C_k\left[\cos\lambda_k^{1/4}x - \cosh\lambda_k^{1/4}x + \alpha_k(\sin\lambda_k^{1/4}x - \sinh\lambda_k^{1/4}x)\right]$ $\alpha_k = \frac{\sin\lambda_k^{1/4} - \sinh\lambda_k^{1/4}}{\cos\lambda_k^{1/4} + \cosh\lambda_k^{1/4}}$	$\int_0^1 X_j(x)X_k(x)\,dx$ $+\beta X_j(1)X_k(1)$

I The dimensional natural frequencies are obtained by multiplying the given nondimensional natural frequencies by $\sqrt{EI/\rho AL^4}$; for a given beam β is as defined in Table 9.3.

3.4.4 Vibrations of beams

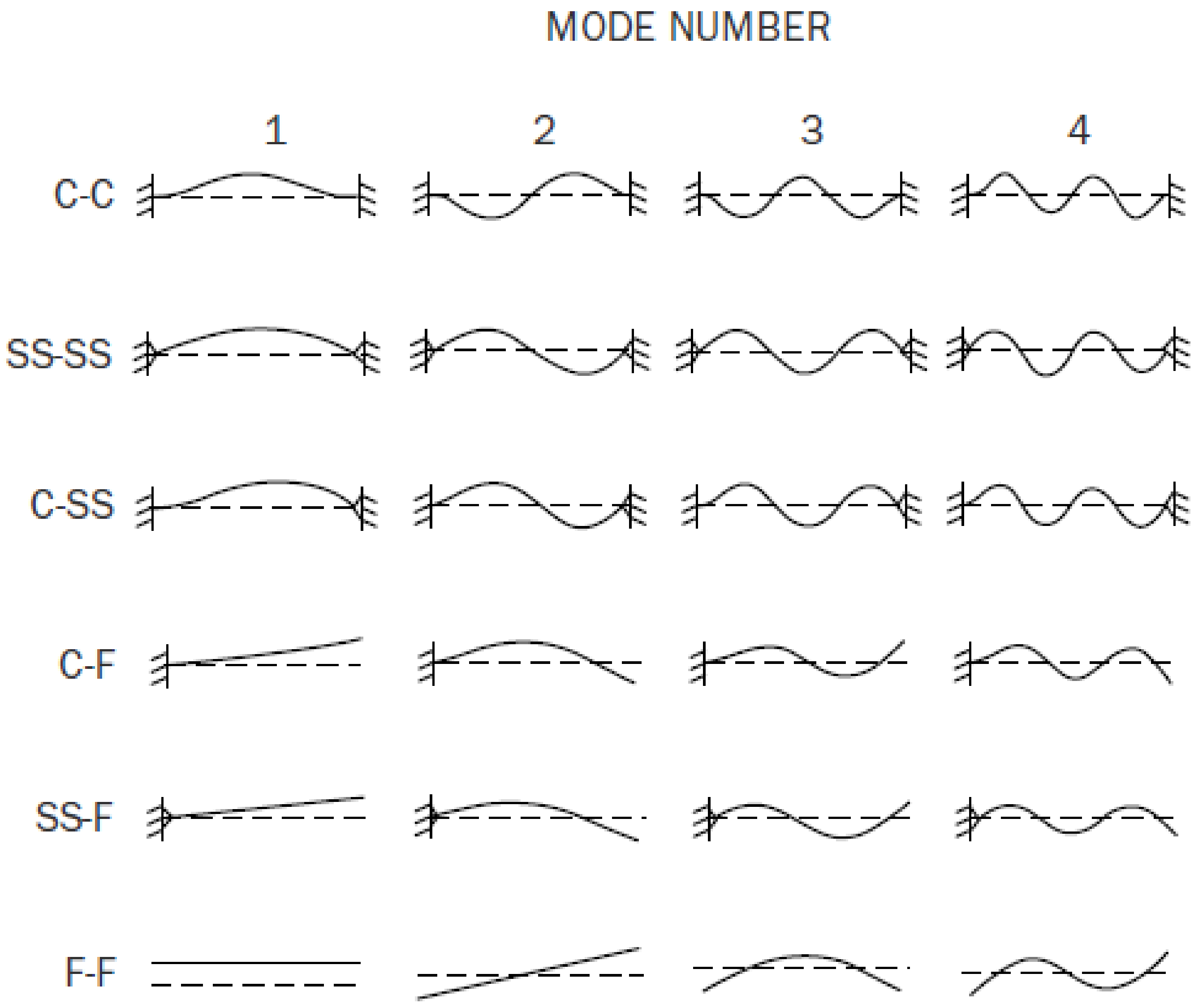


Figure 4.3 The first four mode shapes for beams with different boundaries.

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Mechanical Vibration and Structural Dynamics

Unit 4: Determination of natural frequencies and mode shapes

Contents

4.2 Solution Methods for Eigenproblems

We concentrate on the solution of the eigenproblem

$$K\phi = \lambda M\phi \qquad (1)$$

and., in particular, on the calculation of the smallest eigenvalues $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_p$ and corresponding eigenvectors $\phi_1, \phi_2, \phi_3, \ldots, \phi_p$.

The solution methods that we considered here first can be subdivided into four groups, corresponding to which basic property is used as the basis of the solution algorithm (Ref. J.H. Wilkinson)

1. Vector Iteration Method

$$K\phi_i = \lambda_i M\phi_i \qquad (2)$$

2. Transformation Method

   First we have to determine mode shapes matrix Φ, such that

$$\Phi^T K\Phi = \Lambda \qquad (3)$$

$$\Phi^T M\Phi = I \qquad (4)$$

4.2 Solution Methods for Eigenproblems

where
$$\Phi = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_n \end{bmatrix}$$
$$\Lambda = diag(\lambda_i), \quad i = 1,2,\cdots,n$$

3. Polynomial Iteration

$$p(\lambda_i) = 0 \tag{5}$$
$$\text{where } p(\lambda) = \det(K - \lambda M) \tag{6}$$

4. Sturm Sequence Property of the Characteristic Polynomials

$$p(\lambda) = \det(K - \lambda M) \tag{7}$$
$$\text{where} \quad p^{(r)}(\lambda^{(r)}) = \det(K^{(r)} - \lambda^{(r)} M^{(r)}) \tag{8}$$

n=1,2,3,...,(n-1)

4.2 Solution Methods for Eigenproblems

$p^{(r)}\left(\lambda^{(r)}\right)$ is the characteristic polynomial of r^{th} associated constraint problem corresponding to $\mathrm{K}\phi = \lambda \mathrm{M}\phi$

5. Lanczos Method and Subspace Iteration Method used combination of above 4 methods

4.3.1 Eigenvalue Extraction Methods in MSC/NASTRAN

In MSC/NASTRAN following Methods are Available for Real Eigenvalue Extraction

1. Transformation Methods
- ❑ Givens Method
- ❑ Householder Method
- ❑ Modified Givens Method
- ❑ Modified Householder Method

2. Tracking Methods
- ❑ Inverse Power Method
- ❑ Sturm Modified Inverse Power Method

Lanczos Method combines the best characteristics of both the tracking and transformation methods.

4.3.1Eigenvalue Extraction Methods in MSC/NASTRAN

Table 3-1 Comparison of Eigenvalue Methods

	Method				
	Givens, Householder	Modified Givens, Householder	Inverse Power	Sturm Modified Inverse Power	Lanczos
Reliability	High	High	Poor (can miss modes)	High	High
Relative Cost: Few Modes Many Modes	 Medium High	 Medium High	 Low High	 Low High	 Medium Medium
Limitations	Cannot analyze singular [M] Expensive for problems that do not fit in memory	Expensive for many modes Expensive for problems that do not fit in memory	Can miss modes Expensive for many modes	Expensive for many modes	Difficulty with massless mechanisms
Best Application	Small, dense matrices that fit in memory Use with dynamic reduction (Chapter 11)	Small, dense matrices that fit in memory Use with dynamic reduction (Chapter 11)	To detemine a few modes	To detemine a few modes Backup method	Medium to large models

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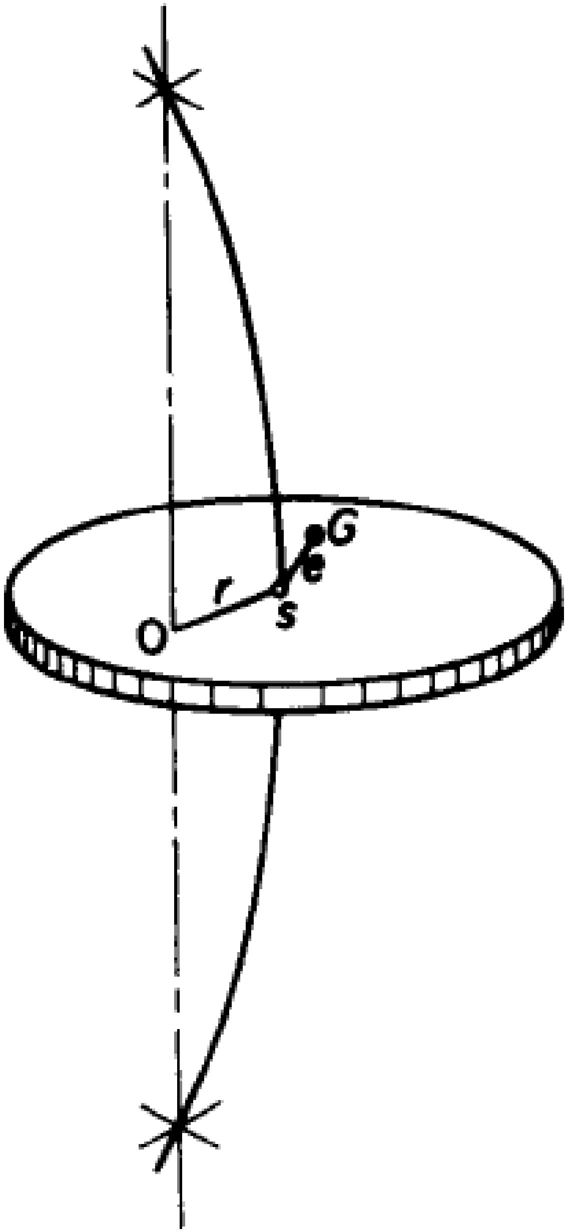
15. Quite big, E:\Library4_Dec11\Engineering\Mechanics of Solids\Structural Dynamics

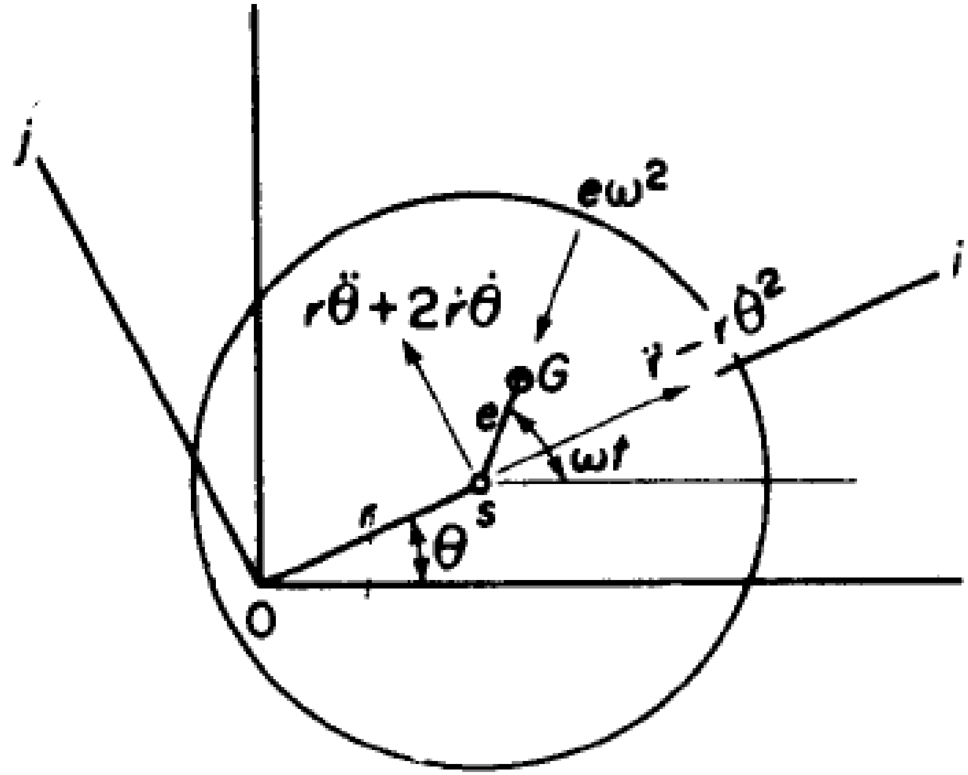
Mechanical Vibration and Structural Dynamics

Unit 5:

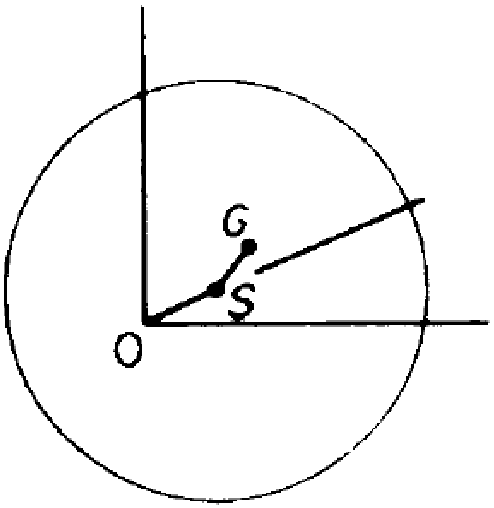
Contents

5.2 Whirling of shafts

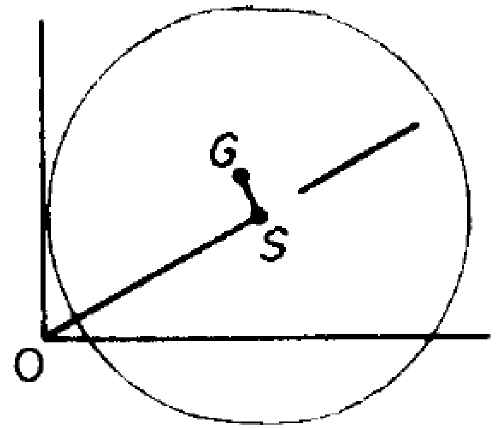




5.2 Whirling of shafts



$\omega << \omega_n$



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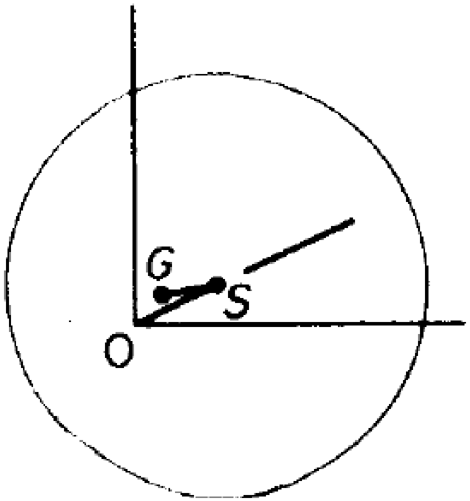

$\omega >> \omega_n$

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1. Clough, R.W., and Penzien, J., *Dynamics of Structures,* McGraw-Hill, Inc., 1975.
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5) Church, A.H., Mechanical Vibrations, John Wiley and Sons, Inc.
6) Timoshenko, S.P., Vibration Problems in Engineering, Ch. V, **Torsional and lateral vibration of shaft**, pp. 253/269.
7) http://www.elmer.unibas.ch/pendulum/nonosc.htm

Question Bank

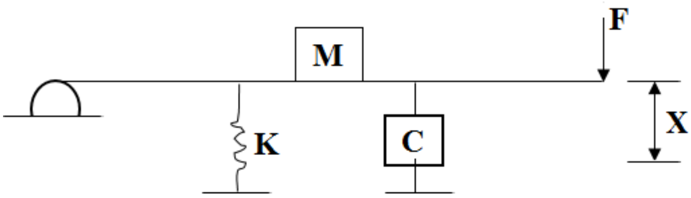
UNIT-I

INTRODUCTION TO STRUCTURAL DYNAMICS

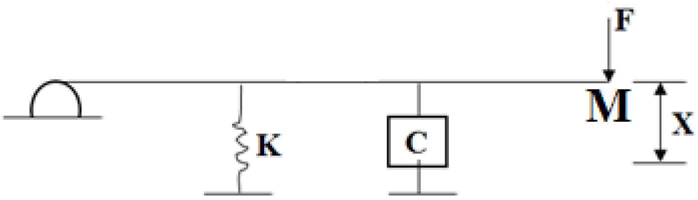
1. a) Explain about lumped mass and Continuous mass system. [6M]

b) Derive the Equation of motion for Undamped single degree of freedom system with forced vibration. [6M]

2. a) Derive the equation of motion for given system [6M]



b) Derive the equation of motion for given system [6M]



3. a) Derive the equation of motion for damped single degree of freedom system with forced vibration. [6M]

b) Briefly explain oscillatory motion. [6M]

4. Explain [12M]

a) Degree of freedom system

b) Harmonic Excitation

c) Simple harmonic motion

d) D'Alemberts principle

5. Briefly explain fundamental objectives of dynamic analysis with example [12M]

6. a) What is mathematical model with specific reference to structural dynamics. [6M]

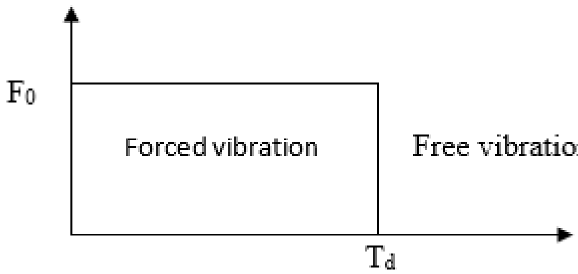
b) Describe various method of discretization analysis of dynamic problem. [6M]

7. Derive the Equation of motion for damped single degree of freedom system with free vibration. [12M]
8. Explain about the D'Alemberts principle with example. [12M]
9. a) Derive the expression for time period of simple harmonic motion [6M]
   b) Derive the Equation of motion for undamped single degree of freedom system with free vibration [6M]
10. Explain different types of vibration problems and derive their equation of motion. [12M]

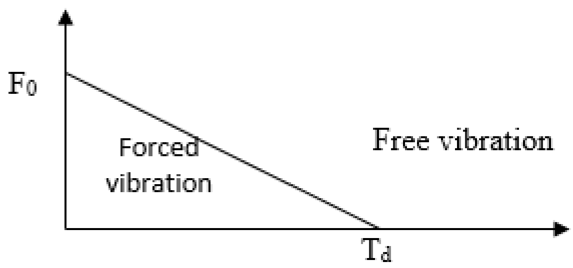
UNIT-II

SINGLE DEGREE OF FREEDOM SYSTEM

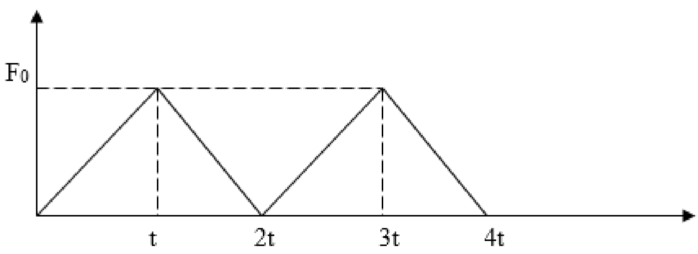
1. Derive the solution for undamped single degree of freedom system with free vibration [12M]
2. Derive the solution for damped single degree of freedom system with free vibration [12M]
3. Derive the solution for undamped single degree of freedom system with forced vibration [12M]
4. Derive the expression for logarithmic decrement for damped free vibration of SDOF for
   a) Two successive cycles [6M]
   b) Two cycles of N cycle apart [6M]
5. Derive expression for Duhamel integral. [12M]
6. Determine the response of SDOF system subjected to rectangular pulse load. [12M]



7. Determine the response of SDOF system subjected to triangle pulse load. [12M]



8. Derive the amplitude of the given problem when time is 4t. [12M]



9. Derive the equation for DMF for undamped single degree of freedom system with forced vibration. [12M]

10. Derive the formula for Damping ratio & Frequency ratio for undamped single degree of freedom system with forced vibration. [12M]

UNIT-III

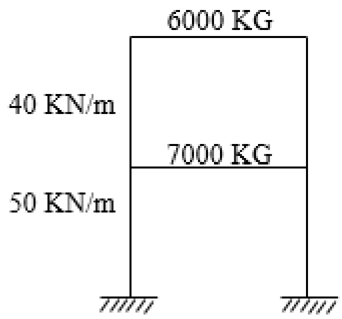
MULTI DEGREE OF FREEDOM SYSTEM

1. Derive the equation of motion for two degree of freedom system in matrix form and also derive the solution for the equation. [12M]

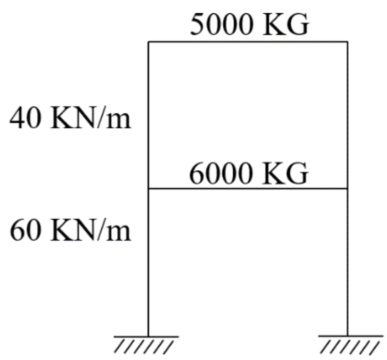
2. Derive the equation of motion for three degree of freedom system in matrix form and also derive the solution for the equation. [12M]

3. Briefly explain orthogonal properties of normal modes. [12M]

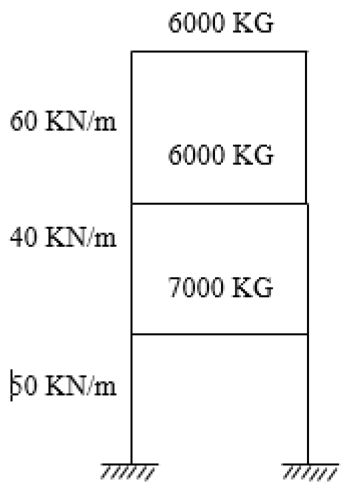
4. Draw the mode shapes for given problem [12M]



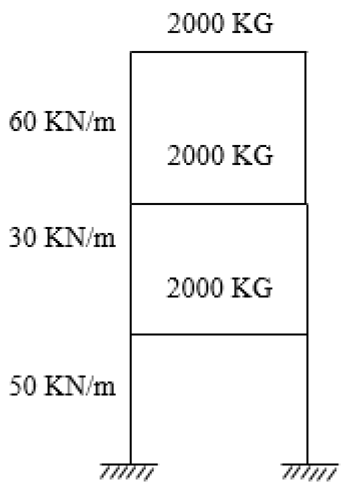
5. Draw the mode shapes for given problem. [12M]



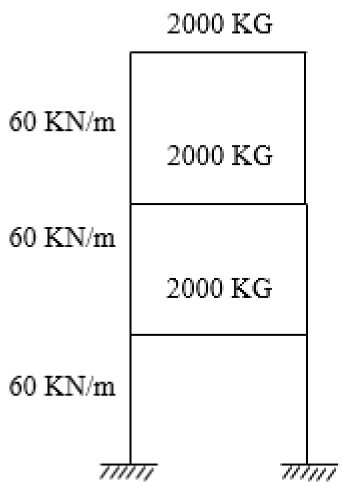
6. Draw the mode shapes for given problem. [12M]



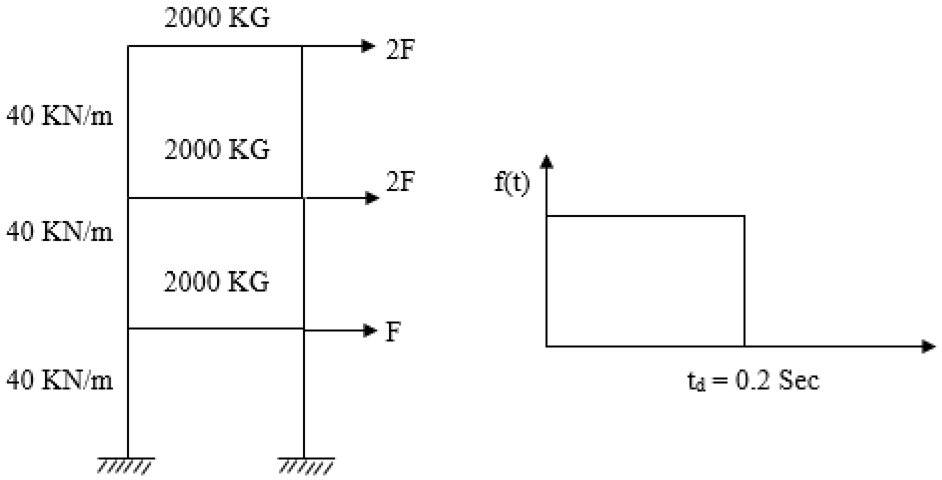
7. Draw the mode shapes for given problem. [12M]



8. Draw the mode shapes for given problem. [12M]

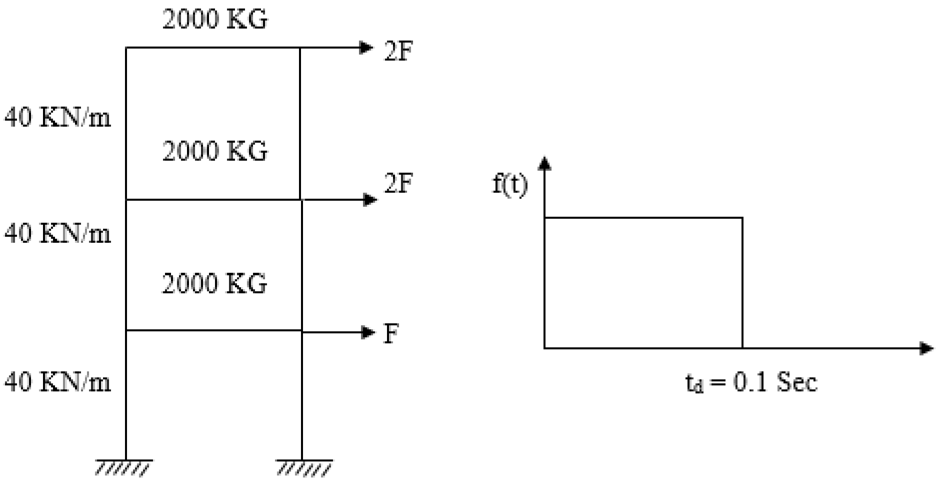


9. Draw the mode shapes for given problem [12M]



F = 2 KN

10. Draw the mode shapes for given problem. [12M]



F = 1.5 KN

UNIT-IV

CONTINUOUS SYSTEM

1. Derive the equation of motion for beam subjected to uniformly distributed load. [12M]
2. Derive the solution of equation of motion for the beam subjected to uniformly distributed load. [12M]
3. Derive the natural frequency and mode shapes for uniform beam having both end simply supported. [12M]
4. Derive the natural frequency and mode shapes for uniform beam having both end free. [12M]

5. Derive the natural frequency and mode shapes for uniform beam having one end fixed other end free. [12M]

6. Derive the natural frequency and mode shapes for uniform beam having one end fixed other end simply supported. [12M]

7. Derive the natural frequency for uniform beam having both end fixed. [12M]

8. Draw the mode shapes for uniform beam having both end fixed. [12M]

9. Draw the mode shapes for uniform beam having one end is fixed other end is simply supported. [12M]

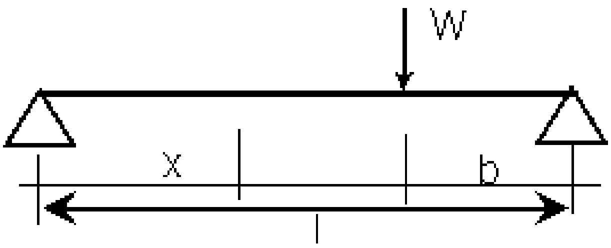
10. Draw the mode shapes for uniform beam having one end fixed other end free. [12M]

UNIT-V

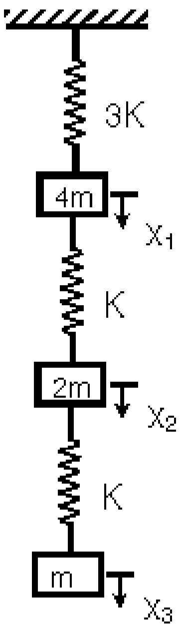
PRACTICAL VIBRATION ANALYSIS

1. Explain step by step procedure of Stodola's method? Derive fundamental natural frequencies and mode shapes? [12M]

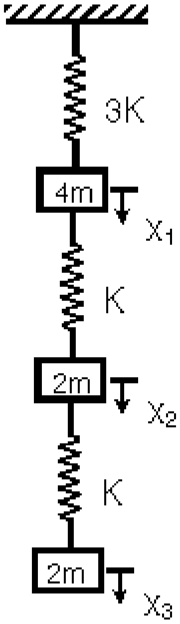
2. Find the fundamental natural frequencies and mode shapes of a vibratory system shown in figure. [12M]



3. For the given system, find the lowest natural frequency by Stodola's method. [12M]

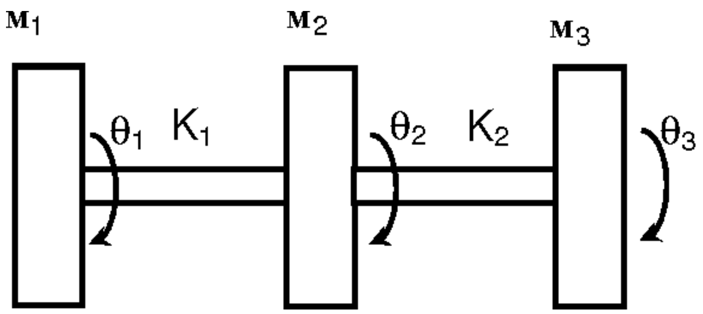


4. Find the fundamental frequencies and mode shapes of a vibratory system shown in figure.[12M]

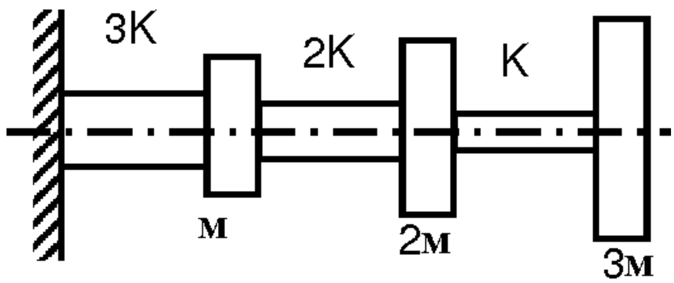


5. Explain step by step procedure of Holzer method? Derive fundamental natural frequencies and mode shapes? [12M]

6. For the system shown in figure, obtain natural frequencies using Holzer's method? [12M]

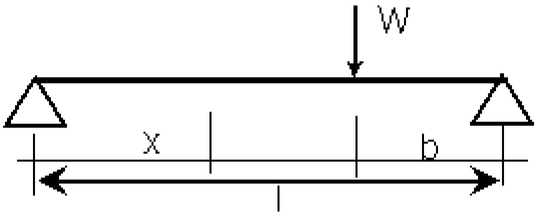


7. Calculate approximate natural frequency of a system by using Holzer's method? [12M]

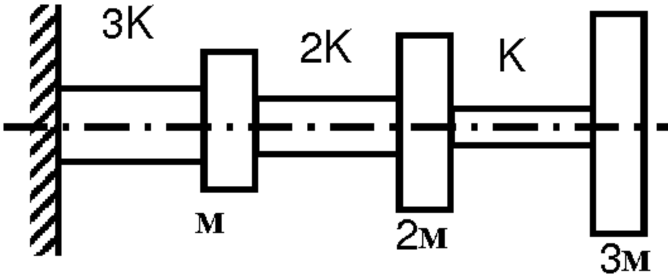


8. Explain step by step procedure of Transfer matrix method? Derive fundamental natural frequencies and mode shapes? [12M]

9. Find the fundamental natural frequencies and mode shapes of a vibratory system shown in figure by using Transfer matrix method. [12M]



10. Calculate approximate natural frequency of a system by using Transfer matrix method?[12M]



CIVIL ENGINEERING DEPARTMENT, I.I.T., KHARAGPUR

Time: 2 Hrs. | Structures | B.Tech IV
Autumn, 2012 | Sub. No. **CE41003** | Marks:30

Sub. **Structural Dynamics and Earthquake Engineering**

Instructions: Answer all questions.

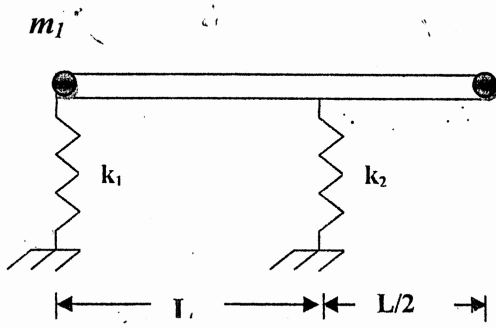
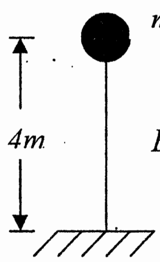


Fig. 1



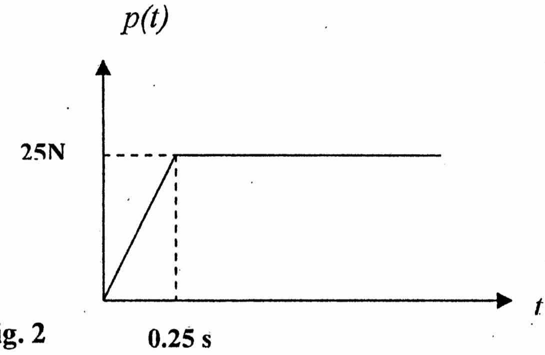


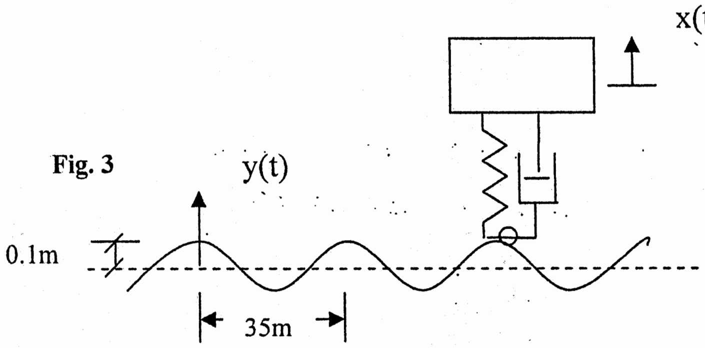
Fig. 2

Q.1 A rigid mass-less bar carrying two masses m_1 and m_2 is supported on two springs as shown in the Fig. 1. Considering vertical displacement of masses m_1 and m_2 as u_1 and u_2 respectively, derive the differential equation of motion assuming small motions and hence estimate the natural frequencies and mode shapes of the system for following parameters, $k_1 = 800$ N/mm; $k_2 = 2000$ N/mm; $m_1 = 20$ kg; $m_2 = 25$ kg; $L = 1000$ mm. **(7.5)**

Q. 2 Idealizing the water tank shown in Fig. 2 as a single-degree-freedom-system, calculate the response of the system to an excitation shown in the figure. Assume that the whole mass of the water tank is lumped at the top of the tank. Neglect damping. Take, $m = 10$ kg; $EI = 160$ N.m^2 Calculate displacement at $t = 1$ s. **(7.5)**

Q. 3 An automobile is modeled as a single degree of freedom system vibrating in the vertical direction. It is driven along a road whose elevation varies sinusoidally as shown in Fig. 3. The distance along the roads between the peaks is 35m. If the natural frequency of the automobile is 2 Hz. and the damping ratio of the shock absorber is 0.15, determine amplitude of vibration of the automobile if the speed of automobile is 60 kms/hr. Evaluate the most unfavourable speed (for the passengers) of the automobile. **(7.5)**

Fig. 3



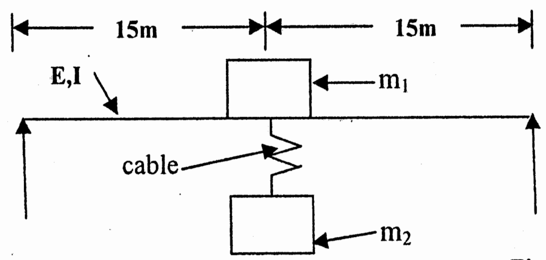


Fig. 4

Q. 4 An electric overhead crane consisting of girder, trolley and cable is shown in Fig. 4. The girder has following properties: span, $L = 30$ m; area moment of inertia, $I = 0.01$ m^4; and modulus of elasticity, E =200 GPa; The mass of the trolley (m_1), the mass being lifted (m_2) are respectively 1000 kg and 4000 kg. The cable has a stiffness 3x10^5 N/m. Determine natural frequencies and mode shapes of the system. **(7.5)**

CIVIL ENGINEERING DEPARTMENT, I.I.T., KHARAGPUR

Time: 3 Hrs. | Structural Engineering | M.Tech, B.Tech(IV)

Autumn, 2012 | Sub. No. **CE41001** | Marks: 50

Sub. **Structural Dynamics and Earthquake Engineering**

Instructions: Answer all questions.

Q.1 (a) For the conservative multi-degree-of-freedom system, show that Rayleigh's quotient has a stationary value in the neighbourhood of the system eigenvector $\{u\}_r$, and the stationary value is the associated eigenvalue λ_r (r =1,2,....,n). **(5)**

Q. 1 (b) Calculate the fundamental frequency of simply supported beam, carrying three masses m_1,m_2, and m_3 (see Fig.1), in lateral vibration by Rayleigh's method. Assume the trial vibration shape of the beam as that produced by the weights ($m_i g$) at the three mass locations. The beam is mass less and flexuaral rigidity of beam is EI = 9.8x10^5 N.m^2, m_1 = 20 kg, m_2 = 30 kg, m_3 = 20kg. **(10)**

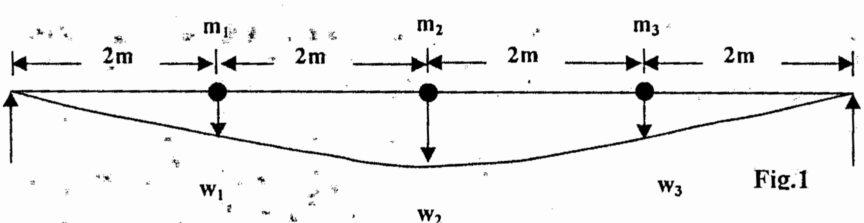


Fig.1

Q. 2 The two storey **shear frame** shown in Fig. 2, is subjected to a horizontal ground motion $\ddot{x}_g(t)$. Determine (a) the modal components of effective earthquake forces, (b) the floor displacements in terms of $D_n(t)$ [where $D_n(t)$ is the n-th mode displacement component of SDOF system to $\ddot{x}_g(t)$], (c) the storey shear response in terms of pseudo-acceleration $A_n(t)$ [$A_n(t)$ is the n-th mode pseudo-acceleration of SDOF system to $\ddot{x}_g(t)$] and (d) the base over turning moment in terms of $A_n(t)$. EI =168.75x10^5 N.m^2, m_1 = 1000 kg and m_2 = 500 kg. **(12)**

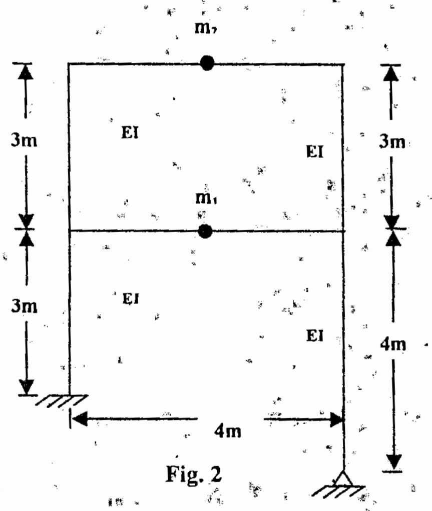


Fig. 2

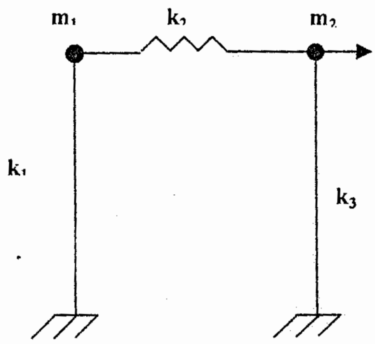


Fig. 3(a)

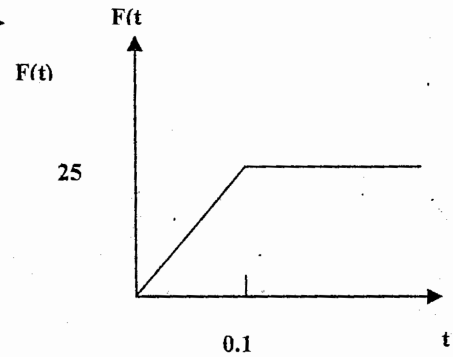


Fig. 3(b)

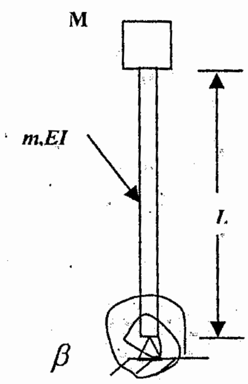
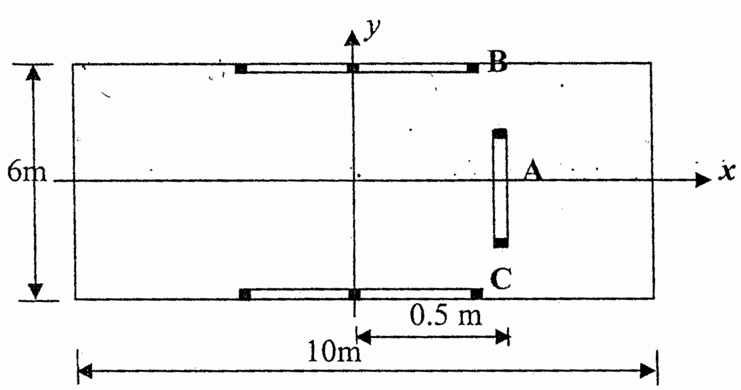


Fig. 4

Q. 3. A two-degree freedom system is shown in Fig. 3 (a). The mass m_2 is acted by a force $F(t)$ as shown in Fig. 3 (b). Estimate the displacement of m_2 at time instant $t = 0.2$s using modal analysis. Assume m_1=20 kg; m_2 = 10 kg; k_1=100 N/m; k_2=50 N/m; k_3 = 150N/m. **(8)**

Q.4 A beam of length L, mass per unit length m and flexural rigidity EI carries a lumped mass M at the top. The beam is supported at the bottom as shown in Fig. 4. Assuming, $y(x,t) = \frac{x}{L} y_1(t) + (\frac{x}{L})^2 y_2(t)$, derive the equation of motion of the system. Calculate frequencies and mode shapes of this two-degree freedom system by assuming $M = mL$ and rotational spring constant, $\beta = EI/L$ N.m/rad **(8)**

Q. 5. Determine the fundamental frequency of a one-storey building consisting of rigid diaphragm supported by three frames A,B, and C as shown in Fig. 5. The lumped mass at the roof level is 150 kg. The lateral stiffness of the frame each idealized as shear frame is $k_{yA} = k_y = 225$ kN/m and $k_{xB} = k_{xC} = k_x = 120$ kN/m. **(7)**



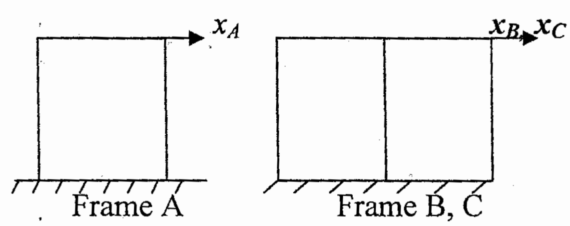


Fig. 5

Figure. 5

CIVIL ENGINEERING DEPARTMENT, I.I.T., KHARAGPUR

Time: 3 Hrs. | Structural Engineering | M.Tech, B.Tech(IV)

Autumn, 2012 | Sub. No. **CE41001** | Marks: 50

Sub. **Structural Dynamics and Earthquake Engineering**

Instructions: Answer all questions.

Q.1 (a) For the conservative multi-degree-of-freedom system, show that Rayleigh's quotient has a stationary value in the neighbourhood of the system eigenvector $\{u\}_r$, and the stationary value is the associated eigenvalue λ_r (r =1,2,….,n). **(5)**

Q. 1 (b) Calculate the fundamental frequency of simply supported beam, carrying three masses m_1,m_2, and m_3 (see Fig.1), in lateral vibration by Rayleigh's method. Assume the trial vibration shape of the beam as that produced by the weights ($m_i g$) at the three mass locations. The beam is mass less and flexuaral rigidity of beam is $EI = 9.8\text{x}10^5$ N.m^2, m_1 = 20 kg, m_2 = 30 kg, m_3 = 20kg. **(10)**

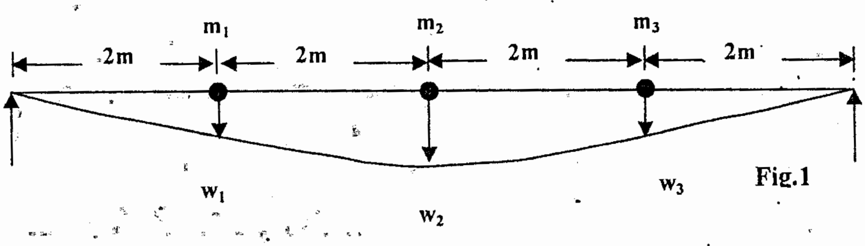


Fig.1

Q. 2 The two storey **shear frame** shown in Fig. 2, is subjected to a horizontal ground motion $\ddot{x}_g(t)$. Determine (a) the modal components of effective earthquake forces, (b) the floor displacements in terms of $D_n(t)$ [where $D_n(t)$ is the n-th mode displacement component of SDOF system to $\ddot{x}_g(t)$], (c) the storey shear response in terms of pseudo-acceleration $A_n(t)$ [$A_n(t)$ is the n-th mode pseudo-acceleration of SDOF system to $\ddot{x}_g(t)$] and (d) the base over turning moment in terms of $A_n(t)$. EI =168.75x10^5 N.m^2, m_1 = 1000 kg and m_2 = 500 kg. **(12)**

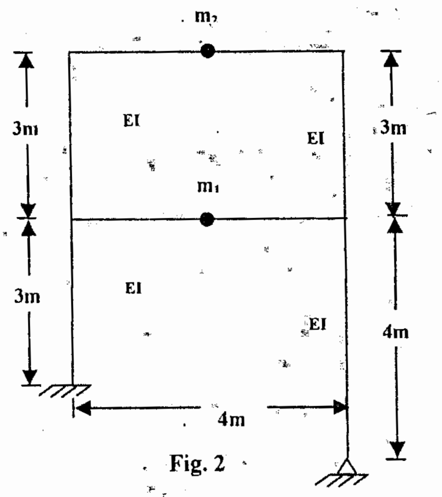


Fig. 2

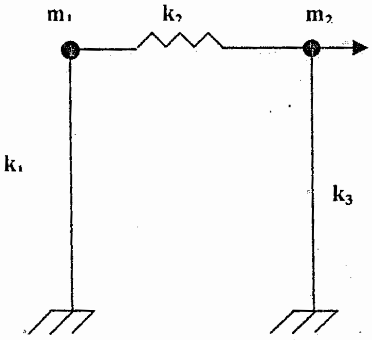


Fig. 3(a)

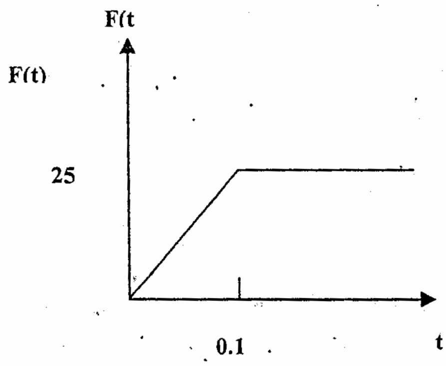


Fig. 3(b)

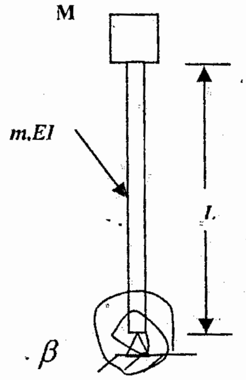


Fig. 4

Q. 3. A two-degree freedom system is shown in Fig. 3 (a). The mass m_2 is acted by a force *F(t)* as shown in Fig. 3 (b). Estimate the displacement of m_2 at time instant $t = 0.2$s using modal analysis. Assume m_1=20 kg; m_2 = 10 kg; k_1=100 N/m; k_2=50 N/m; k_3 = 150N/m. **(8)**

Q.4 A beam of length L, mass per unit length m and flexural rigidity EI carries a lumped mass M at the top. The beam is supported at the bottom as shown in Fig. 4. Assuming, $y(x,t) = \frac{x}{L} y_1(t) + (\frac{x}{L})^2 y_2(t)$, derive the equation of motion of the system. Calculate frequencies and mode shapes of this two-degree freedom system by assuming $M = mL$ and rotational spring constant, $\beta = EI/L$ N.m/rad **(8)**

Q. 5. Determine the fundamental frequency of a one-storey building consisting of rigid diaphragm supported by three frames A,B, and C as shown in Fig. 5. The lumped mass at the roof level is 150 kg. The lateral stiffness of the frame each idealized as shear frame is $k_{yA} = k_y = 225$ kN/m and $k_{xB} = k_{xC} = k_x = 120$ kN/m. **(7)**

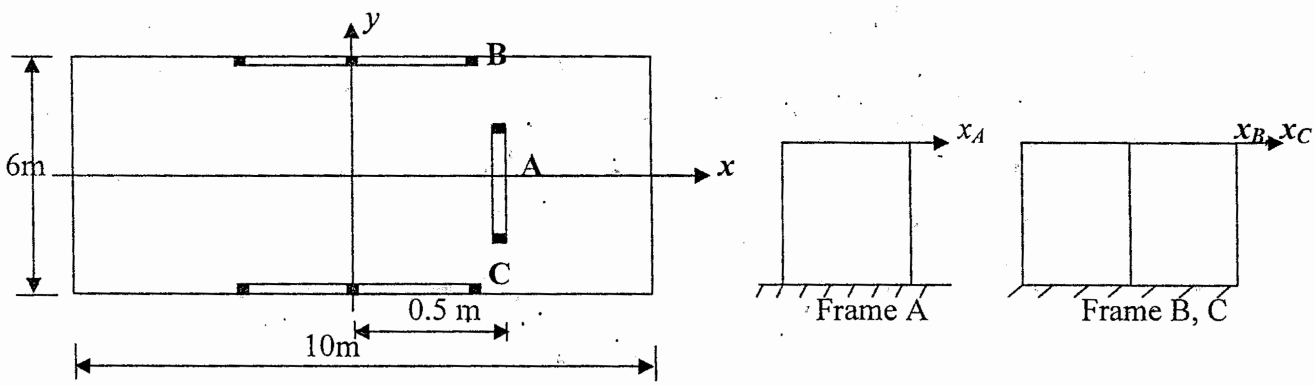


Fig. 5

Figure. 5

CIVIL ENGINEERING DEPARTMENT, I.I.T., KHARAGPUR

Time: 3 Hrs. | Structural Engineering | M.Tech, B.Tech(IV)
Autumn, 2012 | Sub. No. **CE60083/CE41001** | Marks: 50
Sub. **Structural Dynamics and Earthquake Engineering**

Instructions: Answer all questions.

Q.1 (a) For the conservative multi-degree-of-freedom system, show that Rayleigh's quotient has a stationary value in the neighbourhood of the system eigenvector $\{u\}_r$, and the stationary value is the associated eigenvalue $\lambda_r\ (r=1,2,\ldots,n)$. **(5)**

Q.1 (b) Explain Stodola (matrix iteration) method for calculating eigen frequencies of a multi-degree-freedom-system (MDOF) system. Also prove that the Stodola method converges to the first natural frequency when any arbitrary vector is chosen as a trial vector provided the contribution of the first mode is not zero. **(5)**

Q.2. The two storey shear frame shown in Fig. 1, is subjected to a horizontal ground motion $\ddot{x}_g(t)$. Determine (a) the modal components of effective earthquake forces, (b) the floor displacements in terms of $D_n(t)$ [where $D_n(t)$ is the n-th mode displacement component], (c) the storey shear response in terms of pseudo-acceleration $A_n(t)$ and (d) the base over turning moment in terms of $A_n(t)$. Assume **(15)** $m = 5000$ kg; and $k = 15kN/m$.

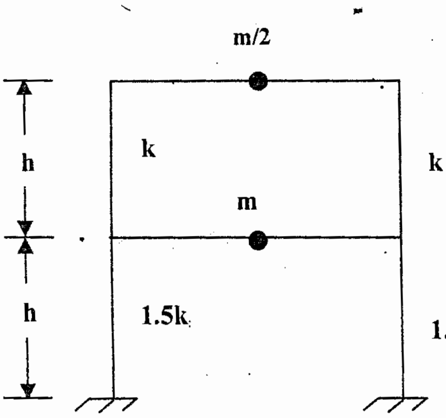


Fig. 1

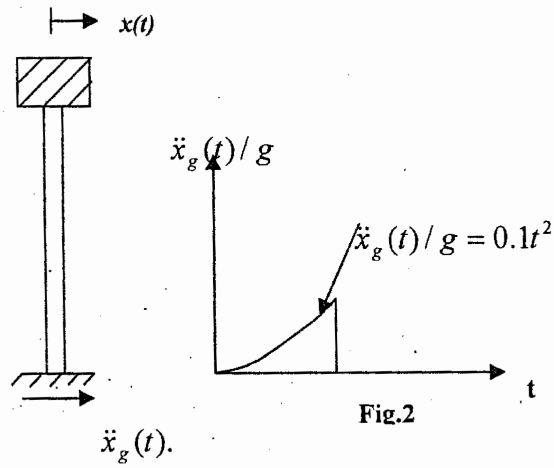


Fig.2

Q.3. A water tank is idealized as shown in the Fig. 2. The column is massless having a stiffness of $k = 180$ kN/m and carrying a mass of 10,000 kg at its top. It is subjected to ground acceleration $\ddot{x}_g(t)$ as shown in the figure. Evaluate the relative response of the water tank at time $t = 0.6$ s. Neglect damping and assume initial conditions as: $x(0) = \dot{x}(0) = 0$. **(10)**

Q.4. A rigid weightless bar of length $2l$ and two masses m each are supported on two identical springs as shown in Fig. 3. Calculate two natural frequencies of this two-degree-freedom system. Also, determine centre of rotation (node) of bar in the two natural modes. **(7)**

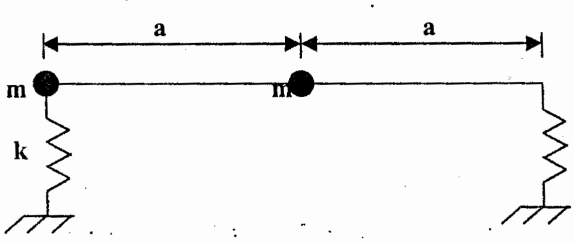


Fig.3

Q. 5. The shear frame shown in Fig. 4 is subjected to a horizontal load of $p(t) = 4\sin(7t)$ kN. Evaluate (a) maximum steady state amplitude of response and (b) maximum dynamic stress in columns assuming 4% damping and $EI = 2.67x10^3$ kN m^2. The mass of the structure is 4000 kg and is lumped at the beam level. Assume the beam to be rigid. **(8)**

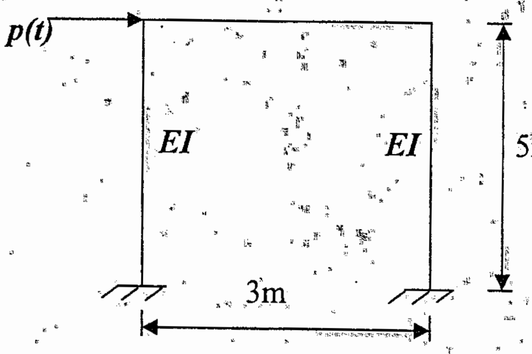


Fig. 4

GR20 2021-22 M.Tech MTECH STE 120, Section: A GR20D5013 Structural Dynamics Sessional Marks

S.No	Roll No	MID-I Marks	MID-II Marks	Tutorial Marks	Assessment Marks	Sessional Marks
1	21241D2001	11	9	5	5	0
2	21241D2002	15	13	5	5	8
3	21241D2003	15	13	5	5	8
4	21241D2004	12	13	5	5	0
5	21241D2005	18	12	5	5	9
6	21241D2006	18	12	5	5	9
7	21241D2007	18	17	5	5	9
8	21241D2008	19	8	5	5	10
9	21241D2009	19	13	5	5	10
10	21241D2010	10	13	5	5	5
11	21241D2011	11	8	5	5	6
12	21241D2012	18	11	5	5	9
13	21241D2013	13	10	5	5	7
14	21241D2014	14	14	5	5	7
15	21241D2015	15	10	5	5	8
16	21241D2016	16	11	5	5	8
17	21241D2017	11	13	5	5	6
18	21241D2018	AB	AB	5	4	0
19	21241D2019	AB	AB	4	4	0
20	21241D2020	AB	12	5	5	0
21	21241D2021	11	9	5	5	0

Faculty Signature

Gokaraju Rangaraju Institute of Engineering and Technology

(Autonomous)

Bachupally, Kukatpally, Hyderabad – 500 090

Direct Internal CO Attainments

Academic Year	2021-22	Department	Civil Engineering	Name of the Programme	MTECH Structural Engi
Year - Semester	I Yr- II Sem	Course Name :	Structural Dynamics	Course Code	GR20D5013
				Section	A

	Mid -I									Mid -II									Assignment Marks					Assessment
	Q.No 1(a)	Q.No 1(b)	Q.No 2(a)	Q.No 2(b)	Q.No 3(a)	Q.No 3(b)			Objective Marks	Q.No 1	Q.No 2	Q.No 3	Q.No 4	Q.No 5	Q.No 6			Objective Marks	I	II	III	IV	V	Marks
Enter CO Number → 1,2,3,4,5	1	1	2	2	3	3			1,2,3	3	3	4	4	5	5			3,4,5	1	2	3	4	5	1,2,3,4,5
Marks →	5	5	5	5	5	5			5	5	5	5	5	5	5			5	5	5	5	5	5	5
S.No/Roll No.	***Note :*** Enter Marks Between Two Green rows. ***Another Note :*** Additional Columns if Required should be inserted after column H and appropriately rename the Q. Nos. For Calculations consult Departments CO-PO Incharge																							
1		2		2	3				4		2		2	2				3	5	5	5	5	5	5
2		2	3			5			5		3	3			5			2	5	5	5	5	5	5
3	4		3		4				4		4	3		3				3	5	5	5	5	5	5
4	3			3		2			4		5		4	3				1	5	5	5	5	5	5
5	4			5	5				4		3		3	3				3	5	5	5	5	5	5
6	5		5		4				4		3		3	2				4	5	5	5	5	5	5
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11	3	2	2		3				4		1			2	2			3	5	5	5	5	5	5
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21		3		3	4				0	3		3			3				5	5	5	5	5	5
if your class strength is > 60 then ***insert rows*** ***above the green row(last record)***, Similarly ***delete the empty rows above green row*** if the class strenght is < 60)																								
Total number of students appeared for the examination (NST)	21	21	21	21	21	21			21	21	21	21	21	21	21			21	21	21	21	21	21	21
Total number of students attempted the question (NSA)	7	12	5	13	13	5			18	3	16	9	9	16	4			18	21	21	21	21	21	21
Attempt % (TAP) = (NSA/NST)*100	33.33	57.14	23.81	61.90	61.90	23.81			85.71	14.29	76.19	42.86	42.86	76.19	19.05			85.71	100.00	100.00	100.00	100.00	100.00	100.00
Total number of Students who got more than 60% marks (NSM)	7	7	4	11	12	4			17	3	10	7	4	9	3			13	21	21	21	21	21	21
Attainment % (TMP) = (NSM/NSA)*100	100.00	58.33	80.00	84.62	92.31	80.00			94.44	100.00	62.50	77.78	44.44	56.25	75.00			72.22	100.00	100.00	100.00	100.00	100.00	100.00
Score(S)	3	2	3	3	3	3			3	3	3	3	1	2	3			3	3	3	3	3	3	3
Note : CO attainment is considered to be zero if the attempt % is less than 30%																								
CO Validation	1	1	2	2	3	3			1,2,3	3	3	4	4	5	5			3,4,5	1	2	3	4	5	1,2,3,4,5
Course Outcome	CO1	CO1	CO2	CO2	CO3	CO3			CO1,CO2, CO3	CO3	CO3	CO4	CO4	CO5	CO5			CO3,CO4, CO5	CO1	CO2	CO3	CO4	CO5	CO1,CO2,CO3,CO4,CO5
Marks (Y)	5	5	5	5	5	5			5	5	5	5	5	5	5			5	5	5	5	5	5	5
No. of COs Shared (Z)	1	1	1	1	1	1			3	1	1	1	1	1	1			3	1	1	1	1	1	5
Y/Z	5	5	5	5	5	5			1.66667	5	5	5	5	5	5			1.66667	5	5	5	5	5	1
S*Y/Z	15	10	15	15	15	15			5	15	15	15	5	10	15			5	15	15	15	15	15	3
CO1	1	1	0	0	0	0			1	0	0	0	0	0	0			0	1	0	0	0	0	1
CO2	0	0	1	1	0	0			1	0	0	0	0	0	0			0	0	1	0	0	0	1
CO3	0	0	0	0	1	1			1	1	1	0	0	0	0			1	0	0	1	0	0	1
CO4	0	0	0	0	0	0			0	0	0	1	1	0	0			1	0	0	0	1	0	1
CO5	0	0	0	0	0	0			0	0	0	0	0	1	1			1	0	0	0	0	1	1

Weighted Average for Attainment relevance (Internal CODn)	CO1	CO2	CO3	CO4	CO5
	2.72	3.00	3.00	2.43	2.72

!!

s < 2.1 should be justified with Remidial Action Report.

Gokaraju Rangaraju Institute of Engineering and Technology

(Autonomous)

Bachupally, Kukatpally, Hyderabad – 500 090

Indirect CO Attainments

Academic Year	2021-22	Department	Civil Engineering	Name of the Programme	MTECH Structural Engineering		
Year - Semester	I Yr- II Sem	Course Name :	Structural Dynamics	Course Code	GR20D5013	Section	A

Course Outcomes survey on Scale 1 (Low) to 5 (High)

Enter Course Outcomes →	Comprehend and model the systems subjected to vibrations and dynamic loads	Analyze and obtain dynamics response of single degree freedom system using fundamental Theory and equations of motion.	Analyze and obtain dynamics response of single degree freedom system using fundamental Theory and equations of motion.	Obtain dynamics response of systems using numerical methods.	To explain the dynamic effects of Wind Loads, Moving Loads and Vibrations caused by Traffic, Blasting and Pile Driving.		
CO Number → 1,2,3,4,5	1	2	3	4	5		
Marks →	5	5	5	5	5		
S.No/Roll No.	Note : Enter Marks Between Two Green rows.						
1	5	5	4	4	3		
2	4	4	4	3	3		
3	4	3	3	4	3		
4	5	5	5	5	5		
5	5	5	5	5	5		
6	5	5	5	5	5		
7	5	5	5	5	5		
8	5	5	5	5	5		
9	5	5	5	5	5		
10	5	5	5	5	5		
11	5	5	5	5	5		
12	5	5	5	5	5		
13	5	5	5	5	5		
14	5	5	5	5	5		
15	5	5	5	5	5		
16	5	5	5	5	5		
17	5	5	5	5	5		
18	5	5	5	5	5		
19	5	5	5	5	5		
20	3	2	3	3	4		
21	3	4	4	3	4		
22	5	5	5	5	5		
23	5	5	5	5	5		
24	3	2	3	3	4		
25	3	4	4	3	4		
26	5	5	5	5	5		
27	5	5	5	5	5		
28	5	5	5	5	5		
29	5	5	5	5	5		
if your class strength is > 60 then *insert rows above the green row(Last Record)* , Similarly *delete the empty rows above green row* if the class strenght is < 60)							
Total number of students appeared for the examination (NST)	29	29	29	29	29		
Total number of students attempted the question (NSA)	29	29	29	29	29		
Attempt % (TAP) = (NSA/NST)*100	100.00	100.00	100.00	100.00	100.00		
Total number of Students who got more than 60% marks (NSM)	29	27	29	29	29		
Attainment % (TMP) = (NSM/NSA)*100	100.00	93.10	100.00	100.00	100.00		
Score(S)	3	3	3	3	3		

CO attainment is considered zero if the attempt % is less than 30%

Indirect CO (COIn)	CO1	CO2	CO3	CO4	CO5
	3	3	3	3	3

!! Caution !! For CO Values < 2.1 should be justified with Remidial Action Report.

Gokaraju Rangaraju Institute of Engineering and Technology

(Autonomous)

Bachupally, Kukatpally, Hyderabad – 500 090

Direct External CO Attainment

Academic Year	2021-22	Department	Civil Engineering	Name of the Programme	MTECH Structural Engin		
Year - Semester	I Yr- II Sem	Course Name :	Structural Dynamics	Course Code	GR20D5013	Section	A

	Part A										Part B																		
	Q.No 1 (a) Marks	Q.No 1 (b) Marks	Q.No 1 (c) Marks	Q.No 1 (d) Marks	Q.No 1 (e) Marks	Q.No 1 (f) Marks	Q.No 1 (g) Marks	Q.No 1 (h) Marks	Q.No 1 (i) Marks	Q.No 1 (j) Marks	Q.No 2 Marks	Q.No 3A Marks	Q.No 3B Marks	Q.No 4A Marks	Q.No 4 B Marks	Q.No 5A Marks	Q.No 5B Marks	Q.No 6A Marks	Q.No 6B Marks	Q.No 7A Marks	Q.No 7B Marks	Q.No 8 A Marks	Q.No 8 B Marks	Q.No 9 A Marks	Q.No 9 B Marks	Q.No 10 A Marks	Q.No 10 B Marks	Q.No 11 A Marks	Q.No 11 B BMarks
Enter CO Number → 1,2,3,4,5,6,7	1	1	2	2	3	3	4	4	5	5	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
Marks →	2	2	2	2	2	2	2	2	2	2	10	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
S.No/Roll No.	***Note :*** Enter Marks Between Two Green rows. ***Another Note :*** Additional Columns if Required should be inserted after column H and appropriately rename the Q. Nos. For Calculations consult Departments CO-PO Incharge																												
1	2	2	2	2	2	2	2			1	4			6				9		7									
2	2	2	2	0	2	2	1	1	1	1	3					2		4				2		5					
3	2	2	2	1	2		2	2	2	2	4			6						6				7		7			
4	2	2	2	2	2	2	2	2	1	1	7			6						9		8				8			
5	2	2	2	2	1	2	1	1	1	1	3	4	1	6				8		3		6		4		6			
6	2	2	1	2	1	2	2	2	2	1	10			9						2		8				4			
7	2	2	2	2	1	2	2	1	1	0	3	3		6				2		3		5		1		5			
8	2	2	1	2	1	2	2	1	1	1		3		6						5		7				6			
9	2	2	2	2	2	2	2	1	2	2	1	4	0	6		0		0		8		0		7		8			
10	2	2	2	2	2	2	2	1	2	2	10	5		9						9		8				8			
11	2	2	1		1	2	2		2	2		2	1							8		4		0		3			
12	2	2	2	2	2	2	2	1	1	1	10			10				8				7				5			
13	2	1	1		1	2	2	1	1	1		2		4						3		5				2			
14	2		1	2	1	1	1		1	1	4	2		6				1		5		4				3			
15	2	2	2	2	2	2	1	2	1	1	5							7				4				5			
16	2	2	1	2	1	1	2	2	1	1	3	3	2	2		2		4				4				5		4	
17	2	2	2	2	2	1	1	2	2	2	3			8				5				0				5			
18	2	2	2	2	2	2	2	1	2	1	9			5						9				7		8			
19	2	1	2	2	2	2	1	1	2	1		4	2	3		3		7				3				6			
20	2	2	2	2	2	2	2	1	2	1	10	4	1	8				10		4				3		9			
21																													
if your class strength is > 60 then *insert rows above the green row* , Similarly *delete the empty rows above green row* if the class strenght is < 60)																													
Total number of students appeared for the examination (NST)	21	21	21	21	21	21	21	21	21	21	21	21	21	21		21		21		21		21		21		21		21	
Total number of students attempted the question (NSA)	20	19	20	18	20	19	20	17	19	20	16	11	6	17		4		12		14		16		8		18		1	
Attempt % (TAP) = (NSA/NST)*100	95.24	90.48	95.24	85.71	95.24	90.48	95.24	80.95	90.48	95.24	76.19	52.38	28.57	80.95		19.05		57.14		66.67		76.19		38.10		85.71		4.76	
Total number of Students who got more than 60% marks (NSM)	20	19	20	17	20	19	20	17	19	19	6	8	0	16		1		9		13		13		6		17		1	
Attainment % (TMP) = (NSM/NSA)*100	100.00	100.00	100.00	94.44	100.00	100.00	100.00	100.00	100.00	95.00	37.50	72.73	0.00	94.12		25.00		75.00		92.86		81.25		75.00		94.44		100.00	
Score(S)	3	3	3	3	3	3	3	3	3	3	1	3	0	3		0		3		3		3		3		3		3	
CO attainment is considered zero if the attempt % is less than 30%																													
CO Validation	1	1	2	2	3	3	4	4	5	5	1	1	1	2		2		3		3		4		4		5		5	
Course Outcome	CO1	CO1	CO2	CO2	CO3	CO3	CO4	CO4	CO5	CO5	CO1	CO1	CO1	CO2		CO2		CO3		CO3		CO4		CO4		CO5		CO5	
Marks (Y)	2	2	2	2	2	2	2	2	2	2	10	5	5	5		5		5		5		5		5		5		5	
No. of COs Shared (Z)	1	1	1	1	1	1	1	1	1	1	1	1	1	1		1		1		1		1		1		1		1	
Y/Z	2	2	2	2	2	2	2	2	2	2	10	5	5	5		5		5		5		5		5		5		5	
S*Y/Z	6	6	6	6	6	6	6	6	6	6	10	15	0	15		0		15		15		15		15		15		15	
CO1	1	1	0	0	0	0	0	0	0	0	1	1	1	0		0		0		0		0		0		0		0	
CO2	0	0	1	1	0	0	0	0	0	0	0	0	0	1		1		0		0		0		0		0		0	
CO3	0	0	0	0	1	1	0	0	0	0	0	0	0	0		0		1		1		0		0		0		0	
CO4	0	0	0	0	0	0	1	1	0	0	0	0	0	0		0		0		0		1		1		0		0	
CO5	0	0	0	0	0	0	0	0	1	1	0	0	0	0		0		0		0		0		0		1		1	
CO6																													
CO7																													

Weighted Average for Attainment relevance	CO1	CO2	CO3	CO4	CO5
	1.54	1.93	3.00	3.00	3.00

!! Caution !! For CO Values < 2.1 should be justified with Remidial Action Report.

Gokaraju Rangaraju Institute of Engineering and Technology
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Bachupally, Kukatpally, Hyderabad – 500 090

Summary Sheet CO Attainments

Academic Year:	2021-22
Course/Subject:	Structural Dynamics
Department:	Civil Engineering
Section	A

Name of the Program:	MTECH Structural E
Course Code:	GR20D5013
Year - Semester :	I Yr- II Sem

Attainment/CO	CO1	CO2	CO3	CO4	CO5
Attainment for Direct Internal CO (Mid I & II, Assignments, Tutorials, Assessments, etc.)	2.72	3.00	3.00	2.43	2.72
Attainment for Direct External CO (End Semester Exam)	1.54	1.93	3.00	3.00	3.00
Direct CO (0.3*Internal + 0.7*External)	1.89	2.25	3.00	2.83	2.92
Indirect CO	3.00	3.00	3.00	3.00	3.00
Final CO (COFn) = (0.9 x Direct CO + 0.1 x Indirect CO)	**2.00**	**2.33**	**3.00**	**2.85**	**2.92**

CO	Course Outcome	Remedial Action for COs Less than 70% (2.10)
CO1	Comprehend and model the systems subjected to vibrations and dynamic loads	Tutorial sessions and Addiitional Assignments will be given
CO2	Analyze and obtain dynamics response of single degree freedom system using fundamental Theory	-
CO3	Analyze and obtain dynamics response of single degree freedom system using fundamental Theory	-
CO4	Obtain dynamics response of systems using numerical methods.	-
CO5	To explain the dynamic effects of Wind Loads, Moving Loads and Vibrations caused by Traffic, Blasting and	-

ID No.	Name of the Faculty	Department	Signature
1117	Dr V Srinivasa Reddy	Civil Engineering	

HOD **DAA**

Copy to: IQAC

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(Autonomous)

Bachupally, Kukatpally, Hyderabad – 500 090

Direct Internal CO Attainments

Academic Year	2021-22
Year - Semester	I Yr- II Sem

Department	Civil Engineering
Course Name :	Structural Dynamics

Name of the Programme	MTECH Structu
Course Code	GR20D5013

P-Outcomes / C-Outcomes	1	2	3	4	5	6
1			H		H	M
2	M		H		H	M
3	M		H		M	M
4	M	M	H	M	M	M
5	M	H		H	M	

Enter H,M, L values of CO-PO Mapping Matrix in blue shaded rows 12 - 18 for seven CO s automatically PO Attainments are Calculated ←

Convert above mappings to scale 1-3

P-Outcomes / C-Outcomes	1	2	3	4	5	6
CO1			3		3	2
CO2	2		3		3	2
CO3	2		3		2	2
CO4	2	2	3	2	2	2
CO5	2	3		3	2	
Expected Attainment	2.00	2.50	3.00	2.50	2.40	2.00

Fill the below table with obtained attainments in mids, external and Tutorial/Attendence

	CO1	CO2	CO3	CO4	CO5
Final Cos CoF	2.00	2.33	3.00	2.85	2.92

	Attained PO A	Attained PO B	Attained PO C	Attained PO D	Attained PO E	Attained PO F
CO1			2.00		2.00	1.34
CO2	1.55		2.33		2.33	1.55
CO3	2.00		3.00		2.00	2.00
CO4	1.90	1.90	2.85	1.90	1.90	1.90
CO5	1.95	2.92		2.92	1.95	
Attained	1.85	2.41	2.54	2.41	2.04	1.70

Note : If Average Attainment of a PO is #Div/0! Relace the corresponding PO with blank.

	A	B	C	D	E	F
	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6
Expected	2.00	2.50	3.00	2.50	2.40	2.00
Attained	1.85	2.41	2.54	2.41	2.04	1.70
	92.46	96.43	84.81	96.43	84.81	84.81

Note : PO is Satisfied if attained PO > 70, U indicates PO Unsatisfied

Faculty Co-Ordinator **HOD**

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GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING & TECHNOLOGY

Department of Civil Engineering

Year: M.Tech I Year - II Sem **Academic Year : 2021-22**

Structural Engineering

Total Strength of the Class:21 **Student's Batch :2021-2023**

S.No	Name of the Subject	Subject Code	No. of students appeared	No. of students Passed	No. of students Failed	GP 10	GP 9	GP 8	GP 7	GP 6	Pass %
Theory											
1	FEM in Structural Engineering	GR20D5012	21	17	4	1	3	4	3	6	80.95
2	Structural Dynamics	GR20D5013	21	19	2	-	5	3	5	6	90.48
3	Design of Formwork	GR20D5015	21	19	2	1	7	7	4	-	90.48
4	Earthquake Resistant Design of Buildings	GR20D5019	21	16	5	1	5	3	3	4	76.19
5	Disaster Management **	GR20D5153	21	20	1	-	9	6	4	1	95.24
Lab											
6	Model Testing Lab	GR20D5020	21	20	1	4	7	5	4	-	95.24
7	Numerical Analysis Lab	GR20D5021	21	20	1	6	2	9	3	-	95.24
8	Mini Project	GR20D5143	21	19	2	2	7	6	1	3	90.48

Grade	Grade Point
O	10
A+	9
A	8
B+	7
B	6

Subjects & Faculty Details

S.No	Name of the Subject		Faculty
1	FEM in Structural Engineering	GR20D5012	Dr. G V V Satyanarayana (842)
2	Structural Dynamics	GR20D5013	Dr. V Srinivasa Reddy (1117)
3	Design of Formwork	GR20D5015	Mrs.K.Hemalatha (1177)
4	Earthquake Resistant Design of Buildings	GR20D5019	Dr.V.Mallikarjuna Reddy(807)
5	Model Testing Lab	GR20D5020	Mr.C.Vanadeep(1645)/Mr.C.Vivek Kumar(1500)
6	Numerical Analysis Lab	GR20D5021	Mr.V.Naresh Kumar Varma (1359)/Mr.V.Ramesh(1646)
7	Mini Project	GR20D5143	Mr.Y.Kamal Raju (929)
8	Disaster Management **	GR20D5153	Mr. Kusuma Veera Babu (1650)

Arrear Position - First Year Second Semester

Arrear Details					
Description	**All Pass**	**One Arrear**	**Two Arrears**	**Three Arrears**	**>Three Arrears**
No. of Students	16	1	1	1	2

Performance

Class Toppers (Three Positions)			
S.No	**Name of the Student**	**Hall Ticket No.**	**% of Marks**
1	MARIYALA VAISHNAVI	21241D2007	9.67
2	MITTAPALLI NAGA ASHWINI	21241D2009	9.17
3	BANDI SRI RAM GOPAL	21241D2002	8.67

Overall Pass :76.19% **Passed in First class :76.19%**

HOD

Gokaraju Rangaraju Institute of Engineering & Technology

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Results

Year: M.Tech I Year - II Sem Academic Year : 2021-22

Structural Engg

S.No	Roll No	GR20D5012	GR20D5013	GR20D5015	GR20D5019	GR20D5020	GR20D5021	GR20D5143	GR20D5153	SGPA	Credits
1	21241D2007	10	9	9	10	10	10	10	9	9.67	18
2	21241D2009	9	9	8	9	10	10	10	9	9.17	18
3	21241D2002	8	9	9	8	9	10	8	9	8.67	18
4	21241D2005	7	8	9	9	9	10	9	9	8.61	18
5	21241D2006	8	9	9	8	9	8	9	9	8.56	18
6	21241D2016	9	6	9	9	9	8	9	9	8.39	18
7	21241D2008	8	9	9	8	8	8	8	8	8.33	18
8	21241D2012	7	7	8	9	10	9	9	8	8.28	18
9	21241D2003	9	7	7	9	9	8	8	8	8.11	18
10	21241D2015	6	6	10	7	10	10	9	9	8.06	18
11	21241D2014	8	7	8	7	9	9	9	9	8.00	18
12	21241D2011	7	7	8	7	8	10	9	8	7.83	18
13	21241D2004	6	8	9	6	8	7	8	9	7.39	18
14	21241D2001	6	8	8	6	8	8	6	8	7.11	18
15	21241D2010	6	6	8	6	7	8	8	7	6.89	18
16	21241D2017	6	6	7	6	8	8	8	6	6.83	18
17	21241D2013	6	7	7	0	9	8	6	7	5.89	15
18	21241D2021	0	6	7	0	7	8	7	7	4.61	12
19	21241D2020	0	0	8	0	7	7	6	8	3.56	9
20	21241D2018	0	6	0	0	7	7	0	7	2.56	7
21	21241D2019	0	0	0	0	0	0	0	0	0.00	0

GR20D5012	FEM in Structural Engineering
GR20D5013	Structural Dynamics
GR20D5015	Design of Formwork
GR20D5019	Earthquake Resistant Design of Buildings
GR20D5020	Model Testing Lab
GR20D5021	Numerical Analysis Lab
GR20D5143	Mini Project
GR20D5153	Disaster Management

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Students Rubric

Academic **Y**ear : 2022-23
Semester : II
Name of the Program: M.Tech. Structural Engineering Year: I
Course/Subject: Structural Dynamics Course Code: **GR22D5013**
Name of the Faculty: Dr.V Srinivasa Reddy. Dept.: Civil engineering
Designation: Professor

		Beginning	**Developing**	**Reflecting Development**	**Accomplished**	**Exemplary**	**Score**
Name of the Student	**Performance Criteria**	**1**	**2**	**3**	**4**	**5**	
	Level of knowledge on Fundamentals of Vibrations						
	Level of knowledge on SDOF and MDOF systems						
	Level of knowledge on Numerical methods						
	Level of knowledge on Fundamentals of Vibrations						
	Level of knowledge on SDOF and MDOF systems						
	Level of knowledge on Numerical methods						
	Level of knowledge on Fundamentals of Vibrations						
	Level of knowledge on SDOF and MDOF systems						
	Level of knowledge on Numerical methods						